



1. Two particles  $A$  and  $B$ , of mass  $5m$  kg and  $2m$  kg respectively, are moving in opposite directions along the same straight horizontal line. The particles collide directly. Immediately before the collision, the speeds of  $A$  and  $B$  are  $3 \text{ m s}^{-1}$  and  $4 \text{ m s}^{-1}$  respectively. The direction of motion of  $A$  is unchanged by the collision. Immediately after the collision, the speed of  $A$  is  $0.8 \text{ m s}^{-1}$ .

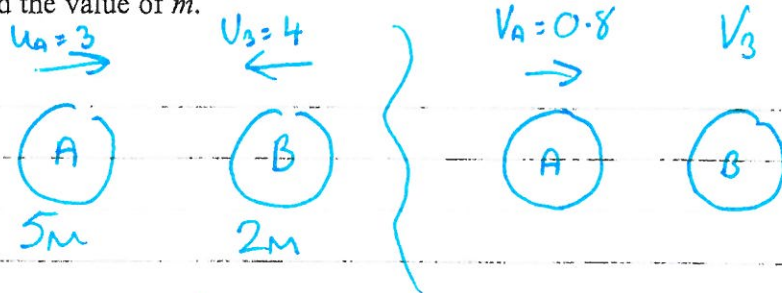
(a) Find the speed of  $B$  immediately after the collision.

(3)

In the collision, the magnitude of the impulse exerted on  $A$  by  $B$  is  $3.3 \text{ N s}$ .

(b) Find the value of  $m$ .

(3)



$$(a) \quad (5m \times 3) + (2m \times -4) = (5m \times 0.8) + 2m v_B$$

$$15m - 8m = 4m + 2m v_B$$

$$7 = 4 + 2v_B$$

$$v_B = \frac{3}{2} = 1.5 \text{ m s}^{-1} \rightarrow$$

(b) Impulse = change in momentum

$$-3.3 = 5m(0.8 - 3)$$

$$-3.3 = -11m$$

$$m = 0.3 \text{ kg}$$



2.

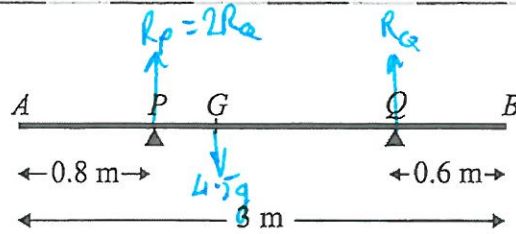


Figure 1

A non-uniform rod  $AB$  has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at  $P$  and at  $Q$ , where  $AP = 0.8$  m and  $QB = 0.6$  m, as shown in Figure 1. The centre of mass of the rod is at  $G$ . Given that the magnitude of the reaction of the support at  $P$  on the rod is twice the magnitude of the reaction of the support at  $Q$  on the rod, find

(a) the magnitude of the reaction of the support at  $Q$  on the rod,

(3)

(b) the distance  $AG$ .

(4)

$$(a) \sum F_y = 0 \quad R_p + R_q - 4.5g = 0$$

$$2R_q + R_q = 4.5g$$

$$3R_q = 4.5g$$

$$R_q = 1.5g \text{ N}$$

$$(b) \sum \tau_A \quad (0.8 \times 3g) + (AG \times -4.5g) + (2.4 \times 1.5g) = 0$$

$$2.4g - 4.5gAG + 3.6g = 0$$

$$4.5AG = 6$$

$$AG = \frac{4}{3} \text{ m.}$$





3.

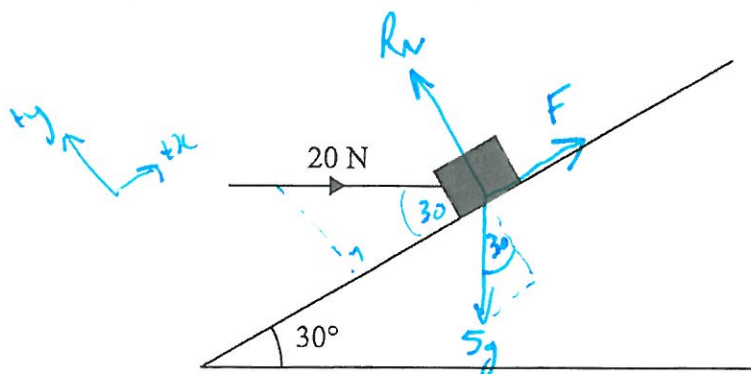


Figure 2

A box of mass 5 kg lies on a rough plane inclined at  $30^\circ$  to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

(a) the magnitude of the normal reaction of the plane on the box,

(4)

(b) the coefficient of friction between the box and the plane.

(5)

$$(a) \sum F_y = 0 \quad R_N - 5g \cos 30 - 20 \sin 30 = 0$$

$$R_N = \frac{\sqrt{3}g}{2} + 10$$

$$(b) \sum F_x = 0 \quad F + 20 \cos 30 - 5g \sin 30 = 0$$

$$F = \frac{5g}{2} - 10\sqrt{3}$$

$$\text{Now } \mu = \frac{F}{R_N} = \frac{2.5g - 10\sqrt{3}}{4.9\sqrt{3} + 10} = 0.388$$



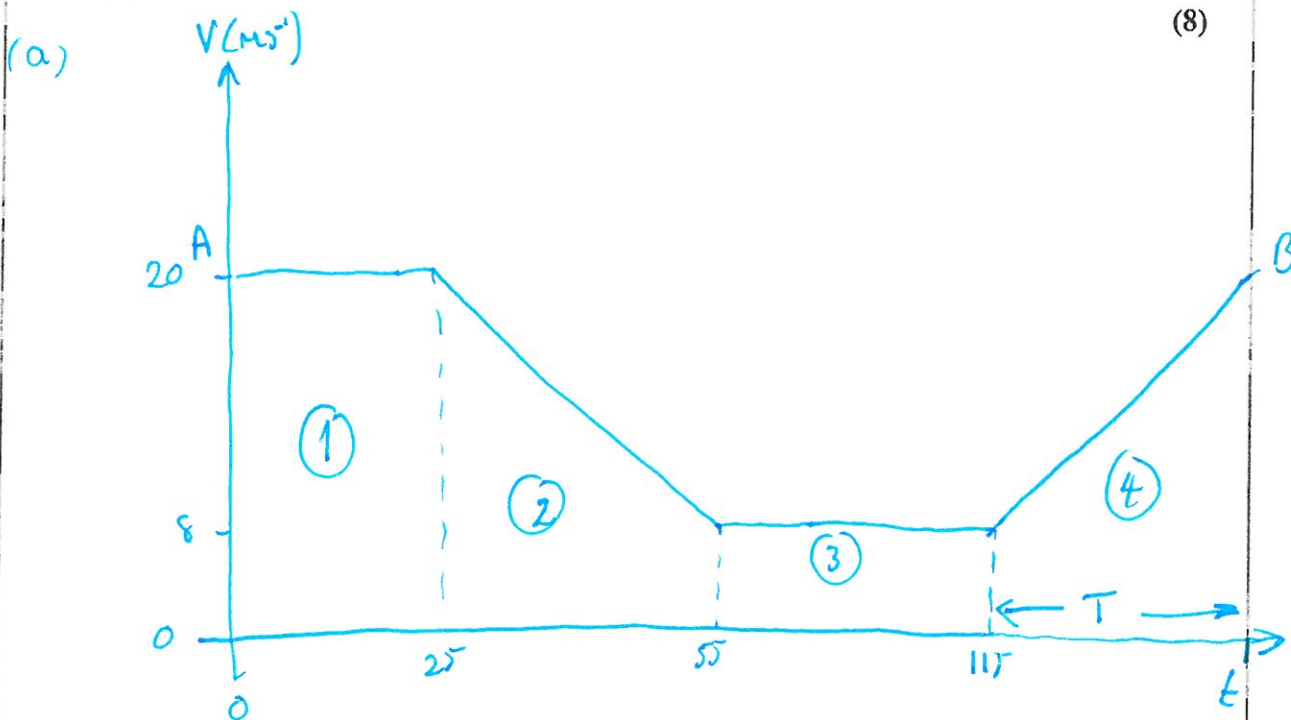
4. A car is moving on a straight horizontal road. At time  $t = 0$ , the car is moving with speed  $20 \text{ m s}^{-1}$  and is at the point  $A$ . The car maintains the speed of  $20 \text{ m s}^{-1}$  for  $25 \text{ s}$ . The car then moves with constant deceleration  $0.4 \text{ m s}^{-2}$ , reducing its speed from  $20 \text{ m s}^{-1}$  to  $8 \text{ m s}^{-1}$ . The car then moves with constant speed  $8 \text{ m s}^{-1}$  for  $60 \text{ s}$ . The car then moves with constant acceleration until it is moving with speed  $20 \text{ m s}^{-1}$  at the point  $B$ .

(a) Sketch a speed-time graph to represent the motion of the car from  $A$  to  $B$ . (3)

(b) Find the time for which the car is decelerating. (2)

Given that the distance from  $A$  to  $B$  is  $1960 \text{ m}$ ,

(c) find the time taken for the car to move from  $A$  to  $B$ . (8)



(b) time of deceleration :  $u=20$   $v=8$   $a=-0.4$   $t=?$

$$v = u + at$$

$$8 = 20 - 0.4t$$

$$0.4t = 12$$

$$t = 30 \text{ sec}$$

(c) distance = Area (1) + (2) + (3) + (4)

$$1960 = \frac{1}{2} (25 \times 20) + \frac{1}{2} (20+8) \times 30 + (60 \times 8) + \frac{1}{2} (8+20) T$$

$$1960 = 500 + 420 + 480 + 14T$$

$$14T = 560 \Rightarrow T = 40 \text{ } \therefore \text{Journey time} = 115 + 40 = 155 \text{ sec.}$$


5. A particle  $P$  is projected vertically upwards from a point  $A$  with speed  $u \text{ m s}^{-1}$ . The point  $A$  is  $17.5 \text{ m}$  above horizontal ground. The particle  $P$  moves freely under gravity until it reaches the ground with speed  $28 \text{ m s}^{-1}$ .

(a) Show that  $u = 21$  (3)

At time  $t$  seconds after projection,  $P$  is  $19 \text{ m}$  above  $A$ .

(b) Find the possible values of  $t$ . (5)

The ground is soft and, after  $P$  reaches the ground,  $P$  sinks vertically downwards into the ground before coming to rest. The mass of  $P$  is  $4 \text{ kg}$  and the ground is assumed to exert a constant resistive force of magnitude  $5000 \text{ N}$  on  $P$ .

(c) Find the vertical distance that  $P$  sinks into the ground before coming to rest. (4)

(a)  $u \uparrow$   $s = 17.5 \downarrow$   $a = 9.8 \downarrow$   $v = 28 \downarrow$   
 $= -u \downarrow$

Using  $v^2 = u^2 + 2as$

$$28^2 = (-u)^2 + 2 \times 9.8 \times 17.5$$

$$28^2 - 343 = u^2$$

$$u^2 = 441$$

$$u = 21 \text{ m s}^{-1} \text{ As required}$$

(b)  $u = 21 \uparrow$   $a = 9.8 \downarrow$   $s = 19 \uparrow$   $t = ?$   
 $= -9.8 \uparrow$

Using  $s = ut + \frac{1}{2}at^2$

$$19 = 21t + \frac{1}{2} \times 9.8 t^2$$

$$4.9t^2 - 21t + 19 = 0$$

$$t = 1.3 \text{ sec or } 3.0 \text{ sec}$$





Question 5 continued

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$$(c) \text{ NZL: } -5000 = 4a$$

$$a = -1250 \text{ m s}^{-2}$$

$$\text{Now } u = 28 \quad v = 0 \quad a = -1250 \quad s = ?$$

$$\text{Using } v^2 = u^2 + 2as$$

$$0^2 = 28^2 + 2 \times (-1250) s$$

$$2500 s = 28^2$$

$$s = 0.3136 \text{ m}$$



6. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

A ship  $S$  is moving with constant velocity  $(-12\mathbf{i} + 7.5\mathbf{j}) \text{ km h}^{-1}$ .

- (a) Find the direction in which  $S$  is moving, giving your answer as a bearing. (3)

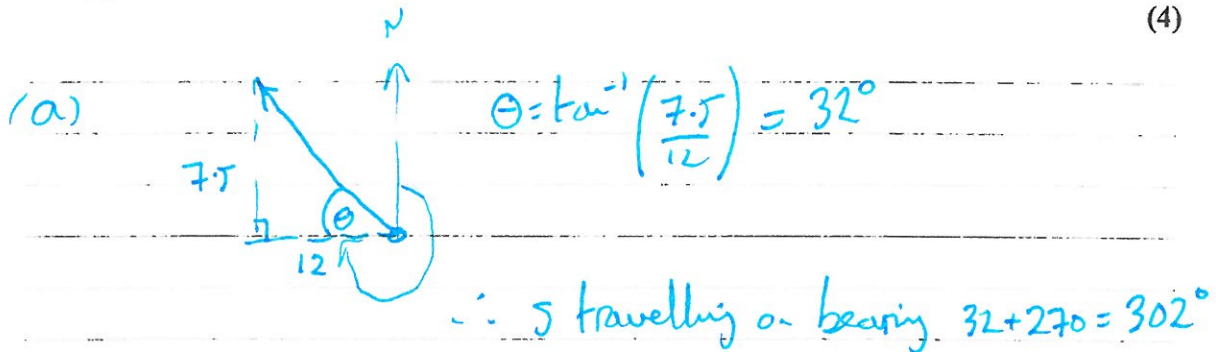
At time  $t$  hours after noon, the position vector of  $S$  is  $\mathbf{s}$  km. When  $t = 0$ ,  $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$ .

- (b) Write down  $\mathbf{s}$  in terms of  $t$ . (2)

A fixed beacon  $B$  is at the point with position vector  $(7\mathbf{i} + 12.5\mathbf{j}) \text{ km}$ .

- (c) Find the distance of  $S$  from  $B$  when  $t = 3$  (4)

- (d) Find the distance of  $S$  from  $B$  when  $S$  is due north of  $B$ . (4)



(b)  $\mathbf{s}_t = \begin{pmatrix} 40 \\ -6 \end{pmatrix} + t \begin{pmatrix} -12 \\ 7.5 \end{pmatrix} = \begin{pmatrix} 40 - 12t \\ 7.5t - 6 \end{pmatrix}$

(c) @  $t=3$   $\mathbf{s} = \begin{pmatrix} 40 \\ -6 \end{pmatrix} + 3 \begin{pmatrix} -12 \\ 7.5 \end{pmatrix} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix}$

$\vec{SB} = -\vec{OS} + \vec{OB} = -\begin{pmatrix} 4 \\ 16.5 \end{pmatrix} + \begin{pmatrix} 7 \\ 12.5 \end{pmatrix} = \begin{pmatrix} 3 \\ -22.5 \end{pmatrix}$

$|\vec{SB}| = \sqrt{3^2 + (-22.5)^2} = 42.5 \text{ km}$

(d) When  $S$  is due north of  $B$ , their  $i$  components will be the same

$\therefore 40 - 12t = 7$   
 $t = \frac{33}{12}$





Question 6 continued

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at this point  $j$  component of  $S$  will be  $7.5\left(\frac{33}{12}\right) - 6 = 14.6$   
So distance between  $S+B$  will be  $14.6 - 12.5 = 2.1 \text{ km.}$



P 4 0 6 8 9 A 0 1 9 2 8

7.

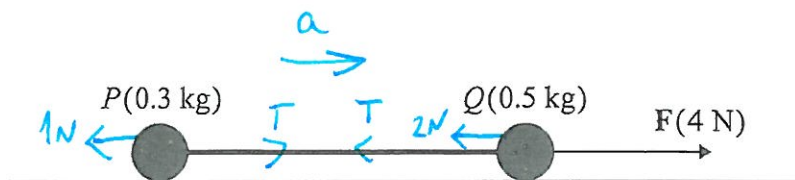


Figure 3

Two particles  $P$  and  $Q$ , of mass  $0.3 \text{ kg}$  and  $0.5 \text{ kg}$  respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time  $t = 0$ , a constant force  $F$  of magnitude  $4 \text{ N}$  is applied to  $Q$  in the direction  $PQ$ , as shown in Figure 3. The system moves under the action of this force until  $t = 6 \text{ s}$ . During the motion, the resistance to the motion of  $P$  has constant magnitude  $1 \text{ N}$  and the resistance to the motion of  $Q$  has constant magnitude  $2 \text{ N}$ .

Find

- (a) the acceleration of the particles as the system moves under the action of  $F$ , (3)
- (b) the speed of the particles at  $t = 6 \text{ s}$ , (2)
- (c) the tension in the rod as the system moves under the action of  $F$ . (3)

At  $t = 6 \text{ s}$ ,  $F$  is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

- (d) the distance moved by  $P$  as the system decelerates, (4)
- (e) the thrust in the rod as the system decelerates. (3)

(a) N2L on whole system:  $4 - 3 = 0.8a$   
 $1 = 0.8a$   
 $a = 1.25 \text{ m s}^{-2}$

(b)  $u = 0$   $v = ?$   $a = 1.25$   $t = 6$   $V = u + at$   
 $V = 0 + 1.25 \times 6 = 7.5 \text{ m s}^{-1}$

(c) N2L on P:  $T - 1 = 0.3a$   
 $T = 0.3 \times 1.25 + 1 = 4.75 \text{ N}$



Question 7 continued

(d) change in accel:  $-3 = 0.8a$

$$a = -3.75 \text{ m/s}^2$$

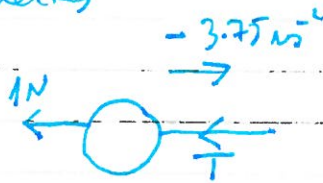
$$U = 7.5 \quad V = 0 \quad a = -3.75 \quad s = ?$$

$$V^2 = U^2 + 2as$$

$$0 = 7.5^2 - 2 \times 3.75 s$$

$$s = 7.5 \text{ metres}$$

(e) N2L on P



$$-T - 1 = 0.3 \times -3.75$$

$$1.125 - 1 = T$$

$$T = 0.125 \text{ N}$$

