Jones Sol's

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Friday 18 June 2010 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
H35386A
W850/R6666/57570 4/5/3/3
H35386A0132

Turn over

edexcel
advancing learning, changing lives

Leave blank

1.

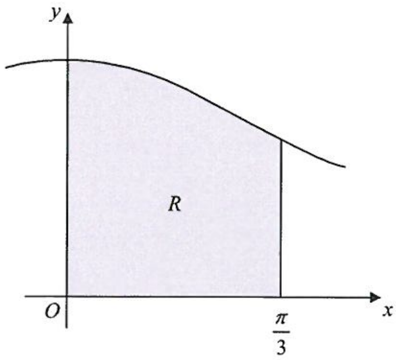


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973	1.2247	1.1180	1

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R. Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

Leave blank

Question 1 continued

(b) (i) $A = \frac{\pi/6}{2}\left[1.3229 + 2(1.2247) + 1\right]$

$= 1.249$

(ii) $A = \frac{\pi/12}{2}\left[1.3229 + 2(1.2973 + 1.2247 + 1.1180) + 1\right]$

$= 1.257$

H35386A0432

Leave blank

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e-1)$$

(6)

$u = \cos x + 1$ when $x = 0$ $u = 2$

$x = \frac{\pi}{2}$ $u = 1$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$

$\therefore I = \int_2^1 e^u \sin x . \frac{du}{-\sin x}$

$I = \int_2^1 -e^u \, du$

$I = \left[-e^u\right]_2^1 = -e^1 - -e^2$

$= e^2 - e$

$= e[e-1]$

H 3 5 3 8 6 A 0 6 3 2

Leave blank

3. A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

(7)

to diff $y = 2^x$ $\quad \ln y = \ln 2^x$

$\ln y = x \ln 2$

$\frac{1}{y}\frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$

$\therefore \quad 2^x + y^2 = 2xy$

$2^x \ln 2 + 2y\frac{dy}{dx} = 2x.1\frac{dy}{dx} + y.2$

$2^x \ln 2 + 2y\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$

$2^x \ln 2 - 2y = 2x\frac{dy}{dx} - 2y\frac{dy}{dx}$

$2^x \ln 2 - 2y = \frac{dy}{dx}[2x - 2y]$

@ $(3,2)$

$2^3 \ln 2 - 4 = \frac{dy}{dx}[6-4]$

$8\ln 2 - 4 = 2\frac{dy}{dx} \implies \frac{dy}{dx} = 4\ln 2 - 2$

H 3 5 3 8 6 A 0 8 3 2

Leave blank

4. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)

(a) $x = (\sin t)^2$

$\frac{dx}{dt} = 2\sin t . \cos t$

$y = 2\tan t$

$\frac{dy}{dt} = 2\sec^2 t$

$\frac{dy}{dx} = \frac{2\sec^2 t}{2\sin t \cos t} = \frac{\sec^2 t}{\sin t \cos t}$

(b) when $t = \frac{\pi}{3}$ $\quad x = \frac{3}{4} \quad y = 2\sqrt{3}$

eq^n of tangent thru $C(3/4, 2\sqrt{3})$

$y - 2\sqrt{3} = \frac{\sec^2(\pi/3)}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}}(x - 3/4)$

H35386A01232

Leave blank

Question 4 continued

$y - 2\sqrt{3} = \dfrac{4}{\frac{\sqrt{3}}{4}}\left(x - \frac{3}{4}\right)$

$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$

$y\sqrt{3} - 6 = 16x - 12$

$y\sqrt{3} = 16x - 6$

Now @ P, $y = 0$

$\therefore \quad 0 = 16x - 6$

$x = 3/8$

H35386A01332

Turn over

Leave blank

5. $$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)

(a) $\frac{A}{1} + \frac{B}{x-1} + \frac{C}{x+2}$

$\frac{A(x-1)(x+2) + B(x+2) + C(x-1)}{(x-1)(x+2)}$

$\frac{A(x^2+x-2) + Bx + 2B + Cx - C}{(x-1)(x+2)}$

Compare Coefficients: x^2: $A = 2$

x^1: $A + B + C = 5 \Rightarrow B + C = 3$ — ①

x^0: $-2A + 2B - C = -10 \Rightarrow 2B - C = -6$ — ②

① + ② $3b = -3$

$b = -1$

in ① $-1 + c = 3$

$c = 4$

$\therefore \frac{2x^2+5x-10}{(x-1)(x+2)} \equiv 2 - \frac{1}{x-1} + \frac{4}{x+2}$

H 3 5 3 8 6 A 0 1 6 3 2

Leave blank

Question 5 continued

(b) $\frac{1}{x-1} = (x-1)^{-1} = \left[-(1-x)^{-1}\right] = -1^{-1}(1-x)^{-1}$

$= -1\left[1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + \ldots\right]$

$= -1\left[1 + x + x^2 + \ldots\right]$

$\frac{1}{x+2} = (2+x)^{-1} = \left[2^{-1}(1+\frac{1}{2}x)^{-1}\right] = \frac{1}{2}\left[1+\frac{1}{2}x\right]^{-1}$

$= \frac{1}{2}\left[1 + (-1)(\frac{1}{2}x) + \frac{(-1)(-2)}{2!}(\frac{1}{2}x)^2 + \ldots\right.$

$= \frac{1}{2}\left[1 - \frac{1}{2}x + \frac{1}{4}x^2 + \ldots\right]$

$\therefore$ $2 - \frac{1}{x-1} + \frac{4}{x+2}$

$= 2 - -1\left[1+x+x^2+\ldots\right] + \frac{4}{2}\left[1 - \frac{1}{2}x + \frac{1}{4}x^2 + \ldots\right]$

$2 + 1 + x + x^2 + 2 - x + \frac{1}{2}x^2 + \ldots$

$5 + \frac{3}{2}x^2 + \ldots$

H35386A01732

Turn over

Leave blank

6.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\, d\theta$.

(7)

(a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - [1 - \cos^2\theta]$

$= 2\cos^2\theta - 1$

So $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\therefore f(\theta) = 4\left[\frac{1}{2} + \frac{1}{2}\cos 2\theta\right] - 3\left[\frac{1}{2} - \frac{1}{2}\cos 2\theta\right]$

$= 2 + 2\cos 2\theta - \frac{3}{2} + \frac{3}{2}\cos 2\theta$

$= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ As required.

Leave blank

Question 6 continued

(b) $\int_0^{\pi/2} \theta f(\theta)\, d\theta$ let $u = \theta$ $\frac{dv}{d\theta} = \frac{1}{2} + \frac{7}{2}\cos 2\theta$

$\frac{du}{d\theta} = 1$ $v = \frac{\theta}{2} + \frac{7}{4}\sin 2\theta$

$I = \frac{\theta^2}{2} + \frac{7\theta}{4}\sin 2\theta - \int \frac{\theta}{2} + \frac{7}{4}\sin 2\theta \, d\theta$

$= \left[\frac{\theta^2}{2} + \frac{7\theta}{4}\sin 2\theta - \frac{\theta^2}{4} + \frac{7}{8}\cos 2\theta\right]_0^{\frac{\pi}{2}}$

$= \frac{\frac{\pi^2}{4}}{2} + \frac{7}{4}\cdot\frac{\pi}{2}\cdot\sin\frac{2\pi}{2} - \frac{\frac{\pi^2}{4}}{4} + \frac{7}{8}\cos\frac{2\pi}{2} - \left[0 + 0 - 0 + \frac{7}{8}\cos 0\right]$

$= \frac{\pi^2}{16} + 0 - \frac{7}{8} - \frac{7}{8}$

$= \frac{\pi^2}{16} - \frac{14}{8}$

$= \frac{\pi^2}{16} - \frac{7}{4}$

H 3 5 3 8 6 A 0 2 1 3 2

Turn over

Leave blank

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)

(a) @C $2+\lambda = 5\mu$ — ①

$3+2\lambda = 9$ — ②

From ① $\lambda = 5\mu - 2$ — ③

in ② $3 + 2(5\mu - 2) = 9$

$10\mu - 4 = 6$

$\mu = 1$

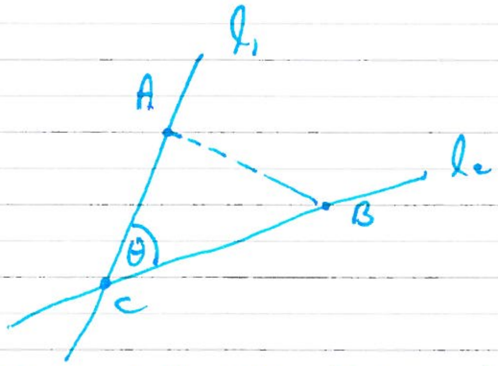
in ③ $\lambda = 3$

$\therefore$ @C $\mathbf{r}_c = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ -1 \end{pmatrix}$

Coords $(5, 9, -1)$

Question 7 continued

(b) $r_A = \begin{pmatrix}2\\3\\-4\end{pmatrix}$ $r_B = \begin{pmatrix}0\\9\\-3\end{pmatrix} + -1\begin{pmatrix}5\\0\\2\end{pmatrix} = \begin{pmatrix}-5\\9\\-5\end{pmatrix}$



$\vec{AC} = -\begin{pmatrix}2\\3\\-4\end{pmatrix} + \begin{pmatrix}5\\9\\-1\end{pmatrix} = \begin{pmatrix}3\\6\\3\end{pmatrix}$ $|\vec{AC}| = \sqrt{3^2+6^2+3^2} = \sqrt{54}$

$\vec{BC} = -\begin{pmatrix}-5\\9\\-5\end{pmatrix} + \begin{pmatrix}5\\9\\-1\end{pmatrix} = \begin{pmatrix}10\\0\\4\end{pmatrix}$ $|\vec{BC}| = \sqrt{10^2+0^2+4^2} = \sqrt{116}$

$\vec{AB} = -\begin{pmatrix}2\\3\\-4\end{pmatrix} + \begin{pmatrix}-5\\9\\-5\end{pmatrix} = \begin{pmatrix}-7\\6\\-1\end{pmatrix}$ $|AB| = \sqrt{7^2+6^2+1^2} = \sqrt{86}$

Now using Cos Rule: $|\vec{AB}|^2 = |\vec{AC}|^2 + |\vec{BC}|^2 - 2|AC||BC|\cos\theta$

$86 = 54 + 116 - 2(\sqrt{54}\sqrt{116})\cos\theta$

$\cos\theta(2\sqrt{6264}) = 54 + 116 - 86$

$\cos\theta = \frac{42}{\sqrt{6264}}$ $\theta = 57.95°$

Turn over

H35386A02532

Leave blank

Question 7 continued

(c) Area $\Delta ABC = \frac{1}{2}|\vec{AC}||\vec{BC}|\sin\theta$

$= \frac{1}{2}\sqrt{54}\sqrt{116}\sin(57.9°)$

$= 33.5$

Leave blank

8.

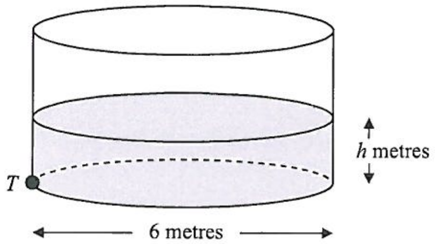


Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that t minutes after the tap has been opened

$$75\frac{dh}{dt} = (4 - 5h)$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

(a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$

$V = \pi r^2 h = 9\pi h$

$\therefore \frac{dV}{dh} = 9\pi$

Now $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

H 3 5 3 8 6 A 0 2 8 3 2

Question 8 continued

So $\frac{dh}{dt} = \frac{1}{9\pi} \times (0.48\pi - 0.6\pi h)$

$\frac{dh}{dt} = \frac{4}{75} - \frac{h}{15}$

$\frac{dh}{dt} = \frac{4}{75} - \frac{5h}{75}$

$\therefore 75\frac{dh}{dt} = 4 - 5h$ as required.

(b) $\int \frac{75}{4-5h}\,dh = \int dt$

$\frac{75}{-5}\ln(4-5h) = t + c$

when $t=0$, $h = 0.2$

$c = -15\ln 3$

$\therefore -15\ln(4-5h) = t - 15\ln 3$

$\therefore t = 15\ln 3 - 15\ln(4-5h)$

When $h = 0.5$

$t = 15\ln 3 - 15\ln(1.5)$

$t = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$

Leave blank

H35386A02932

Turn over