| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
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| Candidate No. | | | | | | 6 | 6 | 6 | 6 | / | 0 | 1 | Signature | |

Paper Reference(s)

# 6666/01

# Edexcel GCE

## Core Mathematics C4

## Advanced Level

Tuesday 23 January 2007 – Afternoon

Time: 1 hour 30 minutes

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.**

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.



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**1.** $$f(x) = (2 - 5x)^{-2}, \qquad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f($x$), in ascending powers of $x$, as far as the term in $x^3$, giving each coefficient as a simplified fraction.

**(5)**

$$\left[2\left(1-\tfrac{5}{2}x\right)\right]^{-2} = 2^{-2}\left(1-\tfrac{5}{2}x\right)^{-2}$$

$$= \frac{1}{4}\left[1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(\frac{-5}{2}x\right)^3\right]$$

$$= \frac{1}{4}\left[1 + 5x + \frac{75}{4}x^2 + \frac{125}{2}x^3 + \dots\right]$$

$$= \frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \frac{125}{8}x^3 + \dots$$

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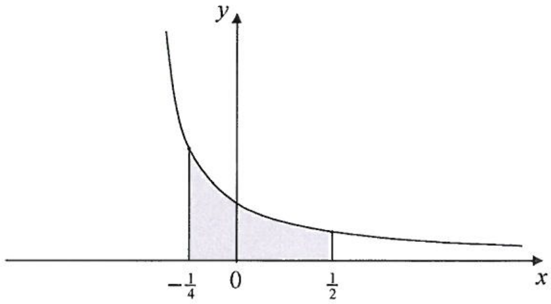
**2.**

**Figure 1**




The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the $x$-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the $x$-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

**(5)**

**Figure 2**

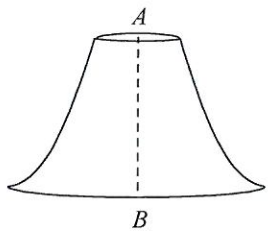



Figure 2 shows a paperweight with axis of symmetry $AB$ where $AB = 3$ cm. $A$ is a point on the top surface of the paperweight, and $B$ is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

**(2)**

(a) $V = \pi \int_{-1/4}^{1/2} \left( \frac{1}{3(1+2x)} \right)^2 dx$

$V = \frac{\pi}{9} \int_{-1/4}^{1/2} (1+2x)^{-2}\, dx$

**Question 2 continued**

$$V = \frac{\pi}{9}\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \frac{\pi}{9}\left[-\frac{1}{2(1+2x)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \frac{\pi}{18}\left[\frac{-1}{1+1} - -\frac{1}{1-\frac{1}{2}}\right]$$

$$= \frac{\pi}{18}\left[-\frac{1}{2} + 2\right]$$

$$= \frac{\pi}{18} \times \frac{3}{2}$$

$$= \frac{\pi}{12}$$

(b)


Scale factor 4

S.F. of volumes $= 4^3 = 64$

So vol of paperweight $= \frac{\pi}{12} \times 64 = \frac{16\pi}{3}$

Leave blank

Q2

**(Total 7 marks)**

Leave blank

**3.** A curve has parametric equations

$$x = 7\cos t - \cos 7t, \quad y = 7\sin t - \sin 7t, \qquad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of $t$. You need not simplify your answer.

**(3)**

(b) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.

Give your answer in its simplest exact form.

**(6)**

(a) $\frac{dx}{dt} = -7\sin t + 7\sin 7t$

$\frac{dy}{dt} = 7\cos t - 7\cos 7t$

$\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{7\sin 7t - 7\sin t}$

(b) @ $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - -\frac{\sqrt{3}}{2}$

$= 4\sqrt{3}$

$y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - -\frac{1}{2} = 4$

$\frac{dy}{dx} = \frac{\frac{7\sqrt{3}}{2} - -\frac{7\sqrt{3}}{2}}{-\frac{7}{2} - \frac{7}{2}} = \frac{7\sqrt{3}}{-7} = -\frac{\sqrt{3}}{1}$

$\therefore$ Mnormal $= +\frac{1}{\sqrt{3}}$

eq$^n$ normal through $(4\sqrt{3}, 4)$

$y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$

$y\sqrt{3} - 4\sqrt{3} = x - 4\sqrt{3} \Rightarrow y = \frac{1}{\sqrt{3}}x$



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4. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

**(3)**

(b) Given that $x \geqslant 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y.$$

**(5)**

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$.

**(4)**

(a) $\dfrac{2x-1}{(x-1)(2x-3)} = \dfrac{A}{x-1} + \dfrac{B}{2x-3}$

$= \dfrac{A(2x-3) + B(x-1)}{(x-1)(2x-3)}$

$= \dfrac{2Ax - 3A + Bx - B}{(x-1)(2x-3)}$

Compare Coeficients : $x^1$ : $2A + B = 2$ — (1)

$x^0$ : $-3A - B = -1$ — (2)

add $-A = 1$

$A = -1$

in (1) $2(-1) + B = 2$

$-2 + B = 2$

$B = 4$

$\therefore \dfrac{2x-1}{(x-1)(2x-3)} = \dfrac{4}{2x-3} - \dfrac{1}{x-1}$



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**Question 4 continued**

(b) $\cancel{I}$ $(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y$

$$\int \frac{1}{y}\, dy = \int \frac{2x-1}{(2x-3)(x-1)}\, dx$$

$$\cancel{\ln y} \int \frac{1}{y}\, dy = \int \frac{4}{(2x-3)} - \frac{1}{(x-1)}\, dx$$

$$\ln y = \frac{4}{2}\ln(2x-3) - \ln(x-1) + c$$

$$\ln y = 2\ln(2x-3) - \ln(x-1) + c$$

(c) @ $(2,10)$

$$\ln 10 = 2\ln(4-3) - \ln(2-1) + c$$

$$c = \ln 10$$

$$\ln y = 2\ln(2x-3) - \ln(x-1) + \ln 10$$

$$\ln y = \ln\left(\frac{(2x-3)^2 \cdot 10}{(x-1)}\right)$$

$$y = \frac{10(2x-3)^2}{(x-1)}$$

Q4

**(Total 12 marks)**

Leave blank

**5.** A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$.

**(2)**

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

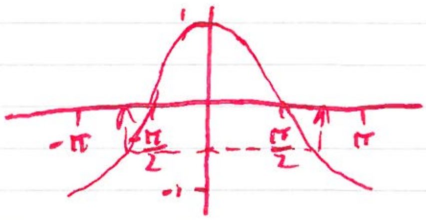
(b) find the coordinates of the points where $\frac{dy}{dx} = 0$.

**(5)**

(a) $\cos x - \sin y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{\cos x}{\sin y}$

(b) $\frac{\cos x}{\sin y} = 0$




$\cos x = 0$

$x = -\frac{\pi}{2}, \frac{\pi}{2}$

So for y: $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$

$-1 + \cos y = 0.5$

$\cos y = \frac{3}{2}$

$y =$ no solution

$\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$

$1 + \cos y = 0.5$

$\cos y = -0.5$

$y = \frac{2\pi}{3}$ & $-\frac{2\pi}{3}$ $\therefore$ Coords $\left(\frac{\pi}{2}, \pm\frac{2\pi}{3}\right)$



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6. (a) Given that $y = 2^x$, and using the result $2^x = e^{x\ln 2}$, or otherwise, show that $\frac{dy}{dx} = 2^x \ln 2$.

**(2)**

(b) Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2,16).

**(4)**

(a) $y = 2^x$

$\ln y = \ln 2^x$

$\ln y = x \ln 2$

$\frac{1}{y}\frac{dy}{dx} = \ln 2$

$\frac{dy}{dx} = y \ln 2$

$\therefore \frac{dy}{dx} = 2^x \ln 2$ as required.

(b) $y = 2^{(x^2)}$

$\ln y = x^2 \ln 2$

$\frac{1}{y}\frac{dy}{dx} = x^2 . 0 + \ln 2 . 2x$

$\frac{dy}{dx} = 2xy \ln 2$

@ (2,16) $\frac{dy}{dx} = 2 \times 2 \times 16 \times \ln 2 = 64 \ln 2$

Leave blank

7. The point $A$ has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point $B$ has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin $O$.

(a) Find the position vector of the point $C$, with position vector $\mathbf{c}$, given by

$$\mathbf{c} = \mathbf{a} + \mathbf{b}.$$

**(1)**

(b) Show that $OACB$ is a rectangle, and find its exact area.

**(6)**

The diagonals of the rectangle, $AB$ and $OC$, meet at the point $D$.

(c) Write down the position vector of the point $D$.

**(1)**

(d) Find the size of the angle $ADC$.

**(6)**

(a) $C = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$

(b)


for a rectangle $OA \cdot OB = 0$

$BO \cdot BC = 0$

$AO \cdot AC = 0$

$CB \cdot CA = 0$

And $\left(|\vec{OB}| = |\vec{AC}|\right) \neq \left(|\vec{OA}| = |\vec{BC}|\right)$



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**Question 7 continued**

$\overrightarrow{OA} \cdot \overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = 2+2-4 = 0$ ✓

$\overrightarrow{BO} \cdot \overrightarrow{BC} = -\begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \cdot \left[ -\begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix} \right] = \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$= -2-2+4 = 0$ ✓

$\overrightarrow{AO} \cdot \overrightarrow{AC} = -\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \left[ -\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix} \right] = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix}$

$= -2 -2 +4 = 0$ ✓

$\overrightarrow{CB} \cdot \overrightarrow{CA} = -\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot -\begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ +4 \end{pmatrix} = 2+2-4$

$= 0$ ✓

So all angles are $90^\circ$

$|\overrightarrow{OB}| = \sqrt{1^2+1^2+-4^2} = \sqrt{18}$

$|\overrightarrow{AC}| = \sqrt{1^2+1^2+-4^2} = \sqrt{18}$

$=$ ✓

$\neq$ ✓

$|\overrightarrow{OA}| = \sqrt{2^2+2^2+1^2} = 3$

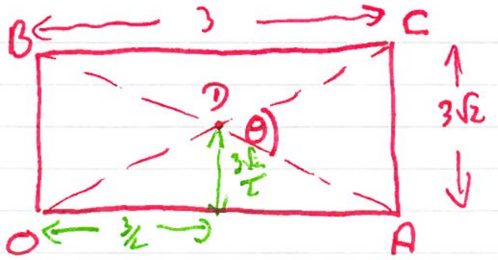
$|\overrightarrow{BC}| = \sqrt{2^2+2^2+1^2} = 3$

$=$ ✓

So OACB is a rectangle Area $3 \times \sqrt{18} = 9\sqrt{2}$

**Question 7 continued**

(c)




$$r_D = \frac{1}{2}\overrightarrow{OC} = \frac{1}{2}\begin{pmatrix}3\\3\\-3\end{pmatrix}$$

(d) $\overrightarrow{DC}\cdot\overrightarrow{DA} = |\overrightarrow{DC}|.|\overrightarrow{DA}|\cos\theta$

$$\overrightarrow{DC} = \begin{pmatrix}3/2\\3/2\\-3/2\end{pmatrix} \qquad |\overrightarrow{DC}| = \sqrt{\tfrac{9}{4}+\tfrac{9}{4}+\tfrac{9}{4}} = \sqrt{\tfrac{27}{4}}$$

$$\overrightarrow{DA} = -\overrightarrow{OD} + \overrightarrow{OA} = -\begin{pmatrix}3/2\\3/2\\-3/2\end{pmatrix} + \begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}1/2\\1/2\\5/2\end{pmatrix}$$

$$|\overrightarrow{DA}| = \sqrt{\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{25}{4}} = \sqrt{\tfrac{27}{4}}$$

$$\begin{pmatrix}3/2\\3/2\\-3/2\end{pmatrix}\cdot\begin{pmatrix}1/2\\1/2\\5/2\end{pmatrix} = \frac{\sqrt{27}}{2}.\frac{\sqrt{27}}{2}\cos\theta$$

$$\frac{3}{4}+\frac{3}{4}-\frac{15}{4} = \frac{27}{4}\cos\theta$$

$$-\frac{9}{4} = \frac{27}{4}\cos\theta$$



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**Question 7 continued**

$\cos\theta = \frac{-9}{27} = -\frac{1}{3}$

$\theta = 109.5°$ (1dp)

Q7

**(Total 14 marks)**

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**8.**

$$I = \int_0^5 e^{\sqrt{(3x+1)}}\,dx.$$

(a) Given that $y = e^{\sqrt{(3x+1)}}$, complete the table with the values of $y$ corresponding to $x = 2$, 3 and 4.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | $e^1$ | $e^2$ | $e^{\sqrt{7}}$ | $e^{\sqrt{10}}$ | $e^{\sqrt{13}}$ | $e^4$ |

**(2)**

(b) Use the trapezium rule, with all the values of $y$ in the completed table, to obtain an estimate for the original integral $I$, giving your answer to 4 significant figures.

**(3)**

(c) Use the substitution $t = \sqrt{(3x+1)}$ to show that $I$ may be expressed as $\int_a^b kte^t\,dt$, giving the values of $a$, $b$ and $k$.

**(5)**

(d) Use integration by parts to evaluate this integral, and hence find the value of $I$ correct to 4 significant figures, showing all the steps in your working.

**(5)**

(b) $I \approx \frac{1}{2}\left[e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4\right]$

$\approx 110.6$ (4 s.f.)

(c) $t = \sqrt{(3x+1)}$ when $x=0$ ~~t=1~~ $t=1$

$x=5$ $t=4$

$t = (3x+1)^{\frac{1}{2}}$

$\frac{dt}{dx} = \frac{1}{2}(3x+1)^{-\frac{1}{2}}.3 = \frac{3}{2\sqrt{3x+1}}$

$dx = \frac{2\sqrt{3x+1}}{3}dt = \frac{2}{3}t\,dt$

$\therefore I = \int_1^4 e^t.\frac{2}{3}t\,dt = \frac{2}{3}\int_1^4 te^t\,dt$



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**Question 8 continued**

(d) let $u = t$ $\quad \frac{dv}{dt} = e^t$

$\frac{du}{dt} = 1 \quad V = e^t$

$I = te^t - \int e^t dt$

$I = \frac{2}{3}\left[te^t - e^t\right]_1^4$

$I = \frac{2}{3}\left[4e^4 - e^4 - (1e^1 - e^1)\right]$

$= \frac{2}{3} \cdot 3e^4$

$= 2e^4$

$= 109.2$ (4 sf).



N 2 3 5 6 2 A 0 1 9 2 0