C3 - Functions + Inverses ppq's

(1) $f(x) = \frac{8}{x+2}$ dom $[0, \infty)$

$f(0) = 4$, as $x \to \infty$ $f(x) \to 0$ $\therefore$ Range $(0, 4]$

for inverse, let $y = \frac{8}{x+2}$

$y(x+2) = 8$

$yx + 2y = 8$

$yx = 8 - 2y$

$x = \frac{8-2y}{y}$

$\therefore f^{-1}(x) = \frac{8-2x}{x}$

dom $f^{-1}(x)$ = range $f(x)$ = $(0, 4]$

(2)(a) $f(x) = 3x^2 + 4$ dom $[0, \infty)$

$f(0) = 4$, as $x \to \infty$ $f(x) \to \infty$ $\therefore$ Range $[4, \infty)$

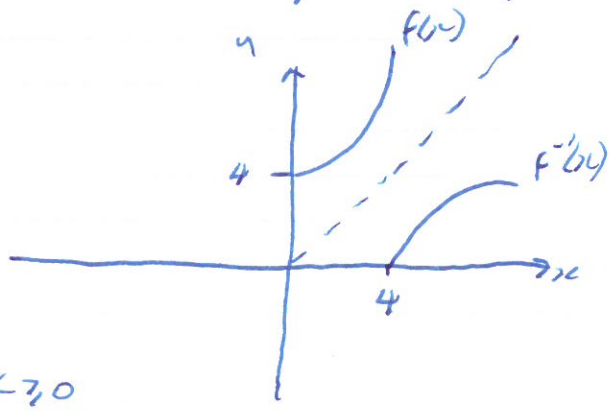
let $y = 3x^2 + 4$

$3x^2 = y - 4$

$x^2 = \frac{y-4}{3}$

$x = \pm\sqrt{\frac{y-4}{3}}$, but $x \geq 0$

$\therefore f^{-1}(x) = \sqrt{\frac{x-4}{3}}$



dom $f^{-1}(x)$ = Range $f(x) = [4, \infty)$

Range $f^{-1}(x)$ = dom $f(x) = [0, \infty)$

(3) $f(x) = \ln(x-2)+3$ dom $(2,\infty)$, range $(3,\infty)$

let $y = \ln(x-2)+3$

$y-3 = \ln(x-2)$

$e^{y-3} = x-2$

$x = e^{y-3}+2$

So $F^{-1}(x) = e^{x-3}+2$

(4) $f(x) = \ln(5x-4)+2$ dom $[1,\infty)$

$f(1) = 2$ as $x \to \infty$ $f(x) \to \infty$ ∴ range $[2,\infty)$

let $y = \ln(5x-4)+2$

$y-2 = \ln(5x-4)$

$e^{y-2} = 5x-4$

$x = \frac{e^{y-2}+4}{5}$

∴ $F^{-1}(x) = \frac{e^{x-2}+4}{5}$

dom $F^{-1}(x)$ = range $F(x)$ = $[2,\infty)$

range $F^{-1}(x)$ = dom $F(x)$ = $[1,\infty)$

(5) $f(x) = \frac{1}{\sqrt{x-2}}$ dom $(2, \infty)$

(a) $f(2) = \infty$ as $x \to 2$ $f(x) \to 0$

$\therefore$ range $(0, \infty)$

(b) let $y = \frac{1}{\sqrt{x-2}}$

$$y^2 = \frac{1}{x-2}$$

$$y^2(x-2) = 1$$

$$xy^2 - 2y^2 = 1$$

$$xy^2 = 1 + 2y^2$$

$$x = \frac{1+2y^2}{y^2}$$

$\therefore F^{-1}(x) = \frac{1+2x^2}{x^2}$ or $\frac{1}{x^2} + 2$

dom $F^{-1}(x)$ = range $F(x) = (0, \infty)$

range $F^{-1}(x)$ = dom $F(x) = (2, \infty)$

(c) $\frac{1+2x^2}{x^2} = -\frac{3}{x}$

$$1 + 2x^2 = -\frac{3}{x} \times x^2$$

$$1 + 2x^2 = -3x$$

$$2x^2 + 3x + 1 = 0$$

either $x = -0.5$ or $x = -1$

both outside domain.

⑥. $F(x) = \sqrt{x+1}$ dom $[0, \infty)$

$F(0) = 1$, as $x \to \infty$ $F(x) \to \infty$ $\therefore$ Rang $[1, \infty)$

(a) let $y = \sqrt{x+1}$

$y^2 = x+1$

$x = y^2 - 1$

$\therefore F^{-1}(x) = x^2 - 1$

(b) dom $F^{-1}(x)$ = range $F(x)$ = $[1, \infty)$

range $F^{-1}(x)$ = dom $F(x)$ = $[0, \infty)$

(c)