Jones Sch

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Thursday 12 June 2008 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
H30427A
W850/R6666/57570 3/3/3/3/

H 3 0 4 2 7 A 0 1 2 8

Turn over

edexcel
advancing learning, changing lives

Leave blank

1.

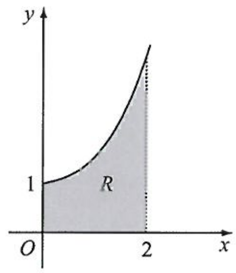


Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

$A \approx \frac{0.4}{2}\left[e^0 + 2\left[e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right] + e^2\right]$

≈ 4.922 (4s.f.)

H30427A0228

Leave blank

2. (a) Use integration by parts to find $\int x\,e^x\,dx$.

(3)

(b) Hence find $\int x^2\,e^x\,dx$.

(3)

(a) $u = x$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 1$ $\quad v = e^x$

$I = xe^x - \int e^x\,dx$

$= xe^x - e^x + c$

(b) $u = x^2$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 2x$ $\quad v = e^x$

$I = x^2e^x - 2\int xe^x\,dx$

$I = x^2e^x - 2[xe^x - e^x] + c$

$I = x^2e^x - 2xe^x + 2e^x + c$

Leave blank

3.

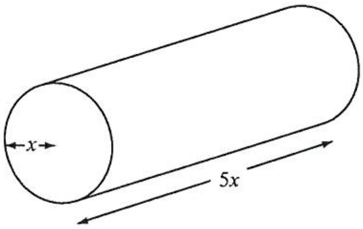


Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of 0.032 cm^2 s^{-1}.

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

$\frac{dA}{dt} = 0.032$

$A = \pi x^2$ $\quad V = \pi x^2 \times 5x$

$V = 5\pi x^3$

(a) $\frac{dx}{dt} = \frac{dx}{dA} \cdot \frac{dA}{dt}$

Now $\frac{dA}{dx} = 2\pi x$

$\therefore \frac{dx}{dt} = \frac{1}{2\pi x} \cdot 0.032 = \frac{0.016}{\pi x}$

when $x=2$ $\quad \frac{dx}{dt} = \frac{0.016}{2\pi} = 0.0255$

H30427A0628

Leave blank

Question 3 continued

(b) $\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$\frac{dV}{dx} = 15\pi x^2$

$\therefore \frac{dV}{dt} = 15\pi x^{\cancel{2}} \cdot \frac{0.016}{\cancel{\pi x}} = 0.24x$

When $x = 2$ $\frac{dV}{dt} = 0.48$

H30427A0728

Leave blank

4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

(a) $6x - 2y\frac{dy}{dx} + \left(x.1\frac{dy}{dx} + y.1\right) = 0$

$6x - 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = 6x + y$

$\frac{dy}{dx} = \frac{6x+y}{2y-x}$

Now @ P+Q $\frac{dy}{dx} = \frac{8}{3}$

$\therefore \frac{6x+y}{2y-x} = \frac{8}{3}$

$3(6x+y) = 8(2y-x)$

$18x + 3y = 16y - 8x$

$13y - 26x = 0$

$y - 2x = 0$ As required.

Question 4 continued

(b) $3x^2 - y^2 + xy = 4$ — (1)

$y - 2x = 0$ — (2)

from (2) $y = 2x$ — (3)

in (1) $3x^2 - 4x^2 + 2x^2 = 4$

$x^2 = 4$

$x = \pm 2$

in (3) $y = \pm 4$

$\therefore$ $P(2, 4)$ $Q(-2, -4)$

Leave blank

5. (a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

(a) $\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}}$

$= \left[4\left(1-\frac{3}{4}x\right)\right]^{-\frac{1}{2}}$

$= 4^{-\frac{1}{2}}\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{3}{4}x\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{4}x\right)^2 / 2! + \dots\right.$

$= \frac{1}{2}\left[1 + \frac{3}{8}x + \frac{27}{128}x^2 + \dots\right]$

$= \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots$

(b) $\frac{x+8}{\sqrt{(4-3x)}} = (x+8)\left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots\right)$

$= \frac{1}{2}x + \frac{3}{16}x^2 + \frac{27}{256}x^3 + 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \dots$

$= 4 + 2x + \frac{33}{32}x^2 + \dots$

Leave blank

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection. **(6)**

(b) Show that l_1 and l_2 are perpendicular to each other. **(2)**

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1. **(1)**

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B. **(3)**

$l_1: \quad r = \begin{pmatrix} -9+2\lambda \\ \lambda \\ 10-\lambda \end{pmatrix}$

$l_2: \quad r = \begin{pmatrix} 3+3\mu \\ 1-\mu \\ 17+5\mu \end{pmatrix}$

if lines meet then $\quad -9+2\lambda = 3+3\mu \quad$ — (1)

$\lambda = 1-\mu \quad$ — (2)

$10-\lambda = 17+5\mu \quad$ — (3)

Sub (2) in (1) $\quad -9+2-2\mu = 3+3\mu$

$-10 = 5\mu$

$\mu = -2$

Question 6 continued

in ② $\lambda = 1 - -2 = 3$

Sub for μ & λ in ③

$10 - 3 = 17 + 5 \times -2$

$7 = 17 - 10$

$7 = 7$ ✓

$\therefore$ lines do meet @ $r = \begin{pmatrix} -9+6 \\ 3 \\ 10-3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$

(b) if l_1 & l_2 are perp then

$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = 0$

$6 - 1 - 5 = 0$ true $\therefore$ lines are perp.

(c) $\vec{OA} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$

if lies on l_1 then $\begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -9+2\lambda \\ \lambda \\ 10-\lambda \end{pmatrix}$ — ① — ② — ③

for some λ : from ② $\lambda = 7$ ✓

in ① $5 = -9 + 14 = 5$ ✓

in ③ $3 = 10 - 7 = 3$ ✓

H30427A01728

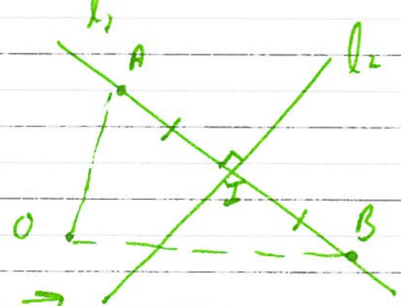
Turn over

Leave blank

Question 6 continued

Leave blank

(d) If intersection of l_1 & l_2 $I = \begin{pmatrix}-3\\3\\7\end{pmatrix}$



$\vec{AB} = 2\vec{AI}$

$\vec{OB} = \vec{OA} + \vec{AB}$

$= \vec{OA} + 2\vec{AI}$ Now $\vec{AI} = -\vec{OA} + \vec{OI}$

$= -\begin{pmatrix}5\\7\\3\end{pmatrix} + \begin{pmatrix}-3\\3\\7\end{pmatrix}$

$= \begin{pmatrix}-8\\-4\\4\end{pmatrix}$

$\therefore \vec{OB} = \begin{pmatrix}5\\7\\3\end{pmatrix} + 2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$

$= \begin{pmatrix}-11\\-1\\11\end{pmatrix}$

H30427A01828

Leave blank

7. (a) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

7(a) $\dfrac{2}{4-y^2} = \dfrac{2}{(2-y)(2+y)}$

$= \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$

$= \dfrac{A(2+y) + B(2-y)}{(2-y)(2+y)}$

Compare Coeff's: $y^1: \quad A - B = 0 \implies A = B$

$y^0: \quad 2A + 2B = 2$

$2A + 2A = 2$

$4A = 2$

$A = \frac{1}{2}$

$\therefore \dfrac{1}{2(2-y)} + \dfrac{1}{2(2+y)}$

H 3 0 4 2 7 A 0 2 0 2 8

Leave blank

Question 7 continued

(b) $2\cot x \frac{dy}{dx} = (4-y^2)$

$\int \frac{2}{4-y^2}\,dy = \int \tan x\,dx$

$\cancel{\tfrac{1}{2}}\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln|\sec x|$

$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln|\sec x| + c$

$y=0 \quad x=\frac{\pi}{3}$

$\frac{1}{2}\ln 1 = \ln\left|\sec\frac{\pi}{3}\right| + c$

$0 = \ln 2 + c$

$c = -\ln 2$

$\therefore \frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln|\sec x| - \ln 2$

$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$

$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$

H30427A02128

Turn over

Leave blank

Question 7 continued

$$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec^2 x}{4}\right)$$

$$\left(\frac{2+y}{2-y}\right) = \frac{\sec^2 x}{4}$$

$$\therefore \sec^2 x = 4\left(\frac{2+y}{2-y}\right)$$

8.

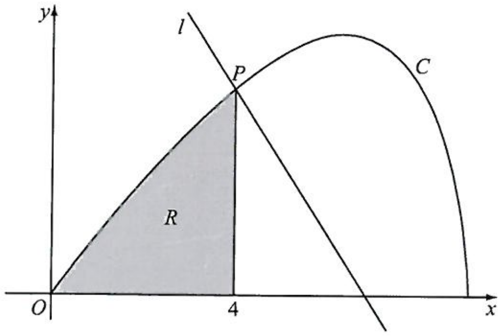


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \tfrac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(a) @ P $\quad 4 = 8\cos t$

$\cos t = \frac{1}{2}$

$t = \frac{\pi}{3}$

H30427A02428

Leave blank

Question 8 continued

(b) $\frac{dx}{dt} = -8\sin t$

$\frac{dy}{dt} = 8\cos 2t$

$\frac{dy}{dx} = \frac{8\cos 2t}{-8\sin t} = -\frac{\cos 2t}{\sin t}$

$M_{norm} = +\frac{\sin t}{\cos 2t}$

eq'n of norm @ P

$y - 2\sqrt{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{2\pi}{3}}(x-4)$

$y - 2\sqrt{3} = \frac{\sqrt{3}/2}{-0.5}(x-4)$

$y - 2\sqrt{3} = -\sqrt{3}(x-4)$

$y - 2\sqrt{3} = -\sqrt{3}x + 4\sqrt{3}$

$y = -x\sqrt{3} + 6\sqrt{3}$ As required.

Leave blank

Question 8 continued

(c) $A = \int y \frac{dx}{dt} . dt$

$= \int 4\sin 2t . -8\sin t \, dt$

$= \int 4(2\sin t \cos t) . -8\sin t \, dt$

$= \int -64\sin^2 t \cos t \, dt$

limits when $x = 0$ $\quad 0 = 8\cos t$

$t = \frac{\pi}{2}$

when $x = 4$ $\quad 4 = 8\cos t$

$\cos t = \frac{1}{2}$

$t = \frac{\pi}{3}$

$\therefore A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64\sin^2 t \cos t \, dt$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} +64\sin^2 t \cos t \, dt$ As required

Question 8 continued

(d) ~~$A = 64\int_{\pi/3}^{\pi/2}\left[\frac{1}{2} - \frac{1}{2}\cos 2t\right]$~~

let $u = \sin t$ $\quad \frac{du}{dt} = \cos t$

When $t = \frac{\pi}{3}$ $\quad u = \frac{\sqrt{3}}{2}$

$t = \frac{\pi}{2}$ $\quad u = 1$ $\quad dt = \frac{du}{\cos t}$

$$64\int_{\sqrt{3}/2}^{1} u^2 . \cancel{\cos t}\,\frac{du}{\cancel{\cos t}}$$

$$64\left[\frac{u^3}{3}\right]_{\sqrt{3}/2}^{1}$$

$$64\left[\frac{1}{3} - \frac{\left(\frac{\sqrt{3}}{2}\right)^3}{3}\right]$$

$$64\left[\frac{1}{3} - \frac{3\sqrt{3}}{24}\right]$$

$$= \frac{64}{3} - 8\sqrt{3}.$$

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

Q8

H30427A02728