Jones Sols

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 19 January 2009 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A curve C has the equation $y^2 - 3y = x^3 + 8$. —(1)

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

(a) $y^2 - 3y = x^3 + 8$

$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx}\left[2y-3\right] = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y-3}$ —(2)

(b) in (1) $y=3$

$3^2 - 3(3) = x^3 + 8$

$0 = x^3 + 8$

$x = \sqrt{-8} = -2$

in (2) $(-2, 3)$

$\frac{dy}{dx} = \frac{3(-2)^2}{2(3)-3} = \frac{12}{3} = 4$

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2.

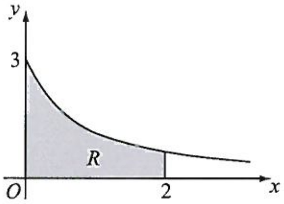

Figure 1

Figure 1 shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

(a) $A = \int_0^2 \frac{3}{\sqrt{1+4x}}\,dx$

$= 3\int_0^2 (1+4x)^{-\frac{1}{2}}\,dx$

$= 3\left[\frac{1}{\frac{1}{2}}(1+4x)^{\frac{1}{2}} \cdot \frac{1}{4}\right]_0^2$

$= 3\left[\frac{2}{4}(1+4x)^{\frac{1}{2}}\right]_0^2$

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Question 2 continued

$$= \frac{3}{2}\left[\sqrt{1+4(2)} - \sqrt{1+0}\right]$$

$$= \frac{3}{2}\left[3-1\right]$$

$$= 3.$$

(b) $$V = \pi\int_0^2 y^2\,dx$$

$$V = \pi\int_0^2 \frac{9}{1+4x}\,dx$$

$$V = 9\pi\left[\frac{1}{4}\ln(1+4x)\right]_0^2$$

$$V = \frac{9\pi}{4}\left[\ln 9 - \ln 1\right]$$

$$V = \frac{9\pi}{4}\ln 9 = \frac{9\pi}{4}\ln 3^2$$

$$= \frac{9\pi}{4}\times 2\ln 3 = \frac{9\pi}{2}\ln 3$$

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3.

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that f(x) can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

(4)

(a) $\frac{27x^2+32x+16}{(3x+2)^2(1-x)} \equiv \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)}$

$\equiv \frac{A(3x+2)+B(1-x)+C(3x+2)^2}{(3x+2)^2(1-x)}$

$\equiv \frac{3Ax+2A+B-Bx+9Cx^2+12Cx+4C}{(3x+2)^2(1-x)}$

$\equiv \frac{9Cx^2+x(3A-B+12C)+2A+B+4C}{(3x+2)^2(1-x)}$

Compare coefficients: x^2: $9C = 27 \Rightarrow C = 3$

x^1: $3A - B + 12C = 32$

$3A - B + 36 = 32$

$3A - B = -4 \Rightarrow B = 3A + 4$

Question 3 continued

$x^0: \quad 2A+B+4C=16$

$2A+(3A+4)+4(3)=16$

$\cancel{2}\; 5A+16=16$

$A=0$

$\therefore\; B=4$

So $f(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{(1-x)}$

$= 4(3x+2)^{-2} + 3(1-x)^{-1}$ — 1

• $(3x+2)^{-2} = \left[2\left(1+\frac{3}{2}x\right)\right]^{-2} = \frac{1}{4}\left(1+\frac{3}{2}x\right)^{-2}$

$= \frac{1}{4}\left(1+(-2)\left(\frac{3}{2}x\right)+\frac{(-2)(-3)}{2!}\left(\frac{3}{2}x\right)^2+\ldots\right]$

$= \frac{1}{4}\left(1-3x+\frac{27}{4}x^2\right)$

• $(1-x)^{-1} = 1+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2+\ldots$

$= 1+x+x^2+\ldots$

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Question 3 continued

So in (1)

$f(x) = 4\left[\frac{1}{4}\left(1 - 3x + \frac{27}{4}x^2 + \ldots\right)\right] + 3\left[1 + x + x^2 + \ldots\right]$

$= 1 - 3x + \frac{27}{4}x^2 + 3 + 3x + 3x^2 + \ldots$

$= 4 + \frac{39}{4}x^2 + \ldots$

(c) So $f(0.2)$ in original $= \dfrac{27(0.2)^2 + 32(0.2) + 16}{(3(0.2)+2)^2(1-0.2)}$

$= \dfrac{587/25}{676/125}$

$= \dfrac{2935}{676}$.

$f(0.2)$ in approx $= 4 + \frac{39}{4}(0.2)^2$

$= 4.39$

So % Error $= \dfrac{\left(4.39 - \frac{2935}{676}\right)}{\frac{2935}{676}} \times 100 = 1.1\%$.

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4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

(a) if perp then $\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} = 0$

$-2q + 2 - 8 = 0$

$2q = -6$

$q = -3$

(b) if intersect then $11 - 2\lambda = -5 - 3\mu$

$2\lambda - 3\mu = 16$ — 1

and $2 + \lambda = 11 + 2\mu$

$\lambda = 9 + 2\mu$ — 2

Question 4 continued

in ① $2(9+2\mu)-3\mu=16$

$18+4\mu-3\mu=16$

$\mu=-2$ in ② $\lambda=5$

and $17-4\lambda=p+2\mu$

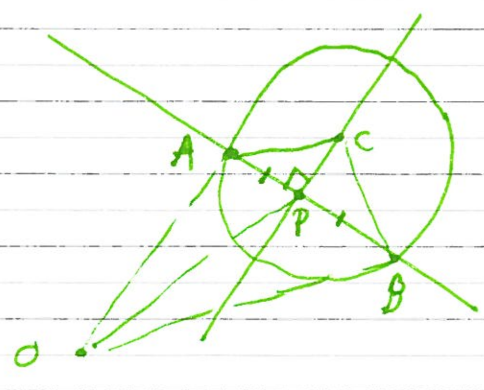
$17-20=p-4$

$p=1$

(c) intersect @ $r=\begin{pmatrix}11\\2\\17\end{pmatrix}+5\begin{pmatrix}-2\\1\\-4\end{pmatrix}=\begin{pmatrix}1\\7\\-3\end{pmatrix}$

Coords $(1,7,-3)$

(d)



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Question 4 continued

$\overrightarrow{AB} = 2\overrightarrow{AP}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP}$

$= -\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$

$\therefore \overrightarrow{AB} = \begin{pmatrix} -16 \\ 8 \\ -32 \end{pmatrix}$

Now $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + \begin{pmatrix} -16 \\ 8 \\ -32 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$.

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5.

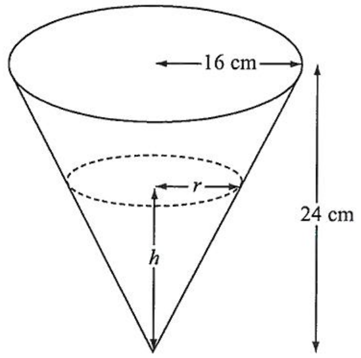


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$.]

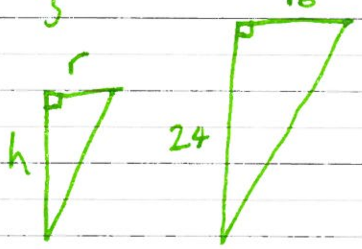
Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

(a) $V = \frac{1}{3}\pi r^2 h$ — (1)

Sim Δ



$\frac{r}{16} = \frac{h}{24}$

$r = \frac{h}{24} \times 16 = \frac{2}{3}h$

in (1) $V = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 . h = \frac{1}{3}\pi . \frac{4}{9}h^3 = \frac{4\pi h^3}{27}$ as required

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Question 5 continued

(b) $\frac{dV}{dt} = 8$

$\frac{dh}{dt} = \frac{dh}{\bigcirc} \cdot \frac{\bigcirc}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

Now $\frac{dV}{dh} = \frac{3.4\pi h^2}{27} = \frac{4\pi h^2}{9}$

$\therefore \frac{dh}{dt} = \frac{9}{4\pi h^2} \cdot 8 = \frac{18}{\pi h^2}$

when $h = 12$ $\frac{dh}{dt} = \frac{18}{144\pi} = \frac{1}{8\pi}$.

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6. (a) Find $\int \tan^2 x \, dx$.

(2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

(4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \, dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant.

(7)

(a) $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx$

$= \tan x - x + c$

(b) $\int x^{-3} \ln x \, dx$ $\quad u = \ln x \quad \frac{dv}{dx} = x^{-3}$

$\frac{du}{dx} = \frac{1}{x} \quad v = -\frac{1}{2}x^{-2}$

$I = -\frac{1}{2x^2}\ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$

$I = -\frac{1}{2x^2}\ln x + \frac{1}{2}\int x^{-3} \, dx$

$= -\frac{1}{2x^2}\ln x + \frac{1}{2} \times \frac{1}{-2} x^{-2}$

Question 6 continued

$I = -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + c$

(c) $\int \frac{e^{3x}}{1+e^x}\,dx$ $\quad u = 1+e^x \implies e^x = u-1$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int \frac{e^{3x}}{u} \cdot \frac{du}{e^x}$

$\int \frac{e^{2x}}{u}\,du = \int \frac{(e^x)^2}{u}\,du$

$= \int \frac{(u-1)^2}{u}\,du$

$= \int \frac{u^2-2u+1}{u}\,du$

$= \int u - 2 + \frac{1}{u}\,du$

$= \frac{u^2}{2} - 2u + \ln u + c$

Question 6 continued

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(c) Cont'd $= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$

$= \frac{1}{2} + \frac{2e^x}{2} + \frac{e^{2x}}{2} - 2 - 2e^x + \ln(1+e^x) + c$

$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + \underbrace{\left(c - \frac{3}{2}\right)}_{k}$

$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$

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7.

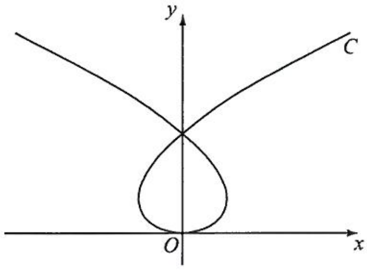

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

(a) @ $t = -1$

$x = (-1)^3 - 8(-1) = -1 + 8 = 7$

$y = (-1)^2 = 1$

$\therefore A(7, 1)$

(b) $\frac{dx}{dt} = 3t^2 - 8 \qquad \frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{2t}{3t^2 - 8}$

Leave blank

Question 7 continued

@A $y - 1 = \frac{2(-1)}{3(-1)^2 - 8}(x - 7)$

$y - 1 = \frac{+2}{+5}(x - 7)$

$5y - 5 = 2x - 14$

$2x - 5y - 9 = 0$ — 1

(c) sub for x & y in 1

$2(t^3 - 8t) - 5(t^2) - 9 = 0$

$2t^3 - 16t - 5t^2 - 9 = 0$

$2t^3 - 5t^2 - 16t - 9 = 0$

if $t = -1$ is a solution, then $(t+1) = 0$

$$\begin{array}{r|l} & 2t^2 - 7t - 9 \\ \hline t+1 & 2t^3 - 5t^2 - 16t - 9 \\ & \underline{2t^3 + 2t^2} \\ & \quad -7t^2 - 16t \\ & \quad \underline{-7t^2 - 7t} \\ & \qquad -9t - 9 \\ & \qquad \underline{-9t - 9} \\ & \qquad \quad \cdot \quad \cdot \end{array}$$

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Question 7 continued

$\therefore \quad (t+1)(2t^2-7t-9)=0$

$(t+1)(t+1)(2t-9)=0$

$\therefore$ other intersection occurs when $t = \frac{9}{2}$

hence $x = \left(\frac{9}{2}\right)^3 - 8\left(\frac{9}{2}\right) = \frac{441}{8}$

$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4}$

$\therefore \quad B\left(\frac{441}{8}, \frac{81}{4}\right).$

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