$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Solving Quadratics

15 A composite shape is made by joining a square and a trapezium as shown. The dimensions are shown on the diagram.

2x cm — (3x+4) cm — 2x — (8x+6) cm — Midpt 5x+3

a) Show that the area of the trapezium is
$$5x^2 + 23x + 12$$

$$\text{Area} = (5x+3)(x+4)$$
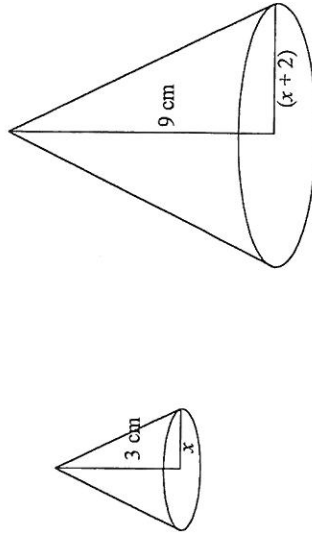$$= 5x^2 + 3x + 20x + 12$$
$$= 5x^2 + 23x + 12$$

[3]

b) The total area of the composite shape is 42 cm². Find x correct to two decimal places.

$$5x^2 + 23x + 12 = 42$$
$$5x^2 + 23x - 30 = 0$$
$$x = \frac{-23 \pm \sqrt{23^2 - 4 \times 5 \times -30}}{10}$$

[4]   2010*

16 Two cones are shown in the diagram below. The radius of the smaller cone is x cm and the height is 3 cm. The radius of the larger cone is $(x+2)$ cm and the height is 9 cm.

3 cm   x

9 cm   (x+2)

a) The total volume of the two cones together is 147π. Show that $4x^2 + 12x - 135 = 0$. [4]
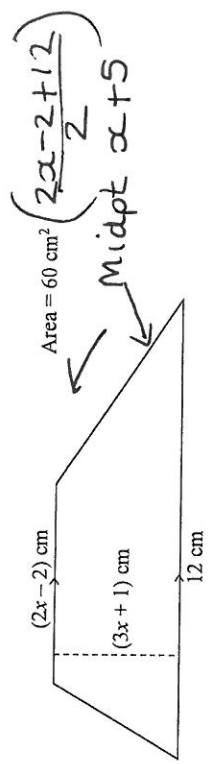
Volume of Cone $= \frac{1}{3}\pi r^2 h$

Small Cone $= \frac{1}{3} \times \pi \times x^2 \times 3$
$= \pi x^2$

Large Cone $= \frac{1}{3} \times \pi \times (x+2)^2 \times 9$
$= 3\pi(x+2)^2$
$= 3\pi(x^2 + 4x + 4)$

$\frac{1}{3} \times 3 = 1$ $\frac{1}{3} \times 9 = 3$

$$\text{Total} = \pi x^2 + 3\pi(x^2 + 4x + 4) = 147\pi \quad \div \pi$$
$$x^2 + 3x^2 + 12x + 12 = 147$$
$$4x^2 + 12x + 12 - 147 = 0$$
$$4x^2 + 12x - 135 = 0$$

b) Calculate the radius of the larger cone. [4]   2009*

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \times 4 \times -135}}{8}$$
$$x = 4.5 \quad \text{or} \quad x = -7.5 \leftarrow \text{can't be length}$$
$$(x+2) = 6.5 \text{ cm}$$

17 The diagram shows a trapezium.

Area = 60 cm²   (2x−2) cm   (3x+1) cm   12 cm   Midpt x+5

The parallel sides of a trapezium are of lengths 12 cm and $(2x-2)$ cm. The height of the trapezium is $(3x+1)$ cm and its area is 60 cm².

(a) Show that $3x^2 + 16x - 55 = 0$.

$$(3x+1)(x+5) = 60$$
$$3x^2 + x + 15x + 5 = 60$$
$$3x^2 + 16x + 5 - 60 = 0$$
$$3x^2 + 16x - 55 = 0$$

[3]

(b) Use the quadratic formula to solve the equation $3x^2 + 16x - 55 = 0$. Give your answers correct to one decimal place.

$$x = \frac{-16 \pm \sqrt{16^2 - 4 \times 3 \times -55}}{6}$$
$$x = 2.4 \, (1dp) \quad \text{or} \quad x = -7.7 \, (1dp) \leftarrow \text{can't be length}$$

[3]

(c) Hence write down the height of the trapezium.

$$\text{Height} = 3x+1$$
$$= 3 \times 2.4 + 1 = 8.2 \text{ cm}$$

[1]   2007*

16 Use the formula method to solve the equation $5x^2 + 27x + 3 = 0$, giving your answer correct to two decimal places.

$$x = \frac{-27 \pm \sqrt{27^2 - 4 \times 5 \times 3}}{10} \quad \text{or} \quad x = \frac{-0.11 \, 2dp}{}$$
$$x = -5.29 \, 2dp$$

2006*

21 Use the formula method to solve the equation $3x^2 + 15x - 3 = 0$, giving the solutions correct to two decimal places.

$$x = \frac{-15 \pm \sqrt{15^2 - 4 \times 3 \times -3}}{6}$$
$$x = 0.19 \, (2dp) \quad \text{or} \quad x = -5.19 \, (2dp)$$

[4]   2005*