Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 25 January 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax+by+c=0$, where a, b and c are integers.

(3)

1a) $2 + 6y\frac{dy}{dx} + 3x^2 . 1\frac{dy}{dx} + y.6x = 8x$

@ $(-1,1)$ $\quad 2 + 6(1)\frac{dy}{dx} + 3(-1)^2\frac{dy}{dx} + 6(1)(-1) = 8(-1)$

$2 + 6\frac{dy}{dx} + 3\frac{dy}{dx} - 6 = -8$

$9\frac{dy}{dx} = -4$

$\frac{dy}{dx} = \frac{-4}{9}$

(b) $M_{norm} = +\frac{9}{4}$

$y - 1 = \frac{9}{4}(x - -1)$

$4y - 4 = 9x + 9$

$9x - 4y + 13 = 0$

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2. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

(a) $\int x \sin 3x \, dx$ let $u = x$ $\frac{dv}{dx} = \sin 3x$

$\frac{du}{dx} = 1$ $v = -\frac{1}{3}\cos 3x$

$I = -\frac{x}{3}\cos 3x - \int -\frac{1}{3}\cos 3x \, dx$

$I = -\frac{x}{3}\cos 3x + \frac{1}{3}\int \cos 3x \, dx$

$= \frac{1}{9}\sin 3x - \frac{x}{3}\cos 3x + c$

(b) $\int x^2 \cos 3x \, dx$ let $u = x^2$ $\frac{dv}{dx} = \cos 3x$

$\frac{du}{dx} = 2x$ $v = \frac{1}{3}\sin 3x$

$I = \frac{x^2}{3}\sin 3x - \frac{2}{3}\int x \sin 3x \, dx$

$= \frac{x^2}{3}\sin 3x - \frac{2}{3}\left[\frac{1}{9}\sin 3x - \frac{x}{3}\cos 3x\right] + c$

$= \frac{x^2}{3}\sin 3x + \frac{2x}{9}\cos 3x - \frac{2}{27}\sin 3x + c$

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3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots$$

(b) find the value of the constant k,

(2)

(c) find the value of the constant A.

(2)

(a) $(2-5x)^{-2} = \left[2\left(1-\frac{5x}{2}\right)\right]^{-2} = \frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$

$= \frac{1}{4}\left[1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{-5x}{2}\right)^2 + \ldots\right]$

$= \frac{1}{4}\left[1 + 5x + \frac{75}{4}x^2 + \ldots\right]$

$= \frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \ldots$

(b) $(2+kx)(2-5x)^{-2}$

$= (2+kx)\left(\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \ldots\right)$

$= \frac{2}{4} + \frac{10}{4}x + \frac{150}{16}x^2 + \frac{kx}{4} + \frac{5k}{4}x^2 + \ldots$

$= \frac{1}{2} + x\left(\frac{5}{2} + \frac{k}{4}\right) + x^2\left(\frac{75}{8} + \frac{5k}{4}\right) + \ldots$

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Question 3 continued

Comparing coeficient x: $\frac{5}{2} + \frac{k}{4} = \frac{7}{4}$

$\frac{k}{4} = -\frac{3}{4}$

$k = -3$

(c) x^2: $A = \frac{75}{8} + \frac{5(-3)}{4} = \frac{75}{8} - \frac{15}{4} = \frac{45}{8}$

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4.

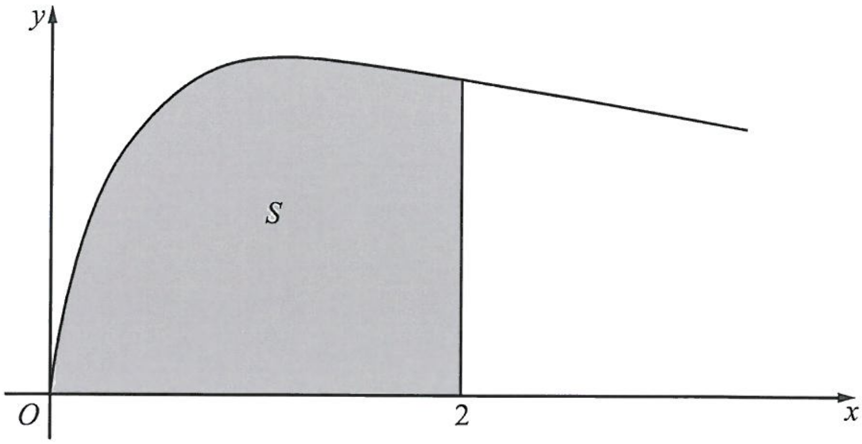

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

$V = \pi \int_0^2 y^2 \, dx$

$= \pi \int_0^2 \frac{2x}{3x^2+4} \, dx$

let $u = 3x^2 + 4$

$\frac{du}{dx} = 6x$

$dx = \frac{du}{6x}$

when $x = 0$ $u = 4$

$x = 2$ $u = 16$

$\therefore V = \pi \int_4^{16} \frac{2\cancel{x}}{u} \cdot \frac{du}{6\cancel{x}}$

$V = \frac{\pi}{3} \int_4^{16} \frac{1}{u} \, dx$

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Question 4 continued

$$V = \frac{\pi}{3}\left[\ln u\right]_4^{16}$$

$$= \frac{\pi}{3}\left[\ln 16 - \ln 4\right]$$

$$= \frac{\pi}{3}\ln\left(\frac{16}{4}\right)$$

$$= \frac{\pi}{3}\ln 4$$

Q4

(Total 5 marks)

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5.

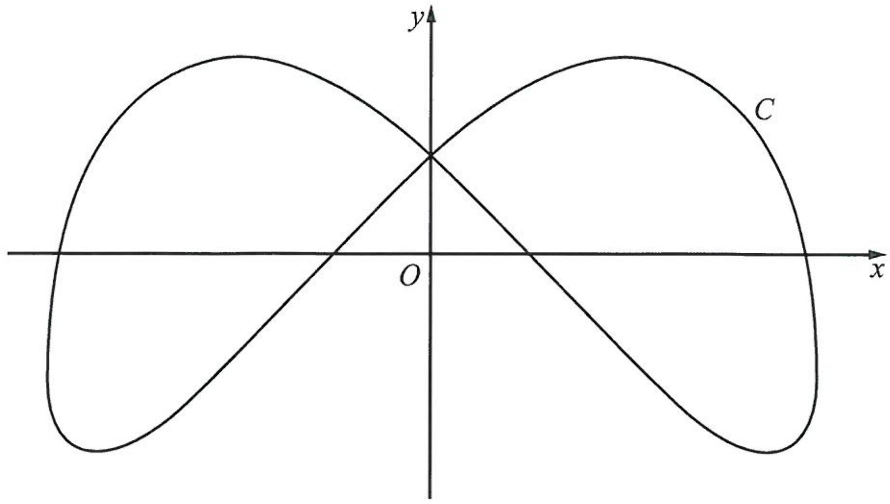

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

(a) $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ $\quad \frac{dy}{dt} = -6\sin 2t$

$\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t+\frac{\pi}{6}\right)}$

(b) $\frac{-6\sin 2t}{4\cos\left(t+\frac{\pi}{6}\right)} = 0 \Rightarrow \sin 2t = 0$

$2t = 0, \pi, 2\pi, 3\pi, \cancel{4\pi}$

$t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \cancel{2\pi}$

$t=0 \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \; y = 3 \quad (2, 3)$

$t = \frac{\pi}{2} \quad x = 4\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = 2\sqrt{3} \quad y = -3 \quad (2\sqrt{3}, -3)$

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Question 5 continued

$t = \pi \quad x = 4\sin\left(\pi + \frac{\pi}{6}\right) = -2 \quad y = 3\cos 2\pi = 3 \quad (-2, 3)$

$t = \frac{3\pi}{2} \quad x = 4\sin\left(\frac{3\pi}{2} + \frac{\pi}{6}\right) = -2\sqrt{3} \quad y = 3\cos 3\pi = -3 \quad (-2\sqrt{3}, -3)$

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Question 6 continued

(b) $I \approx \frac{\pi/8}{2}\left[0+2(0.73508+1.17157+1.02280]+0\right]$

≈ 1.1504 (4dp).

(c) $u = 1+\cos x$ $\quad \frac{dy}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

when $x=0$ $u=2$

$x=\frac{\pi}{2}$ $u=1$

$\therefore$ $I = \int \frac{2.2\cancel{\sin x}\cos x}{u} \cdot \frac{du}{-\cancel{\sin x}}$

$I = -4\int \frac{\cos x}{u}\, du$

but $\cos x = u-1$

$I = -4\int \frac{u-1}{u}\, du = -4\int 1-\frac{1}{u}\, du$

$= -4\left(u - \ln u\right) + c$

$= 4\ln(1+\cos x) - 4\cos x -4 + c$ k

(d) now if $\int_2^1$ $\quad I = -4\left[u - \ln u\right]_2^1$

$= \cancel{-4\left[1-\ln 2\right] = 4\ln 2 - 4}$

$= -4\left(1-\cancel{\ln 1} - 2+\ln 2\right) = 4 - 4\ln 2$

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Question 6 continued

$$\% \text{ error} = \frac{(4-4\ln 2)-1.1504}{4-4\ln 2} \times 100$$

$$= 6.3\%.$$

Leave blank

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.
(2)

(b) Find a vector equation for the line l.
(2)

(c) Show that the size of the angle BAD is 109°, to the nearest degree.
(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.
(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.
(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.
(2)

(a) $r_a = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ $\quad r_b = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix}$

$\overrightarrow{AB} = -r_a + r_b = -\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

(b) $r =$ pos vect on line $+ \lambda$ direct vect

$= \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

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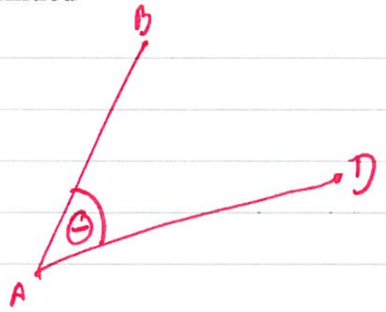
Question 7 continued

(c)



$\vec{AB} \cdot \vec{AD} = |AB||AD| \cos\theta$

$\vec{AD} = -r_A + r_D = -\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$

$\therefore \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} = \sqrt{3^2+3^2+5^2} \sqrt{-3^2+2^2+-1^2} \cos\theta$

$-9+6-5 = \sqrt{43}\sqrt{14} \cos\theta$

$\cos\theta = \frac{-8}{\sqrt{43}\sqrt{14}}$

$\theta = 109°$

(d) if $\vec{AB} = \vec{DC}$

then $r_C = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$

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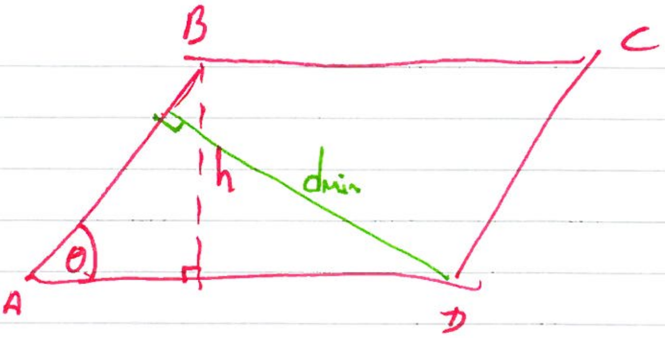
Question 7 continued

(e)



$$\text{Area} = |\overrightarrow{AD}| \times h = |\overrightarrow{AD}| \times |\overrightarrow{AB}| \sin\theta$$

$$= \sqrt{14} \times \sqrt{43} \times \sin 109$$

$$= 23.2 \text{ (to 2sf)}$$

(f) $\dfrac{d_{min}}{|\overrightarrow{AD}|} = \sin\theta$

$d_{min} = \sqrt{14}\,\sin 109 = 3.54$ (3s.f.).

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8. (a) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \qquad t \geqslant 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(c) Hence show that the population cannot exceed 5000

(1)

(a) $\dfrac{1}{p(5-p)} = \dfrac{A}{p} + \dfrac{B}{5-p}$

$= \dfrac{A(5-p) + Bp}{p(5-p)}$

$= \dfrac{5A - Ap + Bp}{p(5-p)}$

Compare Coeffs: p^1: $B - A = 0 \Rightarrow A = B$

p^0: $5A = 1$

$A = \frac{1}{5} = B$

P 4 0 0 8 5 A 0 2 4 2 8

Question 8 continued

(b) $\frac{dP}{dt} = \frac{1}{15} P(5-P)$

$\int \frac{1}{P(5-P)} \, dP = \frac{1}{15} \int dt$

$\frac{1}{5} \int \frac{1}{P} + \frac{1}{5-P} \, dP = \frac{1}{15} \int dt$

$3 \int \frac{1}{P} + \frac{1}{5-P} \, dP = \int dt$

$3 \left[\ln P - \ln(5-P) \right] = t + c$

$3 \left[\ln\left(\frac{P}{5-P}\right) \right] = t + c$

when $t = 0$, $P = 1$

$3 \left[\ln\left(\frac{1}{4}\right) \right] = c$

$\therefore \quad 3 \ln\left(\frac{P}{5-P}\right) = t + 3 \ln\left(\frac{1}{4}\right)$

$3 \ln\left(\frac{P}{5-P}\right) - 3 \ln\left(\frac{1}{4}\right) = t$

$\ln\left(\frac{\frac{P}{5-P}}{\frac{1}{4}}\right) = \frac{t}{3}$

$\ln\left(\frac{4P}{5-P}\right) = \frac{t}{3}$

P 4 0 0 8 5 A 0 2 5 2 8

Question 8 continued

$$\frac{4P}{5-P} = e^{\frac{1}{3}t}$$

$$4P = \cancel{4}e^{\frac{1}{3}t}(5-P)$$

$$4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t}$$

$$4P + Pe^{\frac{1}{3}t} = 5e^{\frac{1}{3}t}$$

$$\cancel{P\left[4 + e^{\frac{1}{3}t}\right] = 5e^{\frac{1}{3}t}}$$

$$\cancel{P = \frac{5e^{\frac{1}{3}t}}{}}$$

$\times e^{-\frac{1}{3}t}$: $4Pe^{-\frac{1}{3}t} + P = 5$

$$P\left[1 + 4e^{-\frac{1}{3}t}\right] = 5$$

$$P = \frac{5}{1+4e^{-\frac{1}{3}t}}$$

(c) as $t \to \infty$, $4e^{-\frac{1}{3}t} \to 0$ $\therefore$ $P \to \frac{5}{1+0} \to 5$

hence population will not exceed 5000.