Jones Sols

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 22 January 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

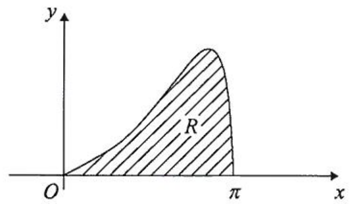

Figure 1

The curve shown in Figure 1 has equation $y = e^x\sqrt{(\sin x)}$, $0 \leqslant x \leqslant \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1.84432	4.81048	8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

$A = \frac{\frac{\pi}{4}}{2}\left[0 + 2(1.84432 + 4.81048 + 8.87207) + 0\right]$

$= 12.1948$ (4dp)

N 2 6 2 8 2 A 0 2 2 4

2. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

(2)

(a) $\left(8\left[1-\frac{3}{8}x\right]\right)^{\frac{1}{3}}$

$= 8^{\frac{1}{3}}\left(1-\frac{3}{8}x\right)^{\frac{1}{3}}$

$= 2\left[1+\left(\frac{1}{3}\right)\left(-\frac{3}{8}\right)x+\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{3}{8}\right)^2\frac{x^2}{2!}\right.$

$\left.+\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{3}{8}\right)^3\frac{x^3}{3!}+\dots\right]$

$= 2\left[1-\frac{x}{8}-\frac{x^2}{64}-\frac{5}{1536}x^3+\dots\right]$

$= 2-\frac{x}{4}-\frac{x^2}{32}-5\frac{x^3}{768}+\dots$ —— 1

(b) $8-3x=7.7$

$8-7.7=3x$

$x=\frac{8-7.7}{3}=0.1$

N 2 6 2 8 2 A 0 4 2 4

Leave blank

Question 2 continued

Subst $x = 0.1$ in ①

$\approx 2 - \frac{0.1}{4} - \frac{(0.1)^2}{32} - \frac{5(0.1)^3}{768}$.

$\simeq 1.9746810$ (7dp)

Q2

(Total 7 marks)

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3.

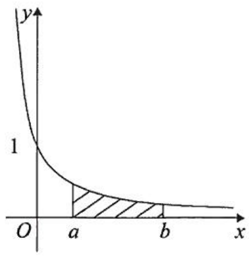

Figure 2

The curve shown in Figure 2 has equation $y = \dfrac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

$$V = \pi \int_a^b y^2 \, dx$$

$$V = \pi \int_a^b \frac{1^2}{(2x+1)^2} \, dx$$

$$V = \pi \int_a^b (2x+1)^{-2} \, dx$$

$$V = \pi \left[\frac{1}{-1} (2x+1)^{-1} \times \frac{1}{2} \right]_a^b$$

$$V = \pi \left[-\frac{1}{2(2x+1)} \right]_a^b$$

N 2 6 2 8 2 A 0 6 2 4

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Question 3 continued

$$V = \frac{\pi}{2}\left[\frac{-1}{2x+1}\right]_a^b$$

$$V = \frac{\pi}{2}\left[-\frac{1}{2b+1} + \frac{1}{2a+1}\right]$$

$$V = \frac{\pi}{2}\left[\frac{-(2a+1) + (2b+1)}{(2b+1)(2a+1)}\right]$$

$$V = \frac{\pi}{\cancel{2}}\left[\frac{-\cancel{2}a + \cancel{2}b}{(2b+1)(2a+1)}\right]$$

$$V = \frac{\pi(-a+b)}{(2b+1)(2a+1)}$$

$$= \frac{\pi(b-a)}{(2b+1)(2a+1)}$$

Q3

(Total 5 marks)

Turn over

N 2 6 2 8 2 A 0 7 2 4

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4. (i) Find $\int \ln(\frac{x}{2})\,dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$.

(5)

(i) $\int 1.\ln\left(\frac{x}{2}\right)$ let $u = \ln\left(\frac{x}{2}\right)$ $\frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{1}{\frac{x}{2}} \times \frac{1}{2}$ $v = x$

$= \frac{1}{x}$

$I = x\ln\left(\frac{x}{2}\right) - \int \frac{1}{x}.x\,dx$

$I = x\ln\left(\frac{x}{2}\right) - \int 1\,dx$

$I = x\ln\left(\frac{x}{2}\right) - x + c$

Leave blank

Question 4 continued

(ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$

$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$

$I = \left[\frac{x}{2} - \frac{1}{4}\sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left(\frac{\pi}{4} - \frac{1}{4}\sin \pi \right) - \left(\frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2} \right)$

$= \frac{\pi}{4} - 0 - \frac{\pi}{8} + \frac{1}{4}$

$= \frac{1}{4} + \frac{\pi}{8}$

Q4

(Total 9 marks)

N 2 6 2 8 2 A 0 9 2 4

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5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$. **(3)**

(b) Find the gradient of the curve at each of these points. **(6)**

(a) when $x = -8$

$(-8)^3 - 4y^2 = 12(-8)y$

$~~48~~ \; -512 - 4y^2 = -96y$

$4y^2 - 96y + 512 = 0$

$y^2 - 24y + 128 = 0$

$y = 8, 16$

$\therefore$ points $(-8, 8)$ & $(-8, 16)$

(b) diff implicitly

$x^3 - 4y^2 = 12xy$

$3x^2 - 8y\frac{dy}{dx} = 12x.1\frac{dy}{dx} + y.12$

$3x^2 - 8y\frac{dy}{dx} = 12x\frac{dy}{dx} + 12y$

$3x^2 - 12y = 12x\frac{dy}{dx} + 8y\frac{dy}{dx}$

Question 5 continued

$$3(x^2 - 4y) = 4\frac{dy}{dx}(3x + 2y)$$

$$\frac{dy}{dx} = \frac{3(x^2 - 4y)}{4(3x + 2y)}$$

@ $(-8, 8)$

$$\frac{dy}{dx} = \frac{3((-8)^2 - 4(8))}{4(3(-8) + 2(8))}$$

$$= \frac{3(64 - 32)}{4(-24 + 16)} = \frac{96}{-32} = -3$$

@ $(-8, 16)$

$$\frac{dy}{dx} = \frac{3((-8)^2 - 4(16))}{4(3(-8) + 2(16))} = \frac{3(64 - 64)}{\text{blah}}$$

$$= 0$$

Leave blank

Q5

(Total 9 marks)

Leave blank

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

(a) $r_A = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \quad r_B = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$

$\overrightarrow{AB} = -r_A + r_B = -\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

(b) l_1 : r = point on line + λ direct of line

$r = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

(c) angle between l_1 + l_2 will be same as angle between direct vects

$\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \sqrt{1^2 + (-2)^2 + (2)^2} \sqrt{1^2 + 0^2 + 1^2} \cos\theta$

$1 + 0 + 2 = \sqrt{9}\sqrt{2} \cos\theta$

N 2 6 2 8 2 A 0 1 2 2 4

Question 6 continued

$3 = 3\sqrt{2}\cos\theta$

$\frac{1}{\sqrt{2}} = \cos\theta \qquad \theta = 45^0$

(d) eqⁿ of l_2 $\quad r = \begin{pmatrix}0\\0\\0\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$

@ C $\quad l_1 = l_2$

$\mu = 2+\lambda$ — (1)

$0 = 6-2\lambda$ — (2)

$\mu = -1+2\lambda$ — (3)

from (2) $\lambda = 3$

in (1) $\mu = 5$

in (3) $\mu = 5$ Consistent

$\therefore$ C $(5, 0, 5)$.

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7.

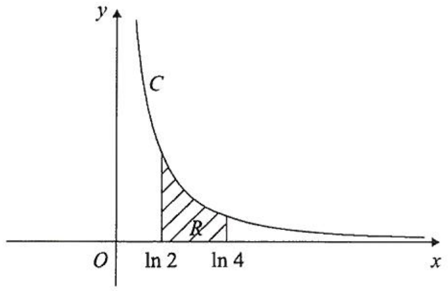

Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,dt.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = f(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

$A = \int y \frac{dx}{dt} . dt$

$\frac{dx}{dt} = \frac{1}{t+2} . 1$

$\therefore A = \int \frac{1}{t+1} . \frac{1}{t+2} \, dt$

N 2 6 2 8 2 A 0 1 6 2 4

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Question 7 continued

when $x = \ln 2$ $\qquad \ln 2 = \ln(t+2)$

$2 = t+2$

$t = 0$

when $x = \ln 4$ $\qquad \ln 4 = \ln(t+2)$

$4 = t+2$

$t = 2$

$$\therefore A = \int_0^2 \frac{1}{(t+1)(t+2)}\,dt$$

(b) $\dfrac{1}{(t+1)(t+2)} = \dfrac{A}{t+1} + \dfrac{B}{t+2}$

$= \dfrac{A(t+2) + B(t+1)}{(t+1)(t+2)}$

Comp coeffs: t: $A+B = 0 \Rightarrow A = -B$

t^0: $2A + B = 1$

$2(-B) + B = 1$

$B = -1$

$\therefore A = 1$.

N 2 6 2 8 2 A 0 1 7 2 4

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Question 7 continued

$\therefore \int_0^2 \frac{1}{t+1} - \frac{1}{t+2}\,dt$

$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$

$= \left[\ln\left(\frac{t+1}{t+2}\right)\right]_0^2$

$= \ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)$

$= \ln\left(\frac{\frac{3}{4}}{\frac{1}{2}}\right)$

$= \ln\left(\frac{3}{2}\right)$

(c) $x = \ln(t+2)$

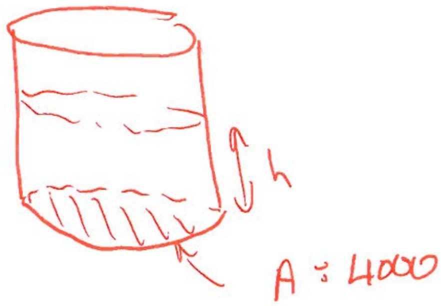
$e^x = t+2$

$t = e^x - 2$

$\therefore y = \frac{1}{t+1}$

$y = \frac{1}{e^x - 2 + 1} = \frac{1}{e^x - 1}$

(d) From graph $x > 0$



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8. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3 s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $cm^3 s^{-1}$.

(b) Show that $k = 0.02$

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

(a) Volume $V = 4000h$

rate of change of vol $\frac{dV}{dt} = 1600 - k\sqrt{h}$ — (1)

vol in   vol out

chain rule $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

N 2 6 2 8 2 A 0 2 0 2 4

Leave blank

Question 8 continued

Now $\frac{dV}{dh} = 4000$

$\therefore \quad \frac{dh}{dt} = \frac{1}{4000}\left[1600 - c\sqrt{h}\right]$

$\frac{dh}{dt} = 0.4 - \underbrace{\frac{c}{4000}}_{=k}\sqrt{h}$

$\frac{dh}{dt} = 0.4 - k\sqrt{h}$ as required.

(b) when $h = 25$ $\quad \cancel{\frac{dh}{dt} = \sqrt{400}} \quad \cancel{-k\sqrt{h} = 400}$

$\cancel{5k = 400}$

$\cancel{\text{Now } \frac{dV}{dt} = 1600 - k\sqrt{h}}$

$\cancel{400 = 1600 - 5k}$

$\cancel{5k = 1200}$

$\cancel{k = 240}$

$\cancel{\frac{k}{4000}\sqrt{h} = 400}$

$\cancel{k}$

From (1) $\quad c\sqrt{h} = 400$

$c\sqrt{25} = 400$

$c = \frac{400}{5} = 80$

Now $\frac{c}{4000} = k \quad \therefore k = \frac{80}{4000} = 0.02$

Question 8 continued

(c) $\frac{dh}{dt} = 0.4 - 0.02\sqrt{h}$

$\frac{dh}{dt} = 0.02(20 - \sqrt{h})$

$\int \frac{1}{0.02(20-\sqrt{h})}\,dh = \int dt$

$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh = t$

(d) $h = (20-x)^2$

$\frac{dh}{dx} = 2(20-x)^1 \cdot -1 = 2x - 40$

$dh = 2(x-20)\,dx$

Change limits: when $h=0$, $x=20$

$h=100$, $x=10$

$\therefore \int_{20}^{10} \frac{50}{20-(20-x)} \cdot 2(x-20)\,dx$

$= \int_{20}^{10} \frac{50}{x} \cdot 2(x-20)\,dx$

Leave blank

Question 8 continued

$$\int_{20}^{10} 100 - \frac{2000}{x} \, dx$$

$$\left[100x - 2000 \ln x \right]_{20}^{10}$$

$1000 - 2000\ln 10 - 2000 + 2000\ln 20$

$-1000 + 2000[\ln 20 - \ln 10]$

$= 2000\ln 2 - 1000$

$= 1000[2\ln 2 - 1]$

(e) $= 386.29$ sec

$\div 60$ min

$= 6.438$ min

6 min 26 sec.

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END