Jones Sfb

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Thursday 15 June 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

Printer's Log. No.
N23563A
W850/R6666/57570 3/3/3/3/3/3/

N 2 3 5 6 3 A 0 1 2 0

Turn over

edexcel

1. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

$6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$

$6x + 2 = \frac{dy}{dx}(4y + 3)$

$\frac{dy}{dx} = \frac{6x+2}{4y+3}$

@ $(0,1)$ $\quad \frac{dy}{dx} = \frac{2}{7}$

$\therefore M_{norm} = -\frac{7}{2}$

eq^n of norm thru $(0,1)$

$y - 1 = -\frac{7}{2}(x - 0)$

$2y - 2 = -7x$

$7x + 2y - 2 = 0$

Leave blank

N 2 3 5 6 3 A 0 2 2 0

Leave blank

2. $$f(x) = \frac{3x-1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(a) find the values of A and B.

(3)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)

(a) $$\frac{A(1-2x)+B}{(1-2x)^2}$$

$$\frac{A-2Ax+B}{(1-2x)^2}$$

Compare Coeficients $\quad -2A = 3$

$$A = -\frac{3}{2}$$

$$A+B = -1$$

$$-\frac{3}{2} + B = -1$$

$$B = \frac{1}{2}$$

$$\therefore \quad f(x) = \frac{1}{2(1-2x)^2} - \frac{3}{2(1-2x)}$$

$$= \frac{1}{2}\left[(1-2x)^{-2} - 3(1-2x)^{-1}\right]$$

Leave blank

Question 2 continued

(b) $(1-2x)^{-2} = 1 + (-2)(-2x) + \frac{(-2)(-3)(-2x)^2}{2!} + \frac{(-2)(-3)(-4)(-2x)^3}{3!} + \ldots$

$= 1 + 4x + 12x^2 + 32x^3 + \ldots$

$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \ldots$

$= 1 + 2x + 4x^2 + 8x^3 + \ldots$

$\therefore f(x) \simeq \frac{1}{2}\left[1+4x+12x^2+32x^3 - 3(1+2x+4x^2+8x^3)\right]$

$\simeq \frac{1}{2}\left[1+4x+12x^2+32x^3 - 3 - 6x - 12x^2 - 24x^3\right]$

$\simeq \frac{1}{2}\left[-2-2x+8x^3\right]$

$= 4x^3 - x - 1.$

Q2

(Total 9 marks)

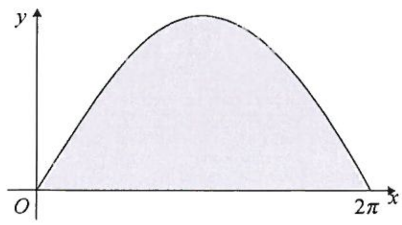
N 2 3 5 6 3 A 0 5 2 0

Turn over

Leave blank

3.

Figure 1



The curve with equation $y = 3\sin\frac{x}{2}$, $0 \leqslant x \leqslant 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)

(a) $A = \int_0^{2\pi} 3\sin\frac{x}{2}\,dx$

$A = 3\int_0^{2\pi} \sin\frac{x}{2}\,dx$

$A = 3\left[-2\cos\frac{x}{2}\right]_0^{2\pi}$

$= -6\left[\cos\pi - \cos 0\right]$

$= -6[-1-1]$

$= 12$

Leave blank

Question 3 continued

(b) $V = \pi \int_0^{2\pi} y^2 \, dx$

$= \pi \int_0^{2\pi} \left(3 \sin \frac{x}{2}\right)^2 dx$

$= 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx$

$= 9\pi \int_0^{2\pi} \frac{1}{2} - \frac{1}{2} \cos 2\left(\frac{x}{2}\right) dx$

$= 9\pi \int_0^{2\pi} \frac{1}{2} - \frac{1}{2} \cos x \, dx$

$= 9\pi \left[\frac{x}{2} - \frac{1}{2} \sin x \right]_0^{2\pi}$

$= 9\pi \left[\pi - \frac{1}{2} \sin 2\pi - (0 - 0) \right]$

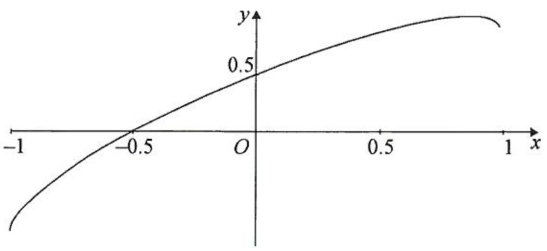
$= 9\pi^2$

Q3

(Total 9 marks)

4.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = \sin t,\ \ y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

(a) $\frac{dx}{dt} = \cos t \qquad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$

$\frac{dy}{dt} = \frac{\cos\left(t + \frac{\pi}{6}\right)}{\cos t}$

When $t = \frac{\pi}{6}$, $x = \sin\frac{\pi}{6} = \frac{1}{2}$

$y = \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Question 4 continued

So eqⁿ of tangent thru $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$

$\times\sqrt{3}$

$$y\sqrt{3} - \frac{3}{2} = x - \frac{1}{2}$$

$\cancel{x = y\sqrt{3}}$ $\cancel{y = \frac{1}{\sqrt{3}}x}$ $y\sqrt{3} = x + 1$

(b) $y = \sin\left(t + \frac{\pi}{6}\right)$

$$y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$$

$$y = \sin t \frac{\sqrt{3}}{2} + \frac{1}{2}\cos t$$

Now if $x = \sin t$ $\quad x^2 = \sin^2 t$

$x^2 = 1 - \cos^2 t$

$\therefore \cos^2 t = 1 - x^2$

$\cos t = \sqrt{1 - x^2}$

$\therefore y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1 - x^2}$ As required.

Leave blank

Q4

(Total 9 marks)

Leave blank

5. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

(a) $r = \begin{pmatrix} 6 \\ 19 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ a \\ b \end{pmatrix}$

$6 + \lambda = 0 \Rightarrow \lambda = -6$

$19 + 4\lambda = a \Rightarrow 19 - 24 = a$

$a = -5$

$-1 - 2\lambda = b \Rightarrow -1 + 12 = b$

$b = 11$

(b) $\vec{OP} \cdot$ direct $l_1 = 0$

$\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0$

$(6+\lambda) + 4(19+4\lambda) - 2(-1-2\lambda) = 0$

$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$

$21\lambda + 84 = 0 \Rightarrow \lambda = -4$

N 2 3 5 6 3 A 0 1 0 2 0

Question 5 continued

(b) $r_P = \begin{pmatrix} 6-4 \\ 19+4(-4) \\ -1-2(-4) \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$

(c) already know that A+P lie on l_1, so if B lies on l_1 then the points are collinear.

$\begin{pmatrix} 5 \\ 15 \\ 1 \end{pmatrix} = \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix}$ $\quad \lambda = -1$, $\lambda = -1$, $\lambda = -1$

λ consistent for B $\therefore$ points are collinear.

$\overrightarrow{AP} = -r_A + r_P = -\begin{pmatrix} 0 \\ -5 \\ 11 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$

$|\overrightarrow{AP}| = \sqrt{2^2 + 8^2 + -4^2} = \sqrt{84} = 2\sqrt{21}$

$\overrightarrow{PB} = -r_P + r_B = -\begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 5 \\ 15 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \\ -6 \end{pmatrix}$

$|\overrightarrow{PB}| = \sqrt{3^2 + 12^2 + -6^2} = \sqrt{189} = 3\sqrt{21}$

$\therefore$ AP : PB $\quad$ ~~$\sqrt{84} : 3\sqrt{21}$~~

$2\sqrt{21} : 3\sqrt{21} \Rightarrow 2 : 3$

N23563A01120

Turn over

Leave blank

6.

Figure 3

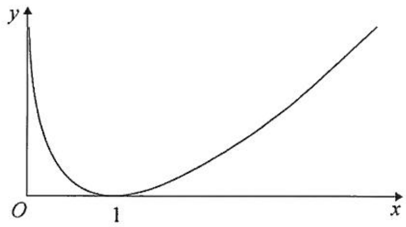


Figure 3 shows a sketch of the curve with equation $y = (x-1)\ln x, \quad x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0	$0.5\ln 1.5$	$\ln 2$	$1.5\ln 2.5$	$2\ln 3$

(1)

Given that $I = \int_1^3 (x-1)\ln x \, dx$,

(b) use the trapezium rule

(i) with values of y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x-1)\ln x \, dx$ is $\frac{3}{2}\ln 3$.

(6)

(b)(i) $I = \frac{1}{2}\left[0 + 2\ln 2 + 2\ln 3\right] = 1.792$

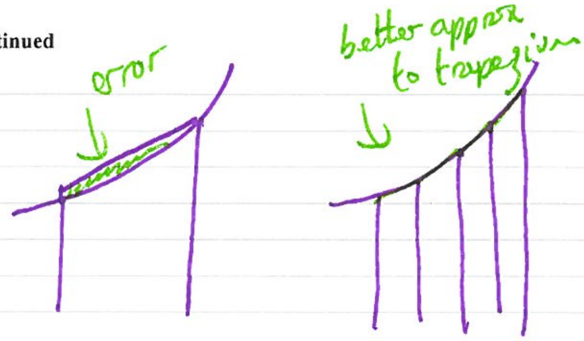
(ii) $I = \frac{0.5}{2}\left[0 + 2\left[(0.5\ln 1.5) + \ln 2 + (1.5\ln 2.5)\right] + 2\ln 3\right]$

$= 1.684$

N 2 3 5 6 3 A 0 1 4 2 0

Question 6 continued

(c)



(d) $\int_1^3 (x-1)\ln x \, dx$ let $u = \ln x$ $\frac{dV}{dx} = x-1$

$\frac{du}{dx} = \frac{1}{x}$ $V = \frac{x^2}{2} - x$

$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$

$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{x}{2} - 1 \, dx$

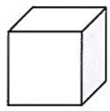
$I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$

$I = \left(\frac{9}{2} - 3\right)\ln 3 - \frac{9}{4} + 3 - \left(0 - \frac{1}{4} + 1\right)$

$= \frac{3}{2}\ln 3 + 0$ As required.

Leave blank

7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\dfrac{dx}{dt} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(b) $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)

$\cancel{8 = x^3}$

(a) $S = 6x^2$ $\qquad V = x^3$

$\dfrac{dS}{dx} = 12x$ $\qquad \dfrac{dV}{dx} = 3x^2$

$\dfrac{dS}{dt} = 8$

Now $\dfrac{dx}{dt} = \dfrac{dx}{\bigcirc} \cdot \dfrac{\bigcirc}{dt} = \dfrac{dx}{dS} \cdot \dfrac{dS}{dt}$

$\dfrac{dx}{dt} = \dfrac{1}{12x} \cdot 8 = \dfrac{2}{3x}$

Question 7 continued

(b) $\frac{dV}{dt} = \frac{dV}{\bigcirc} \cdot \frac{\bigcirc}{dt} = \frac{dV}{dS} \cdot \frac{dS}{dt}$

Now if $S = 6x^2$ $\quad x^2 = \frac{S}{6}$ $\quad x = \left(\frac{S}{6}\right)^{\frac{1}{2}}$

$\therefore \quad V = \left[\left(\frac{S}{6}\right)^{\frac{1}{2}}\right]^3$

$V = \frac{S^{3/2}}{6^{3/2}}$

$\therefore \quad \frac{dV}{dS} = \frac{3}{2} \frac{S^{\frac{1}{2}}}{6^{3/2}}$

Now $\frac{dV}{dS} = \frac{dV}{\bigcirc} \cdot \frac{\bigcirc}{dS} = \frac{dV}{dx} \cdot \frac{dx}{dS}$

$= 3x^2 \cdot \frac{1}{12x} = \frac{x}{4}$

$\therefore \quad \frac{dV}{dt} = \frac{x}{4} \cdot 8 = 2x$

but if $V = x^3$, $x = V^{\frac{1}{3}}$

$\therefore \quad \frac{dV}{dt} = 2V^{\frac{1}{3}}$ As required.

Question 7 continued

(c) $\int V^{-\frac{1}{3}} dV = \int 2\,dt$

$\frac{3}{2} V^{2/3} = 2t + c$

@ (0,8)

$\frac{3}{2}(8)^{2/3} = c$

$\frac{3}{2} \times 4 = c$

$c = 6.$

$\therefore \quad \frac{3}{2} V^{2/3} = 2t + 6$

when $V = 16\sqrt{2}$

$\frac{3}{2}\left[16\sqrt{2}\right]^{2/3} = 2t + 6$

$\frac{3}{2} \times 8 = 2t + 6$

$12 = 2t + 6$

$t = 3$ sec.

Leave blank

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

N23563A02020