Jones Sols

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 18 June 2007 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$\mathrm{f}(x) = (3+2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

(5)

$$f(x) = 3^{-3}\left(1 + \frac{2x}{3}\right)^{-3}$$

$$= \frac{1}{27}\left[1 + (-3)\left(\frac{2}{3}x\right) + \frac{(-3)(-4)}{2!}\left(\frac{2}{3}x\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{2}{3}x\right)^3 + \ldots\right]$$

$$= \frac{1}{27}\left[1 - 2x + \frac{8}{3}x^2 - \frac{80}{27}x^3 + \ldots\right]$$

$$= \frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3 + \ldots$$

Leave blank

2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx.$$

(6)

$u = 2^x$

when $x=0$ $u=1$

$x=1$ $u=2$

$\ln u = x\ln 2$

$\frac{1}{u}\frac{du}{dx} = \ln 2$

$\frac{du}{dx} = u\ln 2$

$dx = \frac{du}{u\ln 2}$

$\therefore I = \int_1^2 \frac{\cancel{u}}{(u+1)^2}\,\frac{du}{\cancel{u}\ln 2}$

$I = \frac{1}{\ln 2}\int_1^2 (u+1)^{-2}\,du$

$I = \frac{1}{\ln 2}\left[-\frac{1}{u+1}\right]_1^2$

$I = \frac{1}{\ln 2}\left[-\frac{1}{3} - -\frac{1}{2}\right]$

$I = \frac{1}{6\ln 2}$

Leave blank

3. (a) Find $\int x\cos 2x \, dx$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x\cos^2 x \, dx$.

(3)

(a) $\int x \cos 2x \, dx$ $\quad u = x \quad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 1 \quad v = \frac{1}{2}\sin 2x$

$I = \frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x \, dx$

$I = \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$

(b) $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

$\therefore \int x\cos^2 x \, dx = \int x\left[\frac{1}{2} + \frac{1}{2}\cos 2x\right] dx$

$= \int \frac{x}{2} \, dx + \frac{1}{2}\int x\cos 2x \, dx$

$= \frac{x^2}{4} + \frac{1}{2}\left[\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\right] + c$

$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + c$

Leave blank

$8x^2+2$

4. $$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

$\frac{A}{1}$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx$ is $2 + \ln k$, giving the value of the constant k.

(6)

(a) $$\frac{A(2x+1)(2x-1) + B(2x-1) + C(2x+1)}{(2x+1)(2x-1)}$$

$$\frac{A(4x^2-1) + B(2x-1) + C(2x+1)}{(2x+1)(2x-1)}$$

Compare Coeficients

x^2: $4A = 8 \Rightarrow A = 2$

x: $2B + 2C = 0 \Rightarrow B = -C$

x^0: $-A - B + C = 2$

$-2 + C + C = 2$

$2C = 4$

$C = 2 \Rightarrow B = -2$

$$\therefore \frac{2(4x^2+1)}{(2x+1)(2x-1)} = 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}$$

Leave blank

Question 4 continued

(b) $I = \int_1^2 2 - \frac{2}{2x+1} + \frac{2}{2x-1} \, dx$

$= 2\left[x - \frac{1}{2}\ln|2x+1| + \frac{1}{2}\ln|2x-1|\right]_1^2$

$= \left[2x - \ln|2x+1| + \ln|2x-1|\right]_1^2$

$= \left[2x + \ln\left|\frac{2x-1}{2x+1}\right|\right]_1^2$

$= 4 + \ln\left(\frac{3}{5}\right) - 2 - \ln\left(\frac{1}{3}\right)$

$= 2 + \ln\left(\frac{3/5}{1/3}\right)$

$= 2 + \ln\left(\frac{9}{5}\right)$

Q4

(Total 10 marks)

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5.

The line l_1 has equation $\mathbf{r} = \begin{pmatrix}1\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\0\end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix}1\\3\\6\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$.

(a) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(b) Find the cosine of the acute angle between AB and l_1.

(6)

(a) $l_1 \quad r = \begin{pmatrix}1+\lambda\\ \lambda\\ -1\end{pmatrix}$

$l_2 \quad r = \begin{pmatrix}1+2\mu\\ 3+\mu\\ 6-\mu\end{pmatrix}$

If lines meet then $1+\lambda = 1+2\mu$ — ①

$\lambda = 3+\mu$ — ②

$-1 = 6-\mu$ — ③

will be consistent for each eqn

from ③ $\mu = 7$

in ② $\lambda = 3+7 = 10$

in ① $1+10 = 1+2(7)$

$11 = 15$ Not true so lines don't meet.

Leave blank

Question 5 continued

$$r_A = \begin{pmatrix}1\\0\\-1\end{pmatrix} + 1\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$$

$$r_B = \begin{pmatrix}1\\3\\6\end{pmatrix} + 2\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}5\\5\\4\end{pmatrix}$$

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} = -\begin{pmatrix}2\\1\\-1\end{pmatrix} + \begin{pmatrix}5\\5\\4\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix}$$

Angle between $\overrightarrow{AB}$ + l_1 will be angle between $\overrightarrow{AB}$ + direct vect of L_1 $\begin{pmatrix}1\\1\\0\end{pmatrix}$

$$\therefore \begin{pmatrix}3\\4\\5\end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\end{pmatrix} = \sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+0^2}\,\cos\theta$$

$$3+4+0 = \sqrt{50}.\sqrt{2}\,\cos\theta$$

$$\therefore \frac{7}{10} = \cos\theta$$

Q5

(Total 10 marks)

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6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer. **(3)**

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined. **(5)**

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$. **(4)**

(a) $x = (\tan t)^2$

$\frac{dx}{dt} = 2\tan t . \sec^2 t$

$y = \sin t$

$\frac{dy}{dt} = \cos t$

Now $\frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t}$.

(b) @ $t = \frac{\pi}{4}$ $\quad x = \left(\tan\frac{\pi}{4}\right)^2 = 1 \quad y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\left(\sec\frac{\pi}{4}\right)^2} = \frac{\sqrt{2}/2}{2(1).\left(\frac{\sqrt{2}}{1}\right)^2}$

$= \frac{\frac{\sqrt{2}}{2}}{4}$

$= \frac{\sqrt{2}}{8}.$

Question 6 continued

(b) Now eq^n of tangent

$$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{8}(x-1)$$

$$8y - 4\sqrt{2} = \sqrt{2}x - \sqrt{2}$$

$$8y = \sqrt{2}x + 3\sqrt{2}$$

$$y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$$

(c) $x = \tan^2 t$

$$x = \frac{\sin^2 t}{\cos^2 t}$$

Now $\sin^2 t = y^2$

& $\sin^2 t + \cos^2 t = 1$

$$y^2 + \cos^2 t = 1$$

$$\cos^2 t = 1 - y^2$$

$\therefore x = \frac{y^2}{1-y^2}$

$x(1-y^2) = y^2$

$x - xy^2 = y^2$

$x = y^2 + xy^2$

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Question 6 continued

$$y^2(1+x) = x$$

$$y^2 = \frac{x}{1+x}$$

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7.

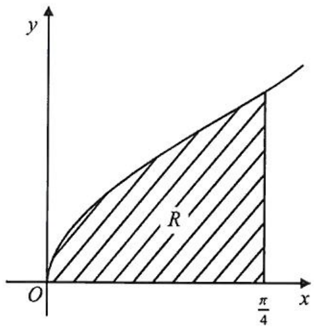

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}$, $\frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0	0.44600	0.64359	0.81742	1

(3)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(c) Use integration to find an exact value for the volume of the solid generated.

(4)

(b) $A = \frac{\frac{\pi}{16}}{2}\left[0 + 2(0.44600 + 0.64359 + 0.81742) + 1\right]$

$= 0.4726$ to 4dp.

N 2 6 1 1 0 A 0 1 6 2 4

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Question 7 continued

(c) $V = \pi \int_0^{\pi/4} y^2 \, dx$

$V = \pi \int_0^{\pi/4} (\sqrt{\tan x})^2 \, dx$

$V = \pi \int_0^{\pi/4} \tan x \, dx$

$V = \pi \Big[\ln|\sec x| \Big]_0^{\pi/4}$

$V = \pi \Big[\ln\left(\sec \frac{\pi}{4}\right) - \ln(\sec 0) \Big]$

$V = \pi \Big[\ln(\sqrt{2}) - \ln(1) \Big]$

$V = \pi . \ln 2^{\frac{1}{2}} - 0$

$= \frac{\pi}{2} \ln 2$

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8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.
(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.
(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.
(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.
(3)

(a) $\int \frac{1}{P}\, dP = \int k\, dt$

$\ln P = kt + c$

when $t=0$, $P=P_0$ $\therefore$ $\ln P_0 = c$

$\ln P = kt + \ln P_0$

$\ln\left(\frac{P}{P_0}\right) = kt$

$\frac{P}{P_0} = e^{kt} \Rightarrow P = P_0 e^{kt}$

Leave blank

Question 8 continued

(b) if $k = 2.5$ and $P = 2P_0$

$2P_0 = P_0 e^{2.5t}$

$2 = e^{2.5t}$

$\ln 2 = 2.5t$

$t = \frac{\ln 2}{2.5} = 0.277\ldots$ days

$\times 24$ hours

$\times 60$ mins

$= 399$ minutes

(c) $\frac{dP}{dt} = \lambda P \cos \lambda t$

$\int \frac{1}{P}\, dP = \int \lambda \cos \lambda t \, dt$

$\ln P = \frac{\lambda}{\lambda} \sin \lambda t + c$

$\ln P = \sin \lambda t + c$

when $t = 0$, $P = P_0$

$\ln P_0 = c$

$\therefore \ln P = \sin \lambda t + \ln P_0$

Question 8 continued

$\ln \frac{P}{P_0} = \sin \lambda t$

$\frac{P}{P_0} = e^{\sin \lambda t}$

$P = P_0 e^{\sin \lambda t}$

(d) Given $\lambda = 2.5$ $P = 2P_0$

$2P_0 = P_0 e^{\sin 2.5t}$

$2 = e^{\sin 2.5t}$

$\ln 2 = \sin 2.5t$

$2.5t = \sin^{-1}(\ln 2)$

$t = \frac{1}{2.5} \sin^{-1}(\ln 2)$

$t = 0.30633\ldots$ days

$\times 24$ hours

$\times 60$ mins

$= 441$ minutes.