FP2 – Inequalities

The story so far…

At key stage three inequality questions looked like this:

Place the correct symbol, <, ≤, =, ≥, > between the following pairs of numbers

(a) 9 > 0 (b) –4 < 4 (c) –3 > -7 (d) (3 – 8) = (-9 - -4)

During GCSE we learned how to solve inequations:

(a) Solve 3x – 5 > 7 (b) List the integers which satisfy $1 \le 2x - 7 < 8$

(a) $3x > 12$
$x > 4$

(b) $1 \le 2x - 7$ $\quad 2x - 7 < 8$
$8 \le 2x$ $\quad 2x < 15$
$4 \le x$ $\quad x < \frac{15}{2}$

$4 \le x < 7.5 \quad \therefore x = 4, 5, 6, 7$

We also learned a VERY important rule about multiplying or dividing by a negative number:

If $-4x > 24$
Then

$x < \frac{24}{-4}$

$x < {}^{-}6$

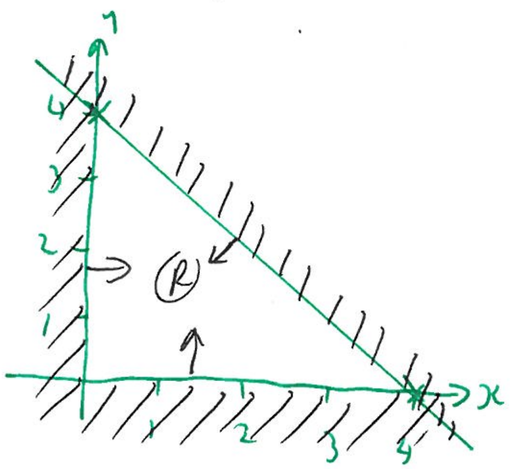
If $\frac{x}{-5} < 6$

$x > 6 \times -5$

$x > 30$

In the Higher Tier we dealt with the graphical representation of inequalities:

Sketch a graph to show clearly the region which satisfies the following inequalities:
$x + y \le 4$, $x \ge 0$, $y \ge 0$



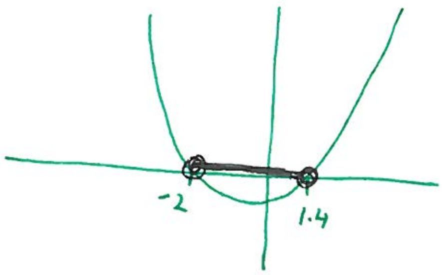
We meet inequalities again at AS level in C1:

Find the set of values of x for which $5x^2 + 3x \leq 14$

$$5x^2 + 3x - 14 \leq 0$$

$$(5x-7)(x+2) \leq 0$$

critical values $x = -2, 1.4$



$$-2 \leq x \leq 1.4$$

And now we meet inequalities finally (in school at least!) in FP2. What can there possibly be left to do with the buggers?!

Solving inequalities which have variables in the denominator

Eg1 Find the set of real values of x for which

$$\frac{2x^2+6}{x+6} > 1$$

Eg2 Find the set of real values of x for which

$$\frac{x+3}{x-1} > \frac{x-3}{x+1}$$

Ex 1A Pg 4 Q's 3, 4, 5, 7, 8, 10, 14, 15

Eg1 $\dfrac{2x^2+6}{x+6} > 1$

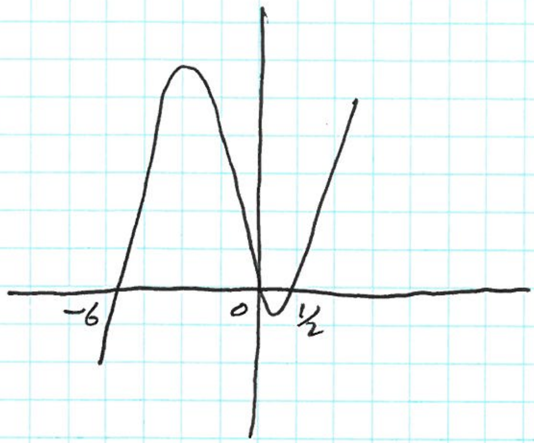
$x+6$ could be $-$ve $\therefore$ Mult by $(x+6)^2$

$\dfrac{2x^2+6}{x+6} \times (x+6)^2 > (x+6)^2$

$2(x^2+3)(x+6) - (x+6)^2 > 0$

$(x+6)[2x^2+6-x-6] > 0$

$x(x+6)(2x-1) > 0$



Critical values $x = -6, 0, \frac{1}{2}$

$-6 < x < 0$ or $x > \frac{1}{2}$

Eg2 $\dfrac{1+x}{1-x} > \dfrac{2-x}{2+x}$

$\dfrac{1+x}{1-x} \times (2+x)^2(1-x)^2 > \dfrac{2-x}{2+x} \times (2+x)^2(1-x)^2$

$(1+x)(1-x)(2+x)^2 > (2-x)(2+x)(1-x)^2$

$(1+x)(1-x)(2+x)^2 - (2-x)(2+x)(1-x)^2 > 0$

$(1-x)(2+x)[(1+x)(2+x) - (2-x)(1-x)] > 0$

$(1-x)(2+x)[2+3x+x^2 - (2-3x+x^2)] > 0$

$(1-x)(2+x)($

Eg2 $\quad \dfrac{x+3}{x-1} > \dfrac{x-3}{x+1}$

$$\frac{x+3}{x-1} \times (x-1)^2(x+1)^2 > \frac{x-3}{x+1} \times (x-1)^2(x+1)^2$$

$$(x+3)(x-1)(x+1)^2 > (x-3)(x+1)(x-1)^2$$

$$(x+3)(x-1)(x+1)^2 - (x-3)(x+1)(x-1)^2 > 0$$

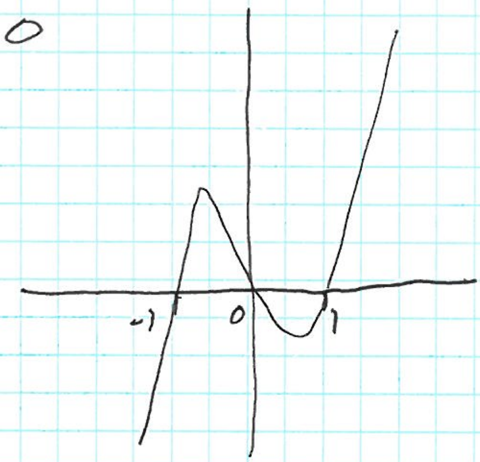
$$(x-1)(x+1)\left[(x+3)(x+1) - (x-3)(x-1)\right] > 0$$

$$(x-1)(x+1)\left[x^2+4x+3 - (x^2-4x+3)\right] > 0$$

$$(x-1)(x+1)(8x) > 0$$

$\therefore$ critical values $x = -1, 0, 1$

$-1 < x < 0$ or $x > 1$



Ex 1A

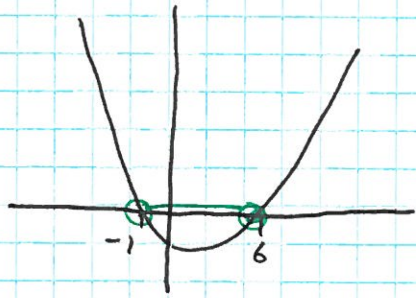
① $x^2 < 5x + 6$

$x^2 - 5x - 6 < 0$

$(x-6)(x+1) < 0$

$\therefore$ critical values $x = -1, 6$

$-1 < x < 6$



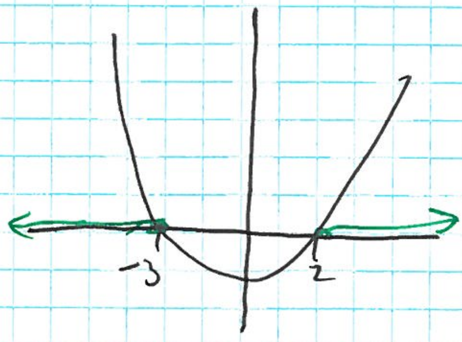
② $x(x+1) \geq 6$

$x^2 + x - 6 \geq 0$

$(x+3)(x-2) \geq 0$

CV $x = -3, 2$

$x \leq -3$ or $x \geq 2$



③ $\dfrac{2}{x^2+1} > 1$

x^2+1 cannot be negative

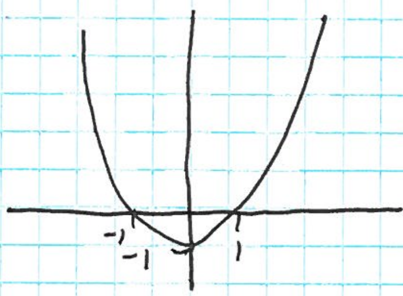
$2 > x^2 + 1$

$0 > x^2 - 1$

$x^2 - 1 < 0$

$x = \pm 1$

$\therefore -1 < x < 1$



④ $\dfrac{2}{x^2-1} > 1$

x^2-1 could be -ve $\therefore \times (x^2-1)^2$

$\dfrac{2}{\cancel{x^2-1}} \times (x^2-1)^{\cancel{2}} > (x^2-1)^2$

$2x^2 - 2 > x^4 - 2x^2 + 1$

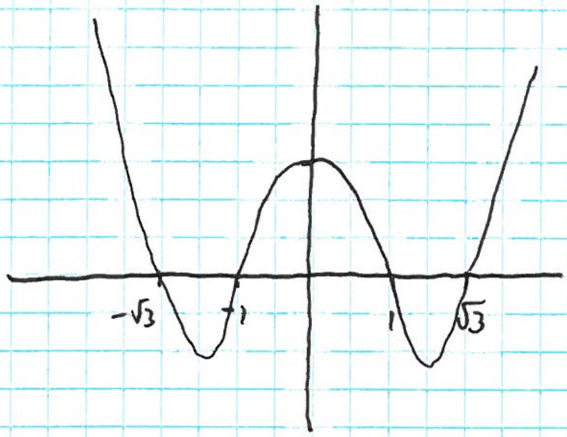
$0 > x^4 - 4x^2 + 3$

~~$x^4 - 4x^2 +$~~ $x^4 - 4x^2 + 3 < 0$

$(x^2-1)(x^2-3) < 0$

④ Cont'd $\quad x^2 - 1 = 0 \quad$ or $\quad x^2 - 3 = 0$

$\quad\quad x = \pm 1 \quad\quad\quad\quad x = \pm\sqrt{3}$



$\cancel{x < -\sqrt{3}, \ -1 < x < 1, \ x > \sqrt{3}}$

$-\sqrt{3} < x < -1 \quad$ or $\quad 1 < x < \sqrt{3}$

⑤ $\quad \dfrac{x}{x-1} \leq 2x$

$$\frac{x}{\cancel{x-1}} \times (x-1)^{\cancel{2}} \leq 2x(x-1)^2$$

$$x^2 - x \leq 2x(x^2 - 2x + 1)$$

$$x^2 - x \leq 2x^3 - 4x^2 + 2x$$

$$0 \leq 2x^3 - 5x^2 + 3x$$

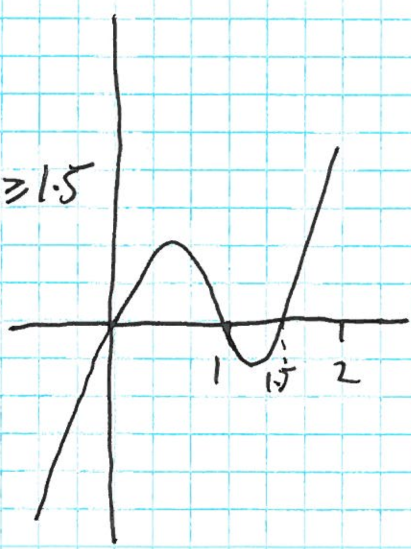
$$2x^3 - 5x^2 + 3x \geq 0$$

$$x(2x^2 - 5x + 3) \geq 0$$

$$x(x-1)(2x-3) \geq 0$$

$\therefore$ critical values @ $x = 0, x = 1, x = 1.5$

$0 \leq x \leq 1$ or $x \geq 1.5$



* Don't know why book Solution here lost "or equal to's"

(6) $\frac{3}{x+1} < \frac{2}{x}$

$\frac{3(x+1)^2x^2}{x+1} < \frac{2}{x}(x+1)^2x^2$

$3x^2(x+1) < 2x(x+1)^2$

$3x^3+3x^2 < 2x(x^2+2x+1)$

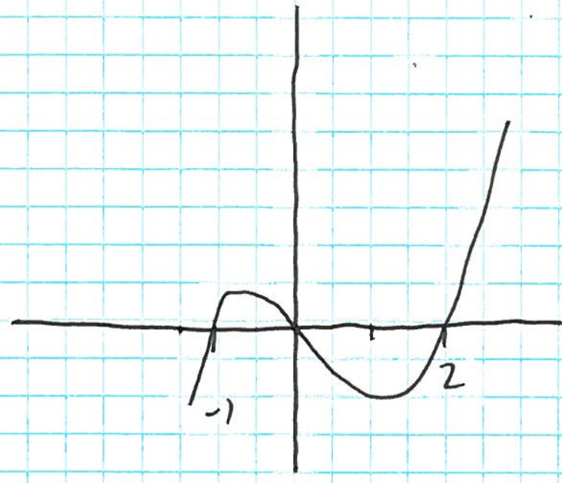
$3x^3+3x^2 < 2x^3+4x^2+2x$

$x^3-x^2-2x < 0$

$x(x^2-x-2) < 0$

$x(x-2)(x+1) < 0$

$\therefore$ critical values @ $x=-1, 0, 2$



$x < -1$ or $0 < x < 2$

(7) $\frac{3}{(x+1)(x-1)} < 1$

$\frac{3}{(\cancel{x+1})(\cancel{x-1})}(\cancel{x+1})^2(\cancel{x-1})^2 < 1(\cancel{x+1})^2(\cancel{x-1})^2$

$3(x+1)(x-1) <$

$\frac{3}{x^2-1} \times (x^2-1)^2 < (x^2-1)^2$

$3(x^2-1) < x^4-2x^2+1$

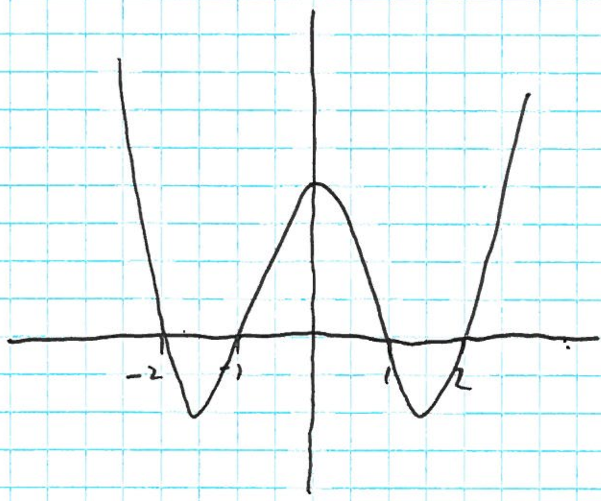
$3x^2-3 < x^4-2x^2+1$

$0 < x^4-5x^2+4$

$x^4-5x^2+4 > 0$

$(x^2-4)(x^2-1) > 0$

$\therefore$ critical values @ $x=\pm 2$, $x=\pm 1$



$x < -2$ or $-1 < x < 1$ or $x > 2$

(8) $\dfrac{2}{x^2} \geqslant \dfrac{3}{(x+1)(x-2)}$

$$\frac{2}{x^2}\cdot x^2(x+1)^2(x-2)^2 \geqslant \frac{3x^2(x+1)^2(x-2)^2}{(x+1)(x-2)}$$

$$2(x^2+2x+1)(x^2-4x+4) \geqslant 3x^2(x+1)(x-2)$$

$$2(x^4-4x^3+4x^2+2x^3-8x^2+8x+x^2-4x+4) \geqslant 3x^2(x^2-x-2)$$

$$2x^4-4x^3-6x^2+8x+8 \geqslant 3x^4-3x^3-6x^2$$

$$0 \geqslant x^4+x^3-8x-8$$

$$x^4+x^3-8x-8 \leqslant 0$$

$$2(x+1)^2(x-2)^2 \geqslant 3x^2(x+1)(x-2)$$

$$2(x+1)^2(x-2)^2-3x^2(x+1)(x-2) \geqslant 0$$

$$(x+1)(x-2)\left[2(x+1)(x-2)-3x^2\right] \geqslant 0$$

$$(x+1)(x-2)\left[2x^2-2x-4-3x^2\right] \geqslant 0$$

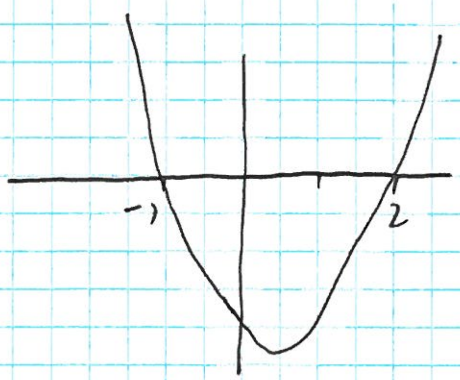
$$(x+1)(x-2)(-x^2-2x-4) \geqslant 0$$

$$-1(x+1)(x-2)(x^2+2x+4) \geqslant 0$$

$$(x+1)(x-2)(x^2+2x+4) \leqslant 0$$

x^2+2x+4 has no real roots

$\therefore$ Critical values $x = -1, 2$



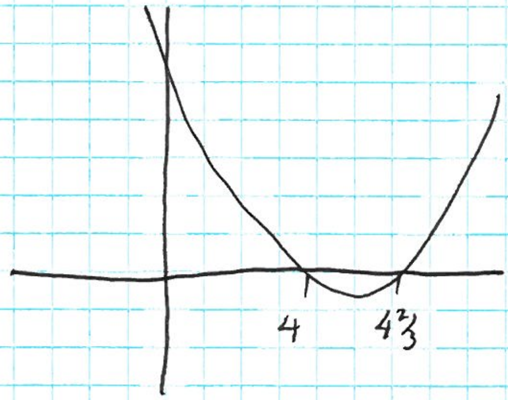
In original expression, $x \neq 0, -1, 2$

$\therefore \quad -1 < x < 2, \quad x \neq 0$

or can be written...

$-1 < x < 0$ <u>OR</u> $0 < x < 2$

(9) $\dfrac{2}{x-4} < 3$



$$\frac{2}{x-4}\cdot(x-4)^2 < 3(x-4)^2$$

$$2x-8 < 3x^2-24x+48$$

$$0 < 3x^2-26x+56$$

$$3x^2-26x+56 > 0$$

$$(3x-14)(x-4) > 0$$

Critical values $x = 4,\ 4\frac{2}{3}$

$x < 4$ or $x > 4\frac{2}{3}$

(10) $\dfrac{3}{x+2} > \dfrac{1}{x-5}$

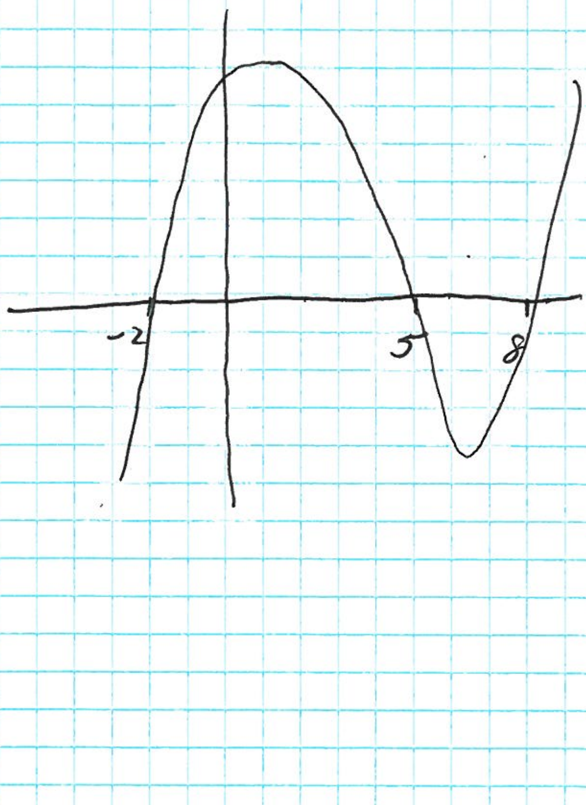
$$\frac{3(x+2)^2(x-5)^2}{(x+2)} > \frac{1}{x-5}(x+2)^2(x-5)^2$$

$$3(x+2)(x-5)^2 > (x+2)^2(x-5)$$

$$3(x+2)(x-5)^2 - (x+2)^2(x-5) > 0$$

$$(x+2)(x-5)[3(x-5)-(x+2)] > 0$$

$$(x+2)(x-5)(2x-17) > 0$$



$\therefore$ critical values $x = -2, 5, 8.5$

$-2 < x < 5$ or $x > 8.5$

(11) $\dfrac{3x^2+5}{x+5} > 1$

$\dfrac{3x^2+5}{x+5}(x+5)^2 > (x+5)^2$

$(3x^2+5)(x+5) > (x+5)^2$

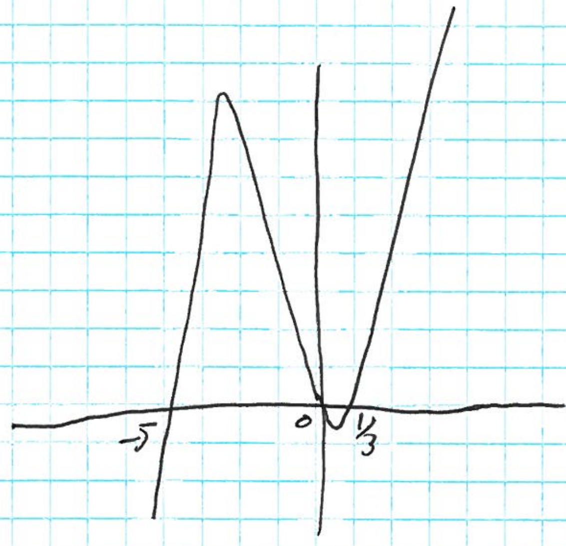
$(3x^2+5)(x+5) - (x+5)^2 > 0$

$(x+5)\left[3x^2+5 - (x+5)\right] > 0$

$(x+5)(3x^2-x) > 0$

$x(x+5)(3x-1) > 0$

$\therefore$ critical values $x = -5, 0, \tfrac{1}{3}$



$-5 < x < 0$ or $x > \tfrac{1}{3}$

(12) $\dfrac{3x}{x-2} > x$

$3x(x-2) > x(x-2)^2$

$3x(x-2) - x(x-2)^2 > 0$

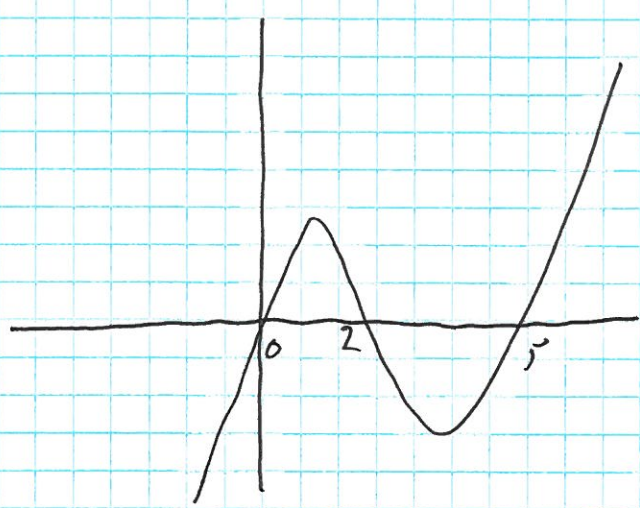
$(x-2)\left[3x - x(x-2)\right] > 0$

$(x-2)\left[5x - x^2\right] > 0$

$-x(x-2)(x-5) > 0$

$x(x-2)(x-5) < 0$

$\therefore$ critical values $x = 0, 2, 5$



$x < 0$ or $2 < x < 5$

(13) $\dfrac{1+x}{1-x} > \dfrac{2-x}{2+x}$

$\dfrac{(1+x)}{(1-x)}(1-x)^2(2+x)^2 > \dfrac{2-x}{2+x}(1-x)^2(2+x)^2$

$(1+x)(1-x)(2+x)^2 > (2-x)(1-x)^2(2+x)$

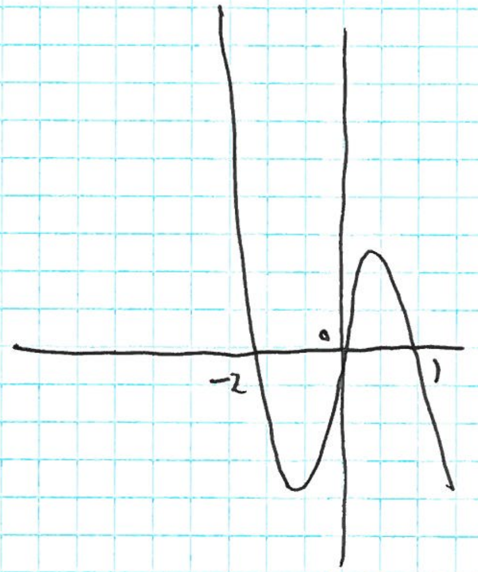
$(1+x)(1-x)(2+x)^2 - (2-x)(1-x)^2(2+x) > 0$

$(1-x)(2+x)\left[(1+x)(2+x) - (2-x)(1-x)\right] > 0$

$(1-x)(2+x)\left[2+3x+x^2 - (2-3x+x^2)\right] > 0$

$(1-x)(2+x)\left[6x\right] > 0$

$\therefore$ critical value $x = -2, 0, 1$



$x < -2$ or $0 < x < 1$

(14)

$\dfrac{x^2+7x+10}{x+1} > 2x+7$

$(x^2+7x+10)(x+1) > (2x+7)(x+1)^2$

$(x^2+7x+10)(x+1) - (2x+7)(x+1)^2 > 0$

$(x+1)\left[x^2+7x+10 - (2x^2+9x+7)\right] > 0$

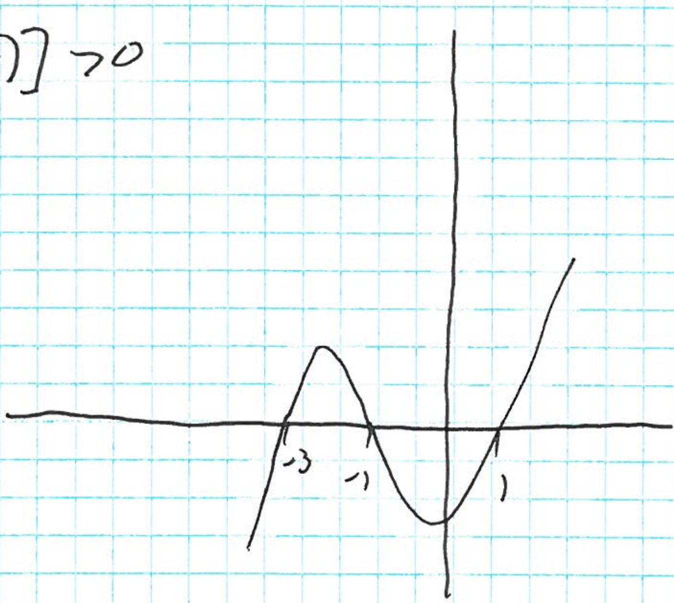
$(x+1)\left[-x^2-2x+3\right] > 0$

$-1(x+1)(x^2+2x-3) > 0$

$(x+1)(x+3)(x-1) < 0$

$\therefore$ c.v.'s $x = -3, -1, 1$

$x < -3$ or $-1 < x < 1$



(15) (a) $\frac{x+1}{x^2} > 6$

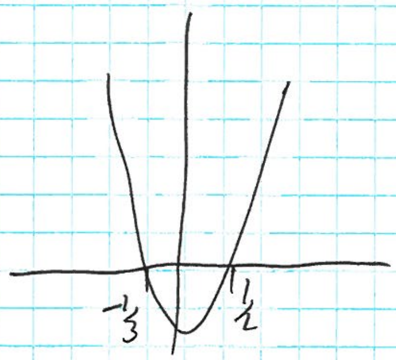
$x+1 > 6x^2$

$0 > 6x^2 - x - 1$

$6x^2 - x - 1 < 0$

$(2x-1)(3x+1) < 0$

CV's $x = -\frac{1}{3}, \frac{1}{2}$



$-\frac{1}{3} < x < \frac{1}{2}$, $x \neq 0$

$\therefore$ $-\frac{1}{3} < x < 0$ or $0 < x < \frac{1}{2}$

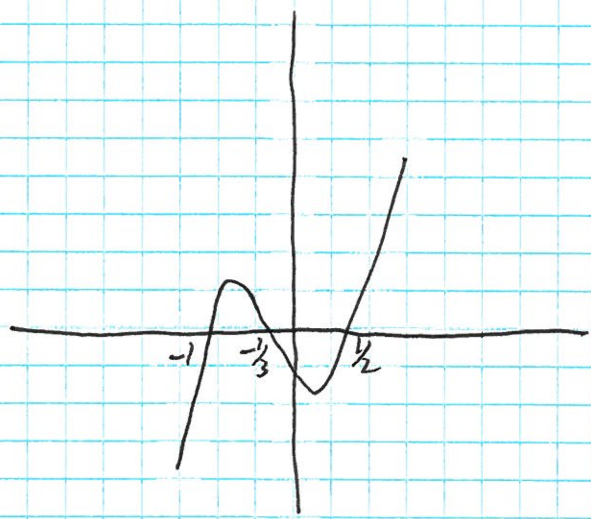
(b) $\frac{x^2}{x+1} > \frac{1}{6}$

$\frac{6x^2(x+1)^2}{x+1} > (x+1)^2$

$6x^2(x+1) - (x+1)^2 > 0$

$(x+1)[6x^2 - x - 1] > 0$

$(x+1)(2x-1)(3x+1) > 0$



$\therefore$ CV's @ $x = -\frac{1}{3}, -1, \frac{1}{2}$

$-1 < x < -\frac{1}{3}$ or $x > \frac{1}{2}$

The Modulus Function |x|

Suppose that on the real number line the point O represents the number zero and the point A the number A. The modulus of a , |a| is the distance OA. That is |20| = 20 and |-12| is 12.

Note that $|a| = \begin{cases} +a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases}$

A modulus function is such that $y = |f(x)|$

Graphs of the form y = |f(x)|

The 'input', x, can be any real number. However the 'output', y, must be positive. Therefore the graph 'bounces' off the x axis.

Eg1 Sketch the graph of $y = |2x - 1|$

Eg2 (a) Sketch on the same axis, the graphs of $y = x^2 - 5x + 4$ and $y = |x^2 - 5x + 4|$.

(b) Use your graph to help solve the equation $|x^2 - 5x + 4| = 1$

(c) Use your graph to help solve the inequality $|x^2 - 5x + 4| > 1$

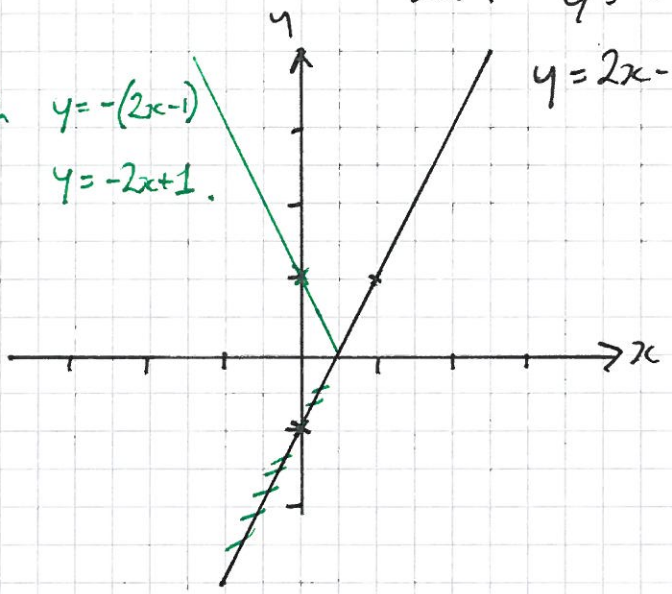
Exercise 1C FP1 Page 8

eg1 Consider $y = 2x - 1$ $\quad x=0 \quad y=-1 \quad (0,-1)$

$x=1 \quad y=1 \quad (1,1)$

equation of reflection $y = -(2x-1)$

$y = -2x+1.$



when $y<0$

Forced >0 by modulus.

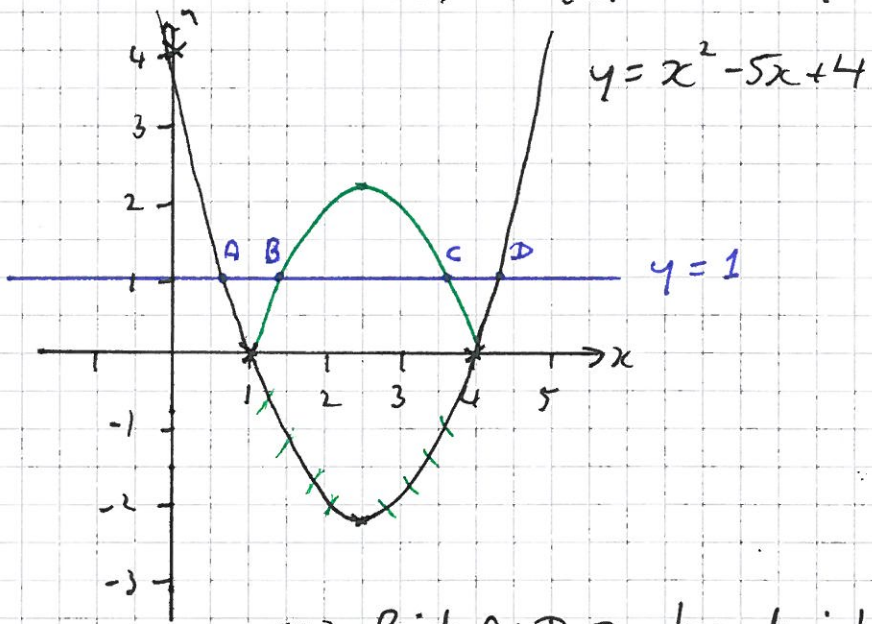
eg2 (a) $y = x^2 - 5x + 4$ $\quad$ when $x=0 \quad y=4 \quad (0,4)$

when $y=0 \quad (x-1)(x-4)=0 \quad (1,0) + (4,0)$

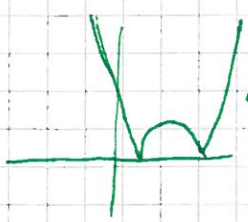
Min $\dfrac{dy}{dx} = 2x - 5 = 0$

$x = \frac{5}{2} \Rightarrow y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 4 = \frac{25}{4} - \frac{25}{2} + 4 = -2\frac{1}{4}$

$\therefore$ Min @ $\left(\frac{5}{2}, -2\frac{1}{4}\right)$



(b)



above.

(c) Points A + D @ intersection between

$x^2 - 5x + 4 = 1$

$x^2 - 5x + 3 = 0$

$x = \dfrac{5 \pm \sqrt{25-12}}{2} = \dfrac{5 \pm \sqrt{13}}{2}$

So A $\quad x = \frac{1}{2}(5 - \sqrt{13})$

And D $\quad x = \frac{1}{2}(5 + \sqrt{13})$

eg 2 (c) contd Points B & C @ intersection between

$$-(x^2-5x+4)=1$$

$$-x^2+5x-5=0$$

$$x=\frac{-5\pm\sqrt{25-20}}{-2}$$

$$x=\frac{-5\pm\sqrt{5}}{-2}$$

$$x=\frac{1}{2}(5\mp\sqrt{5})$$

$\therefore$ @ B $x=\frac{1}{2}(5-\sqrt{5})$

@ C $x=\frac{1}{2}(5+\sqrt{5})$.

(d) $|x^2-5x+4|>1$.

Regions of graph where curve is above ~~and~~ $y=1$.

ie to the left of A, between B & C and to the right of D.

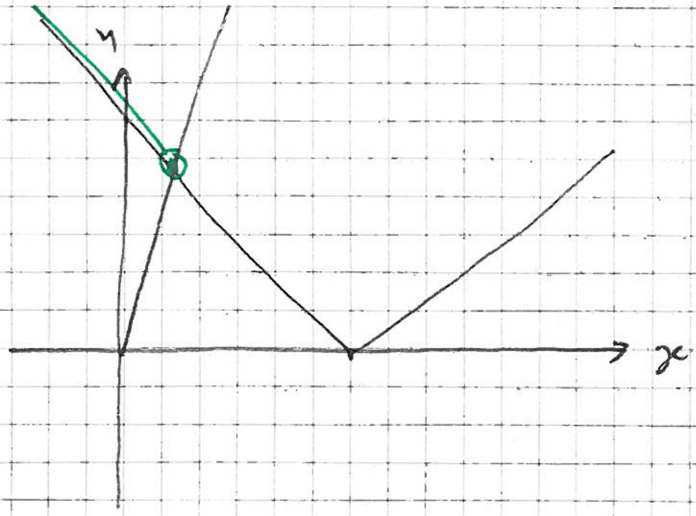
$\therefore$ $x<\frac{1}{2}(5-\sqrt{13})$

$\frac{1}{2}(5-\sqrt{5})<x<\frac{1}{2}(5+\sqrt{5})$

$x>\frac{1}{2}(5+\sqrt{13})$

Ex 1B

① $|x-6| > 6x$



critical value

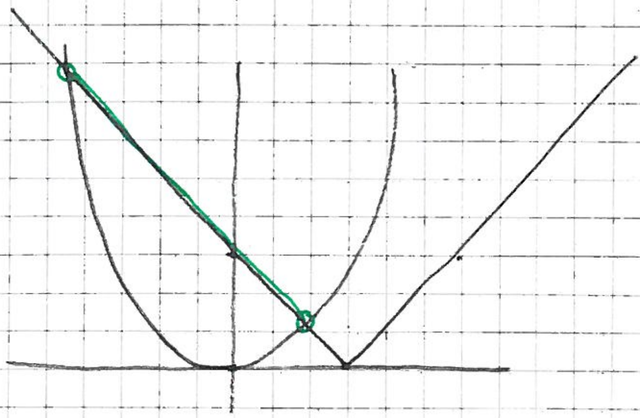
when $-(x-6) = 6x$

$-x+6 = 6x$

$x = \frac{6}{7}$

$x < \frac{6}{7}$

② $|t-3| > t^2$



cuts in two places

$-(t-3) = t^2$

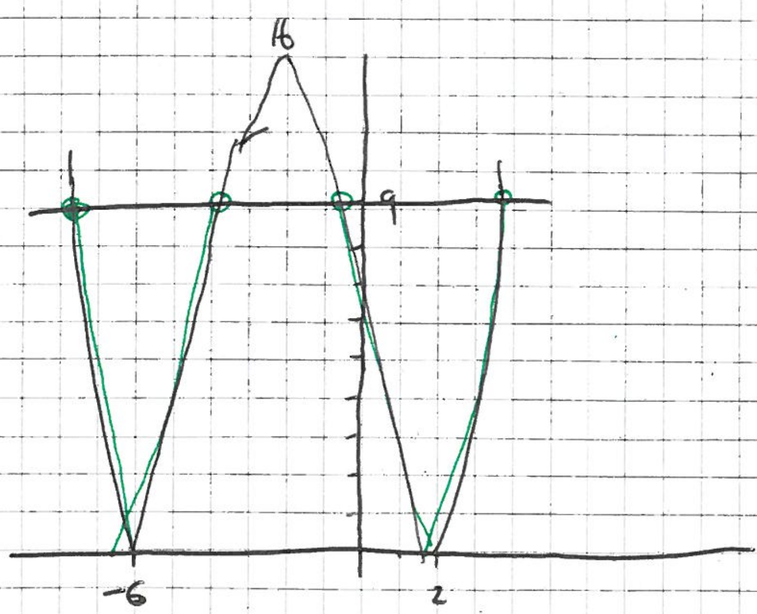
$t^2 + t - 3 = 0$

$t = \frac{-1 \pm \sqrt{1+12}}{2}$

$t = \frac{-1 \pm \sqrt{13}}{2}$

$\therefore \quad \frac{-1-\sqrt{13}}{2} < t < \frac{-1+\sqrt{13}}{2}$

③ $|(x-2)(x+6)| < 9$



$y = (x-2)(x+6)$

$\frac{dy}{dx} = (x-2) + (x+6) = 2x+4 = 0$

$x = -2$

$y = -16$

(3) contd 4 critical values

$(x-2)(x+6)=9$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x=3,-7$

$-(x-2)(x+6)=9$

$x^2+4x-12=-9$

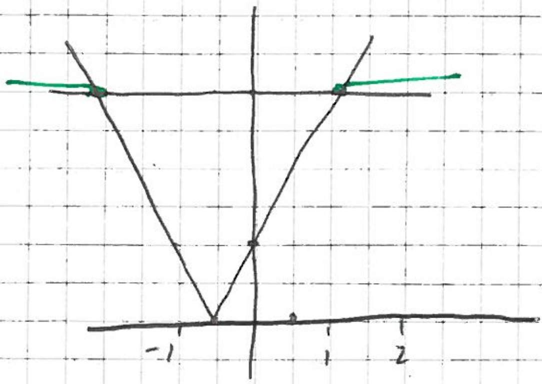
$x^2+4x-3=0$

$(x+2)^2-7=0$

$x=-2\pm\sqrt{7}$

$-7<x<(-2-\sqrt{7})$ or $-2+\sqrt{7}<x<3$

(4) $|2x+1|\geq 3$



critical values when

$2x+1=3$

$x=1.$

$-(2x+1)=3$

$-2x=4$

$x=-2$

$x\leq -2$ or $x\geq 1$

(5) $|2x|+x>3$

$|2x|>3-x$

critical values when

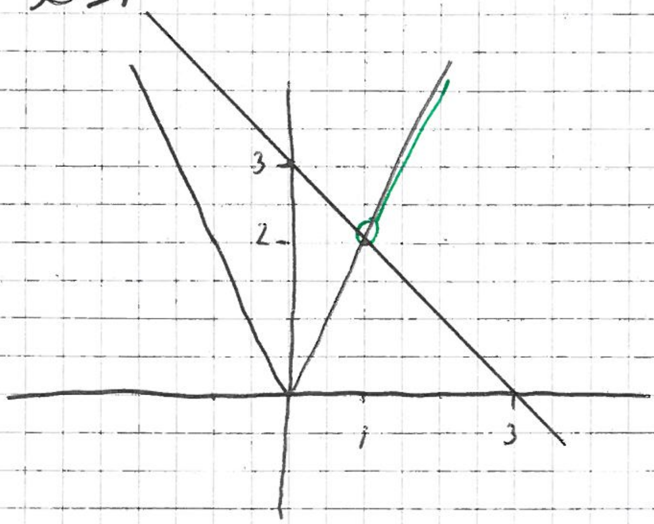
$2x=3-x$

$3x=3$

$x=1.$

$-2x=3-x$

$x=-3$



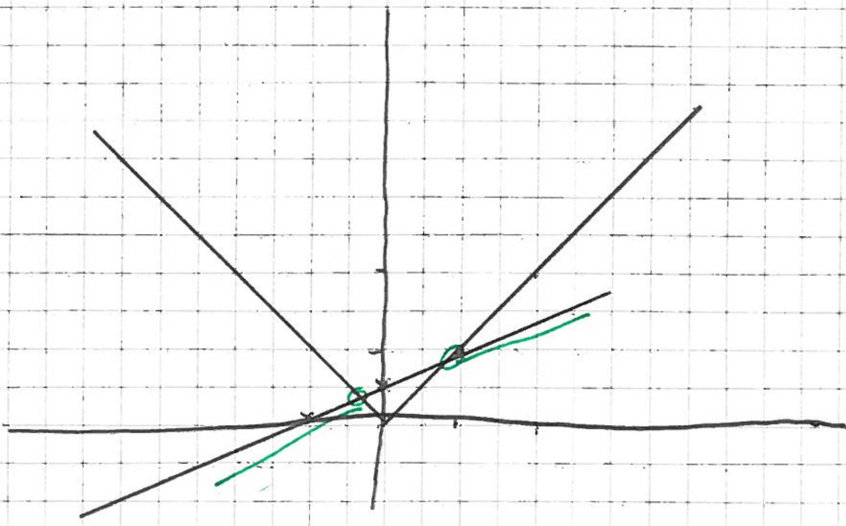
$x<-3$ or $x>1$

⑥ $\frac{x+3}{|x|+1} < 2$

$x+3 < 2|x|+2$

$x+1 < 2|x|$

$\frac{1}{2}x+\frac{1}{2} < |x|$



Critical Values when

$\frac{1}{2}x+\frac{1}{2} = x$

$\frac{1}{2} = \frac{1}{2}x$

$x = 1$

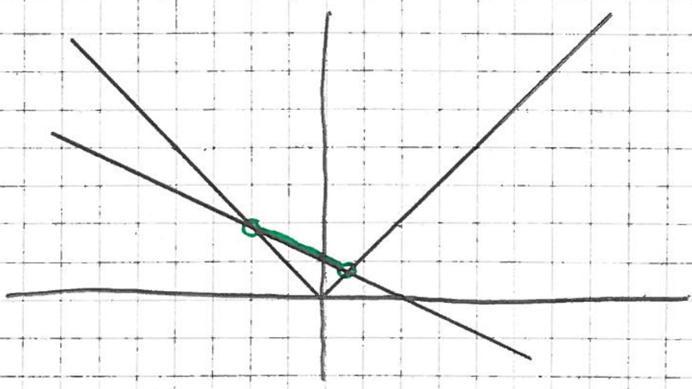
$\frac{1}{2}x+\frac{1}{2} = -x$

$\frac{3}{2}x = -\frac{1}{2}$

$x = -\frac{1}{3}$

$x < -\frac{1}{3}$ OR $x > 1$

⑦ $\frac{3-x}{|x|+1} > 2$



$3-x > 2|x|+2$

$1-x > 2|x|$

$\frac{1}{2}-\frac{1}{2}x > |x|$

Critical values when

$\frac{1}{2}-\frac{1}{2}x = x$

$\frac{3}{2}x = \frac{1}{2}$

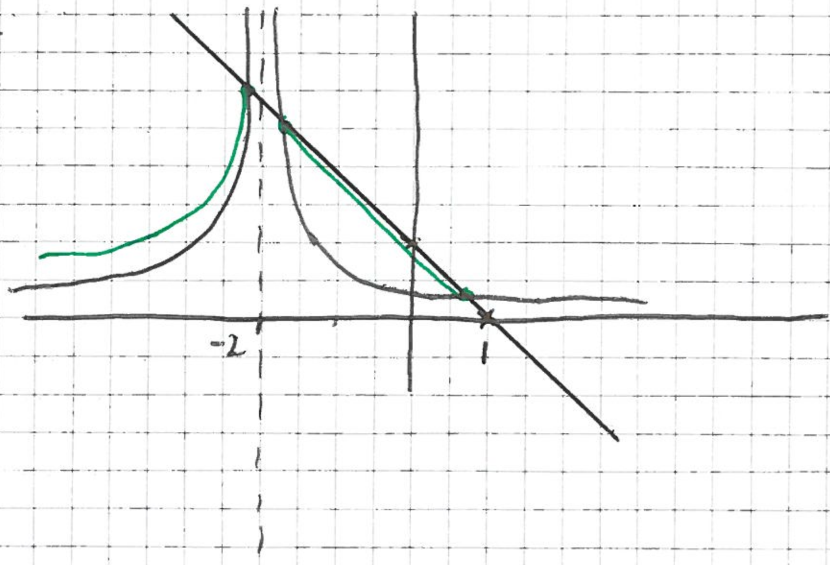
$x = \frac{1}{3}$

$-1 < x < \frac{1}{3}$

$\frac{1}{2}-\frac{1}{2}x = -x$

$x = -1$

⑧ $\left|\dfrac{x}{x+2}\right| < 1-x$



Critical values when

$$\frac{x}{x+2} = 1-x$$

$$x = (1-x)(x+2)$$

$$x = x+2-x^2-2x$$

$$x^2+2x-2=0$$

$$(x+1)^2-3=0$$

$$x = -1 \pm \sqrt{3}$$

$$-\left(\frac{x}{x+2}\right) = 1-x$$

$$\frac{x}{x+2} = x-1$$

$$x = (x-1)(x+2)$$

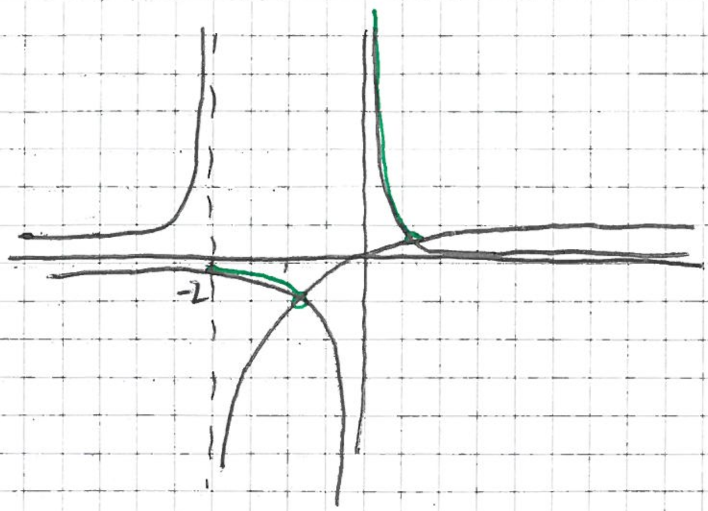
$$x = x^2+x-2$$

$$x = \pm\sqrt{2}$$

$x < -1-\sqrt{3}$

or $-\sqrt{2} < x < -1+\sqrt{3}$.

(9) (a)



(b) and critical values

$$\frac{1}{x} = \frac{x}{x+2}$$

$$x+2 = x^2$$

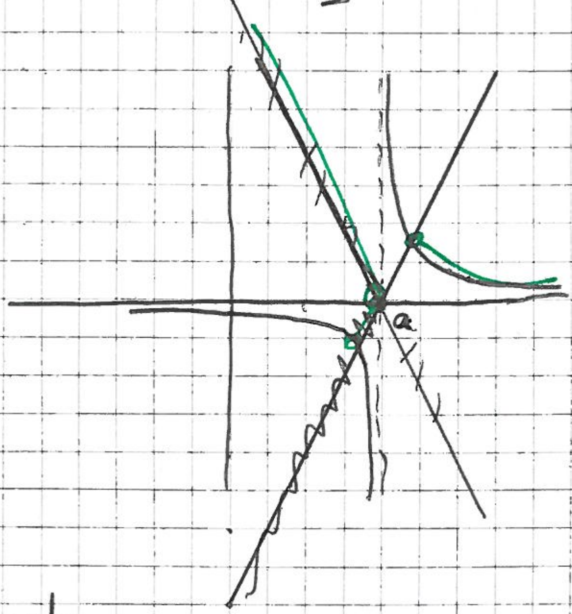
$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = -1, 2$$

$0 < x < 2$ or $-2 < x < -1$

(10)



$y = 4x - 4a$

$0 = 4x - 4a$

$x = a$

$y = -4a$

critical values when

$$\frac{1}{x-a} = 4x - 4a$$

$$1 = 4(x-a)(x-a)$$

$$(x-a)^2 = \frac{1}{4}$$

$$x = a \pm \tfrac{1}{2}$$

$$\frac{1}{x-a} = -4(x-a)$$

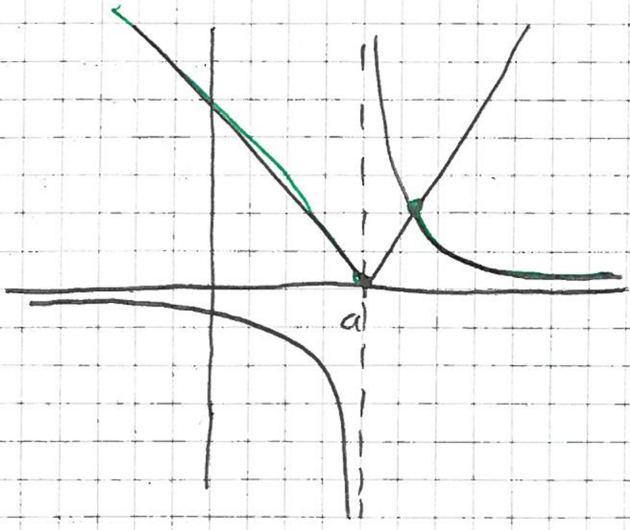
$$-\frac{1}{4} = (x-a)^2$$

no solution

$x > a + \tfrac{1}{2}$

or

10



Ex1C

(1) $|x^2-7| < 3(x+1)$

$y = x^2-7$

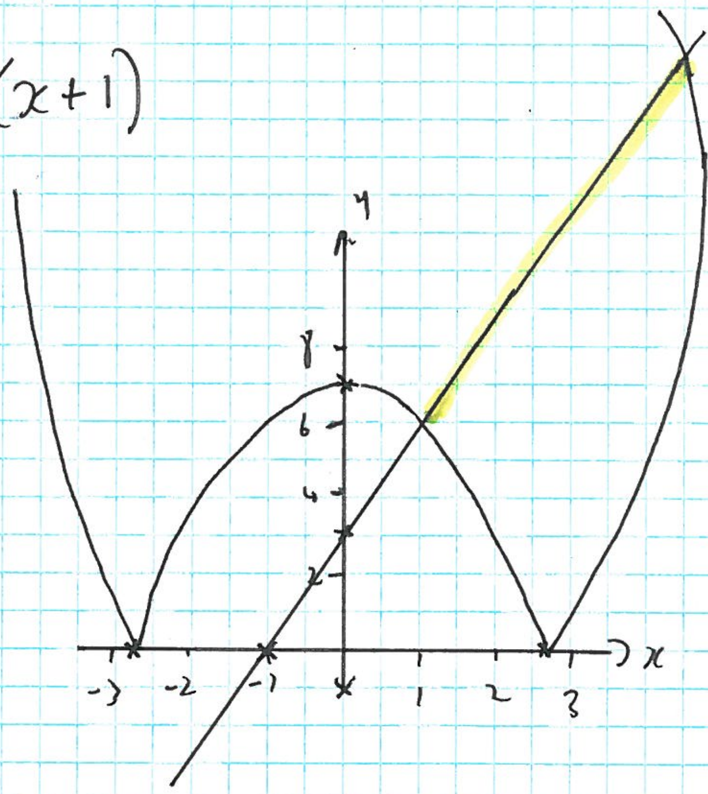
$y=0 \quad x=\pm\sqrt{7}$

$x=0 \quad y=-7$

$y = 3(x+1)$

$y=0 \quad x=-1$

$x=0 \quad y=3$



intersects when $x^2-7 = 3x+3$

$x^2-3x-10=0$

$x=5$ or $x=-2$

and when $-(x^2-7) = 3x+3$

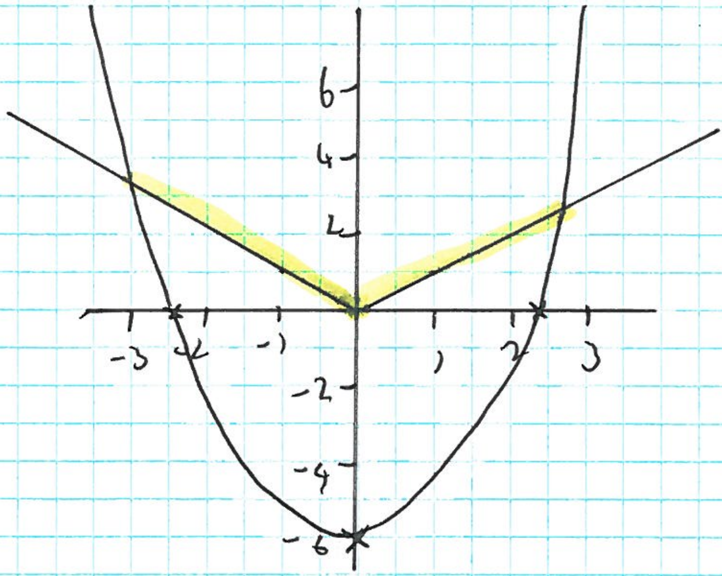
$x^2-7 = -3x-3$

$x^2+3x-4=0$

$x=1$ or $x=-4$

For highlighted region $1 < x < 5$

(2) $\dfrac{x^2}{|x|+6} < 1$



$x^2 < |x|+6$

$x^2-6 < |x|$

$y = x^2-6$

$y=0 \quad x = \pm\sqrt{6}$

$x=0 \quad y=-6$

intersect $\quad x^2-6 = -x$

$x^2+x-6=0$

$x = 2, -3$

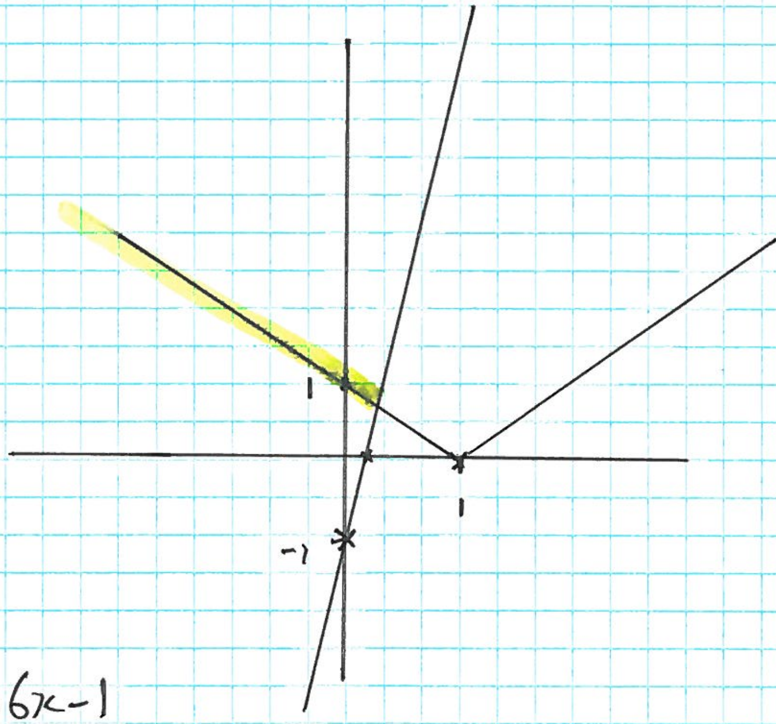
and $\quad x^2-6 = x$

$x^2-x-6=0$

$x = -2, 3$

$\therefore$ from graph $\quad -3 < x < 3$

(3) $|x-1| > 6x-1$



$y = x-1$

$y=0 \quad x=1$

$x=0 \quad y=-1$

$y = 6x-1$

$y=0 \quad x = \frac{1}{6}$

$x=0 \quad y=-1$

intersect when $\quad -(x-1) = 6x-1$

$-x+1 = 6x-1$

$7x = 2$

$x = \frac{2}{7}$

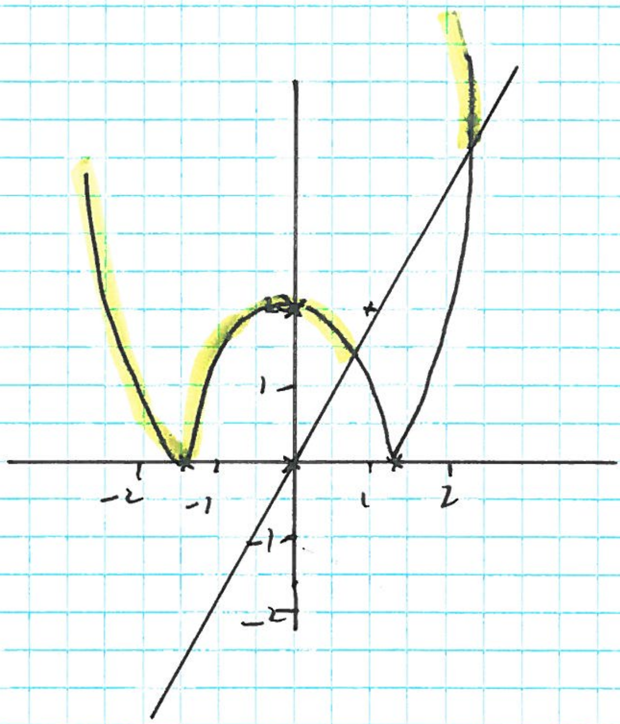
$\therefore \quad x < \frac{2}{7}$ * Wrong answer in textbook

④ $|x^2-2| > 2x$

$y = x^2-2$

$y=0 \quad x = \pm\sqrt{2}$

$x=0 \quad y=-2$



intersects when $x^2-2 = 2x$

$x^2-2x-2=0$

$(x-1)^2-3=0$

$x = 1\pm\sqrt{3}$ for us $x = 1+\sqrt{3}$

and when $-(x^2-2) = 2x$

$-x^2+2=2x$

$x^2+2x-2=0$

$(x+1)^2-3=0$

$x = -1\pm\sqrt{3}$ for us $x = -1+\sqrt{3}$

$\therefore \quad x < -1+\sqrt{3}$ or $x > 1+\sqrt{3}$

(5) $\dfrac{x+1}{2x-3} < \dfrac{1}{x-3}$

$\times (2x-3)^2(x-3)^2$

$(x+1)(2x-3)(x-3)^2 < (x-3)(2x-3)^2$

$(x+1)(2x-3)(x-3)^2 - (x-3)(2x-3)^2 < 0$

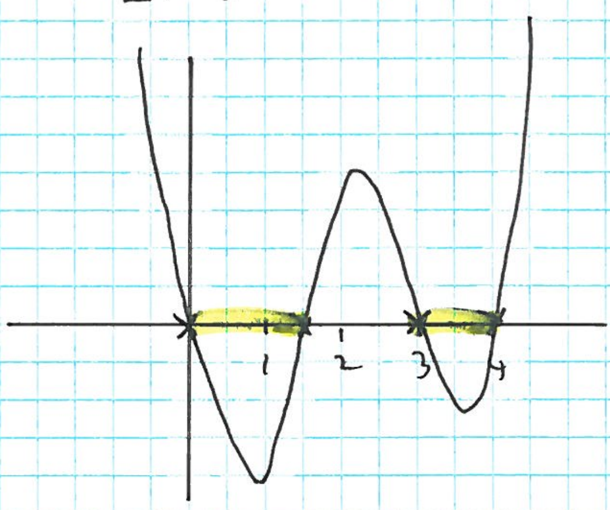
$(2x-3)(x-3)\left[(x+1)(x-3) - (2x-3)\right] < 0$

$(2x-3)(x-3)\left[x^2 - 2x - 3 - 2x + 3\right] < 0$

$(2x-3)(x-3)\left[x^2 - 4x\right] < 0$

$x(2x-3)(x-3)(x-4) < 0$

crosses x axis @ $x = 0, \frac{3}{2}, 3, 4$



$0 < x < \frac{3}{2}$ or $3 < x < 4$

(6) $\dfrac{(x+3)(x+9)}{x-1} > 3x-5$

$\times(x-1)^2$

$(x+3)(x+9)(x-1) > (3x-5)(x-1)^2$

$(x+3)(x+9)(x-1) - (3x-5)(x-1)^2 > 0$

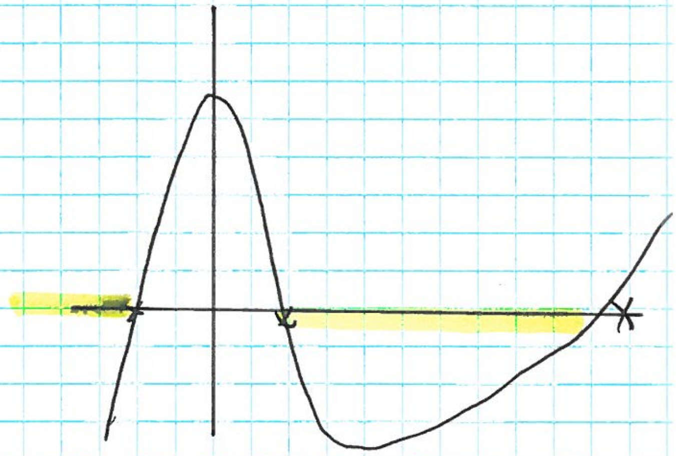
$(x-1)\left[x^2+12x+27-3x^2+8x-5\right] > 0$

$(x-1)\left[-2x^2+20x+22\right] > 0$

$\div -2$

$(x-1)\left[+x^2-10x-11\right] < 0$

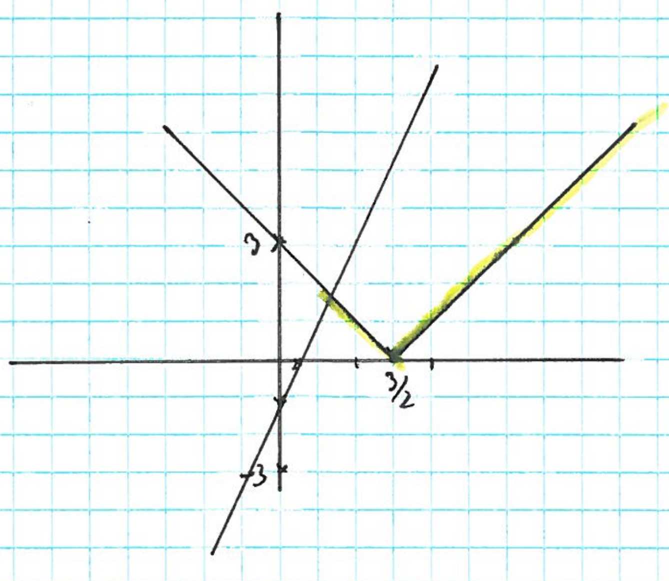
$(x-1)(x-11)(x+1) < 0$



$\therefore$ crosses x axis @ $-1, 1, 11$

$x < -1$ or $1 < x < 11$

(7) $y = |2x-3|$



$y = 2x-3$

when $y=0$ $x = 3/2$

when $x=0$ $y=-3$

$y = 5x-1$

$y=0$ $x = 1/5$

$x=0$ $y=-1$

critical pt when $-(2x-3) = 5x-1$

$-2x+3 = 5x-1$

$7x = 4$

$x = 4/7$

$x > 4/7$

8 (a) $|2x^2+x-6| = 6-3x$

Solution when $-(2x^2+x-6) = 6-3x$

$-2x^2-x+6 = 6-3x$

$2x^2-2x = 0$

$2x(x-1) = 0$

$x = 0$ or 1

and when $2x^2+x-6 = 6-3x$

$2x^2+4x-12 = 0$

$x^2+2x-6 = 0$

$(x+1)^2-7 = 0$

$x = -1 \pm \sqrt{7}$

(b) when $y = 2x^2+x-6$

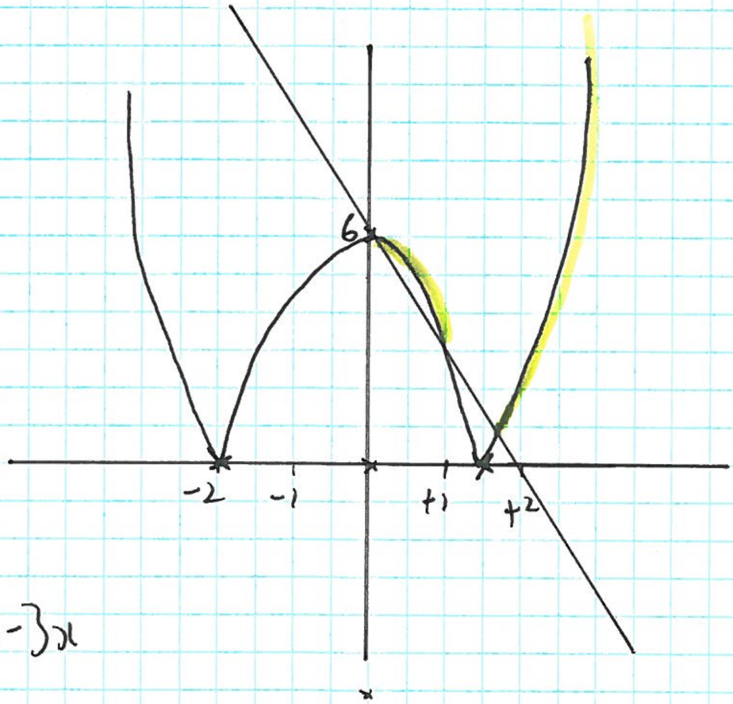
when $y=0$ $x = -2, \frac{3}{2}$

when $x=0$ $y=-6$

when $y = 6-3x$

when $y=0$ $x=2$

$x=0$ $y=6$



Solution for $|2x^2+x-6| > 6-3x$

either $0 < x < 1$

or $x > -1+\sqrt{7}$

or $x < -1-\sqrt{7}$

9

$y = |x^2 - 4|$

$y = x^2 - 4$

when $y = 0$ $x = \pm 2$

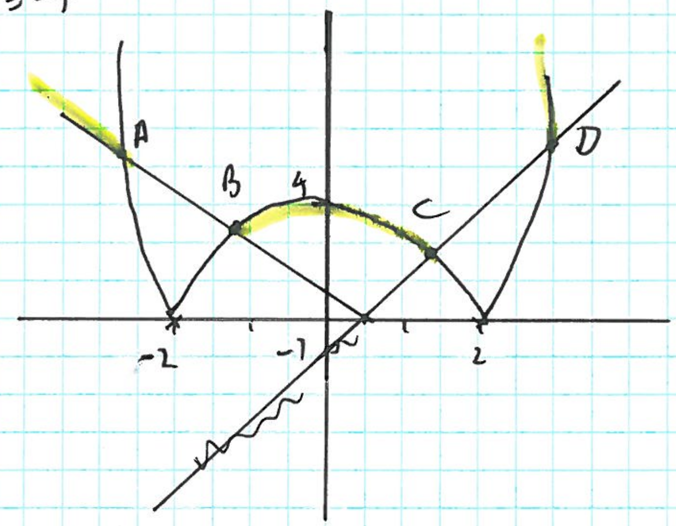
$x = 0$ $y = -4$

$y = |2x - 1|$

$y = 2x - 1$

when $y = 0$ $x = \frac{1}{2}$

$x = 0$ $y = -1$



intersect when (A) $x^2 - 4 = -(2x - 1)$

$x^2 - 4 = -2x + 1$

$x^2 + 2x - 5 = 0$

$(x+1)^2 - 6 = 0$

$x = -1 \pm \sqrt{6}$ @ (A) $-1 - \sqrt{6}$

(B) $-(x^2 - 4) = -(2x - 1)$

$x^2 - 4 = 2x - 1$

$x^2 - 2x - 3 = 0$

$(x-1)^2 - 4 = 0$

$x = 1 \pm 2$ @ (B) $x = -1$

(C) $-(x^2 - 4) = 2x - 1$

$-x^2 + 4 = 2x - 1$

$x^2 + 2x - 5 = 0$

$(x+1)^2 - 6 = 0$

$x = -1 \pm \sqrt{6}$ @ C $x = -1 + \sqrt{6}$

(D) $x^2 - 4 = 2x - 1$

$x^2 - 2x - 3 = 0$

$x = 3$

$\therefore$ Solutions for $|x^2 - 4| > |2x - 1|$

$x < -1 - \sqrt{6}$

or $-1 < x < -1 + \sqrt{6}$

or $x > 3$