Jonas Sob

| Centre No. | | | | | | Paper Reference | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 6 6 6 / 0 1 | Signature | |

Paper Reference(s)

**6666/01**

# Edexcel GCE

## Core Mathematics C4

### Advanced

Monday 25 January 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Pink or Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions to Candidates**

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

**Information for Candidates**

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



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***Turn over***

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1. (a) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of $x$ up to and including the term in $x^3$, simplifying each term.

**(4)**

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

**(2)**

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

**(3)**

(a) $(1-8x)^{\frac{1}{2}}$

$= 1 + \left(\frac{1}{2}\right)(-8x) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{(-8x)^2}{2!} + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{(-8x)^3}{3!} + \ldots$

$= 1 - 4x - \frac{64}{8}x^2 - \frac{1536}{48}x^3 + \ldots$

$= 1 - 4x - 8x^2 - 32x^3 + \ldots$ — 1

(b) when $x = \frac{1}{100}$

$\sqrt{(1-8x)} = \sqrt{1 - \frac{8}{100}} = \sqrt{\frac{100}{100} - \frac{8}{100}} = \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5}$

(c) $\frac{\sqrt{23}}{5} \approx 1 - 4\left(\frac{1}{100}\right) - 8\left(\frac{1}{100}\right)^2 - 32\left(\frac{1}{100}\right)^3$

$\sqrt{23} = 5\left(1 - \frac{4}{100} - \frac{8}{10000} - \frac{32}{1000000}\right)$

$= 4.79584$



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**2.**

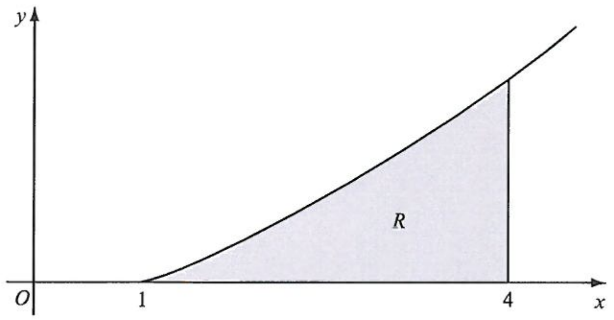


**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geqslant 1$. The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the $x$-axis and the line $x = 4$.

The table shows corresponding values of $x$ and $y$ for $y = x \ln x$.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 0.608 | 1.386 | 2.291 | 3.296 | 4.385 | 5.545 |

(a) Complete the table with the values of $y$ corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.
**(2)**

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate for the area of $R$, giving your answer to 2 decimal places.
**(4)**

(c) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of $R$, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where $a$ and $b$ are integers.
**(7)**

(b) $A = \frac{0.5}{2}\left[0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545\right]$

$= 7.37$



N 3 5 3 8 2 A 0 4 2 8

**Question 2 continued**

(c)(i) $\int x \ln x \, dx$ $\quad u = \ln x \quad \frac{dv}{dx} = x$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^2}{2}$

$I = \frac{x^2}{2}\ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$

(ii) $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4$

$\Rightarrow \frac{16}{2}\ln 4 - \frac{16}{4} - \frac{1}{2}\ln 1 + \frac{1}{4}$

$\Rightarrow 8\ln 4 - 4 - 0 + \frac{1}{4}$

$\Rightarrow 8\ln 2^2 - \frac{15}{4}$

$\Rightarrow 16\ln 2 - \frac{15}{4} \Rightarrow \frac{1}{4}\left[64\ln 2 - 15\right].$

Leave blank

**3.** The curve $C$ has the equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}, \quad 0 \leqslant y \leqslant \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

**(3)**

The point $P$ lies on $C$ where $x = \frac{\pi}{6}$.

(b) Find the value of $y$ at $P$.

**(3)**

(c) Find the equation of the tangent to $C$ at $P$, giving your answer in the form $ax + by + c\pi = 0$, where $a$, $b$ and $c$ are integers.

**(3)**

(a) $\cos 2x + \cos 3y = 1 \quad -(1)$

$-2\sin 2x - 3\sin 3y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$

(b) when $x = \frac{\pi}{6}$ in (1) $\cos 2\frac{\pi}{6} + \cos 3y = 1$

$\frac{1}{2} + \cos 3y = 1$

$\cos 3y = \frac{1}{2}$

$3y = \frac{\pi}{3}$

$y = \frac{\pi}{9}$



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**Question 3 continued**

(c) gradient @P: $\frac{dy}{dx} = -\frac{2\sin\frac{2\pi}{6}}{3\sin\frac{3\pi}{9}} = -\frac{2}{3}$

eqⁿ of tangent $y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$

$\times 18$ $\quad 18y - 2\pi = -12x + 2\pi$

$12x + 18y - 4\pi = 0$

$\div 2$ $\quad 6x + 9y - 2\pi = 0$

Leave blank



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Leave blank

**4.** The line $l_1$ has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line $l_2$ has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where $\lambda$ and $\mu$ are parameters.

The lines $l_1$ and $l_2$ intersect at the point $A$ and the acute angle between $l_1$ and $l_2$ is $\theta$.

(a) Write down the coordinates of $A$.
**(1)**

(b) Find the value of $\cos\theta$.
**(3)**

The point $X$ lies on $l_1$ where $\lambda = 4$.

(c) Find the coordinates of $X$.
**(1)**

(d) Find the vector $\overrightarrow{AX}$.
**(2)**

(e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$.
**(2)**

The point $Y$ lies on $l_2$. Given that the vector $\overrightarrow{YX}$ is perpendicular to $l_1$,

(f) find the length of $AY$, giving your answer to 3 significant figures.
**(3)**

(a) @A: $-6+4\lambda = -6+3\mu$ — (1)

$4-\lambda = 4-4\mu$ — (2)

$-1+3\lambda = -1+\mu$ — (3)

From (2) $\lambda = 4\mu$ — (4)

in (1) $4(4\mu) = 3\mu$

$\mu = 0$

**Question 4 continued**

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(a) $A\ (-6, 4, -1)$

(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\1\end{pmatrix} = \sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$

$12+4+3 = \sqrt{26}\sqrt{26}\cos\theta$

$19 = 26\cos\theta$

$\cos\theta = \dfrac{19}{26}$

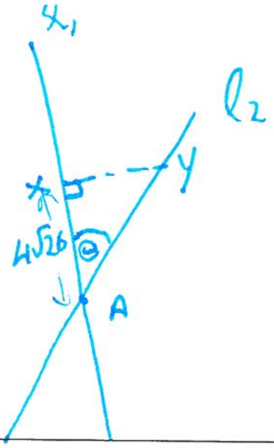
(c) $\lambda = 4$ in $l_1$: $\mathbf{r} = \begin{pmatrix}-6\\4\\-1\end{pmatrix} + 4\begin{pmatrix}4\\-1\\3\end{pmatrix} = \begin{pmatrix}10\\0\\11\end{pmatrix}$

$X\ (10, 0, 11)$

(d) $\overrightarrow{AX} = -\overrightarrow{OA} + \overrightarrow{OX} = -\begin{pmatrix}-6\\4\\-1\end{pmatrix} + \begin{pmatrix}10\\0\\11\end{pmatrix} = \begin{pmatrix}16\\-4\\12\end{pmatrix}$

(e) $|\overrightarrow{AX}| = \sqrt{16^2+(-4)^2+12^2}$

$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$

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**Question 4 continued**

$$\frac{AX}{AY} = \cos\theta$$

$$AY = \frac{AX}{\cos\theta}$$

$$= \frac{4\sqrt{26}}{\frac{19}{26}}$$

$$= 27.9$$

Q4

**(Total 12 marks)**



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**Turn over**

5. (a) Find $\int \frac{9x+6}{x}\,dx$, $x > 0$.

**(2)**

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)\,y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

**(6)**

(a) $\int \frac{9x}{x} + \frac{6}{x}\,dx$

$= \int 9 + \frac{6}{x}\,dx$

$= 9x + 6\ln x + c$

(b) $\frac{dy}{dx} = \frac{(9x+6)y^{1/3}}{x}$

$\int y^{-1/3}\,dy = \int \frac{9x+6}{x}\,dx$

$\frac{y^{2/3}}{2/3} = 9x + 6\ln x + c$

$3y^{2/3} = 18x + 12\ln x + c$

@ $(1,8)$ $3(8)^{2/3} = 18 + 0 + c$

$12 = 18 + c \Rightarrow c = -6$





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Leave blank

**Question 5 continued**

$$3y^{2/3} = 18x + 12\ln x - 6$$

$$y^{2/3} = 6x + 4\ln x - 2$$

cube both sides

$$y^2 = (6x + 4\ln x - 2)^3$$

6. The area $A$ of a circle is increasing at a constant rate of $1.5\,\text{cm}^2\,\text{s}^{-1}$. Find, to 3 significant figures, the rate at which the radius $r$ of the circle is increasing when the area of the circle is $2\,\text{cm}^2$.

**(5)**

$\frac{dA}{dt} = 1.5$ $\quad\quad A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = \frac{dr}{dA} \cdot \frac{dA}{dt}$

$= \frac{1}{2\pi r} \cdot 1.5$

$= \frac{0.75}{\pi r}$

When $A = 2$ $\quad 2 = \pi r^2$ $\quad r = \sqrt{\frac{2}{\pi}}$

$\therefore \frac{dr}{dt} = \frac{0.75}{\pi\sqrt{\frac{2}{\pi}}} = 0.299$ (3 s.f.)

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7.

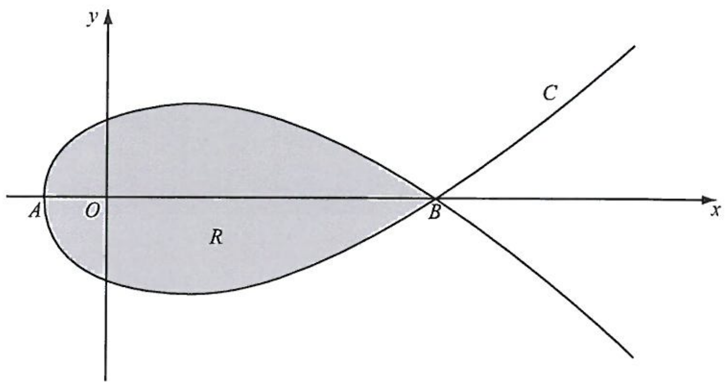


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve $C$ cuts the $x$-axis at the points $A$ and $B$.

(a) Find the $x$-coordinate at the point $A$ and the $x$-coordinate at the point $B$.

**(3)**

The region $R$, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of $R$.

**(6)**

(a) when $y = 0$ $\quad t(9-t^2) = 0$

$\therefore$ either $t = 0$ or $t = \pm 3$

Now when $t = 0$, $\quad x = -4$

$t = \pm 3 \quad x = 5(3)^2 - 4 = 45 - 4 = 41$

$\therefore A(-4, 0)$

$B(41, 0)$

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| Q5 | (a) $\displaystyle\int \frac{9x+6}{x}\,\mathrm{d}x = \int \left(9+\frac{6}{x}\right)\mathrm{d}x$ | | M1 | |
| | $= 9x + 6\ln x \ (+C)$ | | A1 | (2) |
| | (b) $\displaystyle\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$ | Integral signs not necessary | B1 | |
| | $\displaystyle\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$ | | | |
| | $\displaystyle\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \ (+C)$ | $\pm k y^{\frac{2}{3}} =$ their $(\mathrm{a})$ | M1 | |
| | $\displaystyle\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \ (+C)$ | ft their $(\mathrm{a})$ | A1ft | |
| | $y = 8,\ x = 1$ | | | |
| | $\displaystyle\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$ | | M1 | |
| | $C = -3$ | | A1 | |
| | $\displaystyle y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$ | | | |
| | $y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$ | | A1 | (6) |
| | | | | **[8]** |

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| Q6 | $\dfrac{dA}{dt} = 1.5$ | | B1 |
| | $A = \pi r^2 \Rightarrow \dfrac{dA}{dr} = 2\pi r$ | | B1 |
| | When $A = 2$ | | |
| | $2 = \pi r^2 \Rightarrow r = \sqrt{\dfrac{2}{\pi}} \; (= 0.797\,884\,\ldots)$ | | M1 |
| | $\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$ | | |
| | $1.5 = 2\pi r \dfrac{dr}{dt}$ | | M1 |
| | $\dfrac{dr}{dt} = \dfrac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ | awrt 0.299 | A1 |
| | | | **[5]** |

**Question 7 continued**

(b) $A = \int_0^3 y \frac{dx}{dt}\, dt$

$\int_0^3 t(9-t^2) \cdot 10t \, dt$

$10 \int_0^3 \cancel{9t - t^3}\; 9t^2 - t^4 \, dt$

$10 \left[ \frac{9t^3}{3} - \frac{t^5}{5} \right]_0^3$

$10 \left[ 81 - 48.6 \right] = 324$

So Area of whole shape $= 2 \times 324 = 648$



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Turn over

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| Q7 | (a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$ | | | |
| | $t=0, 3, -3$ | Any one correct value | B1 | |
| | At $t=0$, $x=5(0)^2-4=-4$ | Method for finding one value of $x$ | M1 | |
| | At $t=3$, $x=5(3)^2-4=41$ | | | |
| | $\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$ | | | |
| | At $A$, $x=-4$; at $B$, $x=41$ | Both | A1 | (3) |
| | (b) $\dfrac{dx}{dt}=10t$ | Seen or implied | B1 | |
| | $\int y\,dx=\int y\dfrac{dx}{dt}\,dt=\int t(9-t^2)10t\,dt$ | | M1 A1 | |
| | $=\int(90t^2-10t^4)\,dt$ | | | |
| | $=\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\ (+C)\quad \left(=30t^3-2t^5\ (+C)\right)$ | | A1 | |
| | $\left[\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$ | | M1 | |
| | $A=2\int y\,dx=648\quad(\text{units}^2)$ | | A1 | (6) |
| | | | | **[9]** |

Leave blank

**8.** (a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx$$

**(7)**

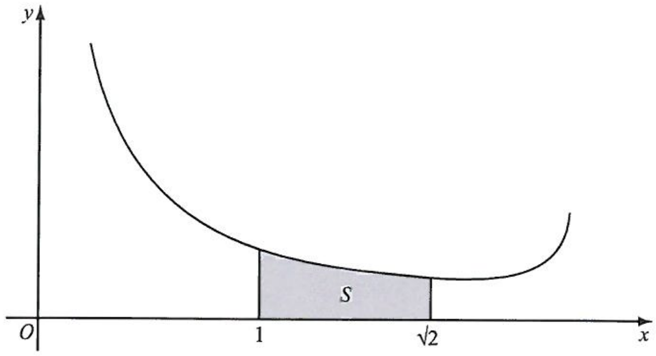


**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $\quad 0 < x < 2$.

The shaded region $S$, shown in Figure 3, is bounded by the curve, the $x$-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region $S$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

**(3)**

(a) $x = 2\cos u$

$\frac{dx}{du} = -2\sin u$

$dx = -2\sin u \, du$

when $x = 1$ $\quad \cos u = \frac{1}{2}$ $\quad u = \frac{\pi}{3}$

when $x = \sqrt{2}$ $\quad \cos u = \frac{\sqrt{2}}{2}$ $\quad u = \frac{\pi}{4}$



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**Question 8 continued**

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$$I = \int_{\frac{\pi}{3}}^{\pi/4} \frac{1}{4\cos^2 u\sqrt{4-4\cos^2 u}} \cdot -2\sin u \, du$$

$$= \int_{\frac{\pi}{3}}^{\pi/4} \frac{1}{4\cos^2 u\sqrt{4(1-\cos^2 u)}} \cdot -2\sin u \, du$$

$$= \int_{\frac{\pi}{3}}^{\pi/4} \frac{-\cancel{2\sin u}}{4\cos^2 u \cdot \cancel{2\sin u}} \, du$$

$$= \frac{1}{4}\int_{\frac{\pi}{3}}^{\pi/4} -\sec^2 u \, du$$

$$= \frac{1}{4}\left[-\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left[-\tan\frac{\pi}{4} - -\tan\frac{\pi}{3}\right]$$

$$= \frac{1}{4}\left[\sqrt{3} - 1\right]$$

Leave blank

**Question 8 continued**

(b) $V = \pi \int_{\sqrt{2}} y^2 \, dx$

$= \pi \int_1^{\sqrt{2}} \left( \frac{4}{x(4-x^2)^{1/4}} \right)^2 dx$

$= 16\pi \int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} \, dx$

$= 16\pi \cdot \frac{1}{4} [\sqrt{3} - 1]$

$= 4\pi [\sqrt{3} - 1]$.

Q8

**(Total 10 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**



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