Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Jones Solo

Wednesday 26 January 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
H35405A
W850/R6666/57570 5/5/5/3

H 3 5 4 0 5 A 0 1 2 4

Turn over

Leave blank

1. Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

let $u = x$ $\quad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 1$ $\quad v = -\frac{1}{2}\cos 2x$

$I = -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x \, dx$

$= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= -\frac{1}{2}.\frac{\pi}{2}.\cos 2\frac{\pi}{2} + \frac{1}{4}\sin 2\frac{\pi}{2} - 0$

$= \frac{\pi}{4}$

Leave blank

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geqslant 0$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

$I = 16 - 16(0.5)^t$

if $u = (0.5)^t$

$\ln u = t \ln 0.5$

$\frac{1}{u}\frac{du}{dt} = \ln 0.5$

$\frac{du}{dt} = u \ln 0.5$

$= (0.5)^t \ln(0.5)$

$\therefore \frac{dI}{dt} = -16(0.5)^t \ln(0.5)$

when $t = 3$ $\frac{dI}{dt} = -16(0.5)^3 \ln(0.5)$

$= -16 \times \frac{1}{8} \ln 0.5$

$= -2 \ln(0.5)$

$= \ln(0.5)^{-2}$

$= \ln 4$

Leave blank

3. (a) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(b) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$.

(3)

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$.

(6)

(a) $\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$

$= \dfrac{A(3x+2) + B(x-1)}{(x-1)(3x+2)}$

Compare coefficient: x: $3A + B = 0$ $\cancel{B=-3A}$

x^0: $2A - B = 5$

add $5A = 5$

$A = 1$ $\therefore$ $B = -3$

(b) $\therefore \displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \frac{1}{x-1} - \frac{3}{3x+2}\,dx$

$= \ln(x-1) - \dfrac{3}{3}\ln(3x+2) + c$

$= \ln\left(\dfrac{x-1}{3x+2}\right) + c$

H 3 5 4 0 5 A 0 6 2 4

Leave blank

Question 3 continued

(c) $(x-1)(3x+2)\frac{dy}{dx} = 5y$

$\int \frac{1}{y}\,dy = \int \frac{5}{(x-1)(3x+2)}\,dx$

$\ln y = \ln\left(\frac{x-1}{3x+2}\right) + c$

when $y=8$, $x=2$

$\ln 8 = \ln\left(\frac{1}{8}\right) + c$

$\ln 8 - \ln\left(\frac{1}{8}\right) = c$

$c = \ln\left(\frac{8}{1/8}\right) = \ln 64$

$\ln y = \ln\left(\frac{x-1}{3x+2}\right) + \ln 64$

$\ln y = \ln\left(\frac{64(x-1)}{3x+2}\right)$

$\therefore\ y = 64\frac{(x-1)}{(3x+2)}$

H35405A0724

Turn over

Leave blank

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to l, find

(c) the value of p,

(4)

(d) the distance AC.

(2)

(a) $r_A = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ $\quad r_B = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$

$\overrightarrow{AB} = -r_A + r_B = -\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix}$

(b) $r = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix}$

(c) $r_C = \begin{pmatrix} 2 \\ p \\ -4 \end{pmatrix}$ $\quad$ if $\overrightarrow{AC}$ is perp to l then $\overrightarrow{AC} . \overrightarrow{AB} = 0$

$\overrightarrow{AC} = -r_A + r_C = -\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ p \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix}$

So $\begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$

$-3 + 5(p+3) + 18 = 0$

$5p + 15 + 15 = 0$

$5p = -30$

$p = -6$

Leave blank

Question 4 continued

(d) $\therefore \overrightarrow{AC} = \begin{pmatrix} 1 \\ -3 \\ -6 \end{pmatrix}$

$|\overrightarrow{AC}| = \sqrt{1^2 + (-3)^2 + (-6)^2} = \sqrt{46}$

H35405A01124

Turn over

Leave blank

5. (a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x|<\frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x)=\frac{a+bx}{(2-3x)^2}, \quad |x|<\frac{2}{3}, \text{ where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) the value of a and the value of b,

(5)

(c) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

(a) $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$

$\frac{1}{4}\left[1+(-2)\left(-\frac{3}{2}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{3}{2}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{3}{2}x\right)^3+\dots\right]$

$\frac{1}{4}\left[1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\dots\right]$

$\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\dots$

(b) $(a+bx)(2-3x)^{-2}$

$=(a+bx)\left[\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\dots\right]$

$=\frac{1}{4}a+\frac{3}{4}ax+\frac{27}{16}ax^2+\frac{27}{8}ax^3+\frac{1}{4}bx+\frac{3}{4}bx^2+\frac{27}{16}bx^3+\frac{27b}{16}x^4$

Coef $x^1=0$ $\therefore \frac{3}{4}a+\frac{1}{4}b=0 \Rightarrow 3a+b=0$ — (1)

$x^2=\frac{9}{16}$ $\therefore \frac{27a}{16}+\frac{3}{4}b=\frac{9}{16} \xRightarrow{\times 16} 27a+12b=9$ — (2)

H 3 5 4 0 5 A 0 1 4 2 4

Leave blank

Question 5 continued

From (1) $b = -3a$

in (2) $27a + 12(-3a) = 9$

$27a - 36a = 9$

$-9a = 9$

$a = -1$

$\therefore b = +3$

(c) $x^3 = \frac{27}{8}a + \frac{27}{16}b$

$= \frac{27}{8}(-1) + \frac{27}{16}(3)$

$= -\frac{27}{8} + \frac{81}{16}$

$= \frac{27}{16}.$

Turn over

H 3 5 4 0 5 A 0 1 5 2 4

Leave blank

6. The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$,

(6)

(b) a cartesian equation of C.

(3)

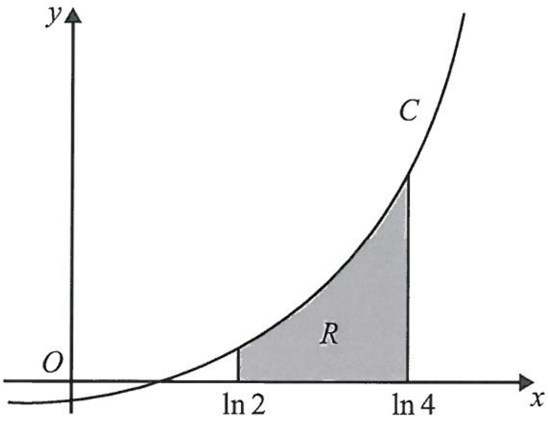

Figure 1

The finite area R, shown in Figure 1, is bounded by C, the x-axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through $360°$ about the x-axis.

(c) Use calculus to find the exact volume of the solid generated.

(6)

(a) $x = \ln t \qquad y = t^2 - 2$

$\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{2t}{\frac{1}{t}} = 2t^2$

when $t = 3 \quad x = \ln 3 \quad y = 9 - 2 = 7 \quad \frac{dy}{dx} = 18$

eqⁿ of normal $y - 7 = -\frac{1}{18}(x - \ln 3)$

H35405A01824

Leave blank

Question 6 continued

(b) $x = \ln t \quad t = e^{x}$

$y = t^2 - 2$

$y = e^{2x} - 2$

(c) $V = \pi \int y^2 \, dx$

$= \pi \int (e^{2x} - 2)^2 \, dx$

$= \pi \int_{\ln 2}^{\ln 4} e^{4x} - 4e^{2x} + 4 \, dx$

$= \pi \left[\frac{1}{4} e^{4x} - \frac{4}{2} e^{2x} + 4x \right]_{\ln 2}^{\ln 4}$

$= \pi \left[\left(\frac{1}{4} e^{4\ln 4} - 2e^{2\ln 4} + 4\ln 4 \right) - \left(\frac{1}{4} e^{4\ln 2} - 2e^{2\ln 2} + 4\ln 2 \right) \right]$

$= \pi \left[\frac{1}{4} . 256 - 2.16 + 4\ln 4 - \frac{1}{4} . 16 + 2.4 - 4\ln 2 \right]$

$= \pi \left[64 - 32 + 4\ln 4 - 4 + 8 - 4\ln 2 \right]$

$= \pi \left[36 - 4\ln 2 \right]$

Leave blank

7.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2	0.1847	0.1745	0.1667

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(c) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

(b) $I = \frac{1}{2}\left[0.2 + 2(0.1847 + 0.1745) + 0.1667\right]$

$= 0.543$

(c) $x = (u-4)^2 + 1$ when $x = 2$ $2 = (u-4)^2 + 1$

$1 = (u-4)^2$

$1 = u - 4$

$u = 5$

when $x = 5$ $5 = (u-4)^2 + 1$

$4 = (u-4)^2$

$2 = u - 4$

$u = 6$

$\frac{dx}{du} = 2(u-4)^1 . 1 = 2(u-4)$

$dx = 2(u-4)\,du$

H 3 5 4 0 5 A 0 2 2 2 4

Question 7 continued

$$\therefore \quad I = \int_5^6 \frac{1}{4+\sqrt{(u-4)^2+1-1}} \; 2(u-4)\,du$$

$$I = \int_5^6 \frac{1}{4+(u-4)} \cdot 2(u-4)\,du$$

$$I = 2\int_5^6 \frac{u-4}{u}\,du$$

$$I = 2\int_5^6 1 - \frac{4}{u}\,du$$

$$= 2\Big[u - 4\ln u\Big]_5^6$$

$$= 2\Big[6 - 4\ln 6 - 5 + 4\ln 5\Big]$$

$$= 2\left[1 + 4\ln\left(\frac{5}{6}\right)\right].$$

$$= 2 + 8\ln\left(\frac{5}{6}\right)$$

Leave blank