

## Functions C3 PP's

### C3 - JANOB

Q9  
3.

$$f(x) = e^x \quad \text{dom } (-\infty, \infty)$$

$$g(x) = \ln(x^2 - 4) \quad \text{dom } (2, \infty)$$

BI

(a)  $f \circ g$  domain  $(2, \infty)$

BI

(b)  $f \circ g(x) = e^{\ln(x^2 - 4)} = x^2 - 4$

$$\therefore x^2 - 4 = 5$$

M/

A/

$$x^2 = 9$$

A/

$$x = \pm 3$$

$x = -3$  outside domain

$$\therefore x = 3 \quad \text{A/}$$

Q

### C3 MAY 13

1/  
2.

$$f(x) = \ln(x) \quad \text{dom } (0, \infty)$$

$$g(x) = \tan x \quad \text{dom } \left(0, \frac{\pi}{4}\right)$$

(a)(i) dom  $f \circ g = \text{dom } g = \left(0, \frac{\pi}{4}\right]$

BI

(ii) range  $f \circ g$

$$f \circ g(x) = \ln(\tan x)$$

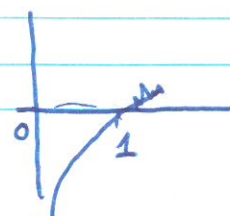
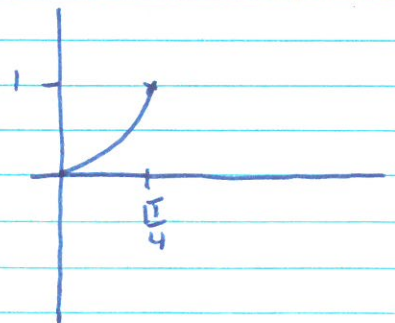
$\tan x$

$$\text{So range } g(x) = (0, 1]$$

$$\text{range } f \circ g(x) = (-\infty, 0]$$

BI BI

$\ln x$



$$(b) (i) \quad f(g(x)) = -0.4$$

$$\ln(\tan x) = -0.4$$

$$\text{take e's} \quad \tan x = e^{-0.4} \quad M1$$

$$x = \tan^{-1}(e^{-0.4})$$

$$x = 0.59 \quad A1$$

(ii) any value  $j - k > 0$  will have no solution B1

### C3 - JAN 13 Q8

$$(a) \quad f(x) = \ln(4x+5) - 2$$

$$\text{dom } f(x) = \ln(4x+5) - 2$$

$$\text{dom } [-1, \infty)$$

$$f(-1) = \ln(-4+5) - 2 = -2$$

as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty \quad \therefore \text{range } [-2, \infty)$

$$(a) \quad \text{let } y = \ln(4x+5) - 2$$

$$(+2) \quad y+2 = \ln(4x+5) \quad B1$$

$$\text{(take e's)} \quad e^{y+2} = 4x+5 \quad M1$$

$$(-x) \quad e^{y+2} - 5 = 4x$$

Did this wrong on video clip!!

$$(\div 4) \quad \frac{e^{y+2} - 5}{4} = x \quad A1$$

$$\text{so } f^{-1}(x) = \frac{e^{x+2} - 5}{4} \quad A1$$

(b) dom = range  $f(x)$

$$= [-2, \infty) \quad B1$$

C3 - May 13 Q10

$$f(x) = e^{5-\frac{x}{2}} + 6$$

$$\text{dom } (-\infty, 10]$$

$$f(10) = e^{5-\frac{10}{2}} + 6 = 7$$

$$\text{range } [7, \infty)$$

$$(a) \text{ let } y = e^{5-\frac{x}{2}} + 6$$

$$(-6) \quad y-6 = e^{5-\frac{x}{2}} \quad \text{BI}$$

$$(ln) \quad \ln(y-6) = 5 - \frac{x}{2}$$

$$\frac{x}{2} + \ln(y-6) = 5$$

$$\frac{x}{2} = 5 - \ln(y-6)$$

$$x = 10 - 2\ln(y-6)$$

MI AI

$$\therefore f^{-1}(x) = 10 - 2\ln(x-6)$$

AI

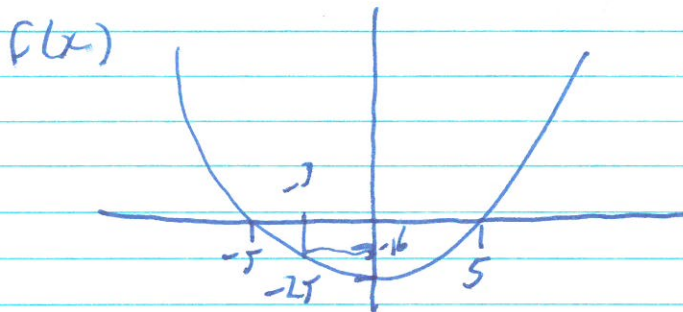
$$(b) \text{ dom } f^{-1} = \text{range } f(x) = [7, \infty) \quad \text{BI} \quad \text{BI}$$

C3 - JAN 13 Q9

(a)  $f(x) = x^2 - 25$        $\text{dom}(-\infty, \infty)$   
 $g(x) = 2x - 3$        $\text{dom}(0, \infty)$

(i)  $\text{dom } f \circ g = \text{dom } g = (0, \infty)$       B1

(ii)  $\text{rang } g(x)$        $f(0) = -3$   
as  $x \rightarrow \infty$ ,  $g(x) \rightarrow \infty$   
So  $\text{range } g(x) = (-3, \infty)$



$$f(-3) = (-3)^2 - 25 = 9 - 25 = -16$$

$\text{range } f \circ g(x) = (-25, \infty)$       B1      B1

(iii)  $f \circ g(x) = (2x - 3)^2 - 25$       B1

(iv)  $f \circ g(x) = 0$

$$(2x - 3)^2 - 25 = 0$$
      M1

$$4x^2 - 12x + 9 - 25 = 0$$

$$4x^2 - 12x - 16 = 0$$

$\div 4$   
 $x^2 - 3x - 4 = 0$

$$(x - 4)(x + 1) = 0 \quad \therefore \text{either } x = 4 \text{ or } x = -1 \quad \text{A1}$$

but  $x > 0 \therefore x = 4$       A1

$$(b)(i) \quad h(x) = \frac{2x+7}{5x-2}$$

$$hh(x) = \left( 2 \left[ \frac{2x+7}{5x-2} \right] + \frac{7}{1} \right) \div \left( 5 \left( \frac{2x+7}{5x-2} \right) - 2 \right) \quad M1$$

$$= \left( \frac{4x+14+7(5x-2)}{5x-2} \right) \div \left( \frac{10x+35-2(5x-2)}{5x-2} \right)$$

$$= \left( \frac{4x+14+35x-14}{5x-2} \right) \times \left( \frac{(10x+35) \cancel{5x-2}}{10x+35-10x+4} \right)$$

$$= \frac{39x}{1} \times \frac{1}{39}$$

$$= x \quad A) \text{ required.} \quad A1$$

$$(ii) \quad h^{-1}(x) = h(x) \quad B1$$

$$y = \frac{2x+7}{5x-2}$$

$$y(5x-2) = 2x+7$$

$$5xy - 2y = 2x+7$$

$$5xy - 2x = 2y+7$$

$$x(5y-2) = 2y+7$$

$$x = \frac{2y+7}{5y-2}$$

$$h^{-1}(x) = \frac{2x+7}{5x-2} = h(x)$$

C3 JANIB Q7

(a)  $|3x-4| > 5$

either  $3x-4 > 5$  or  $-(3x-4) > 5$  M1

$3x > 9$

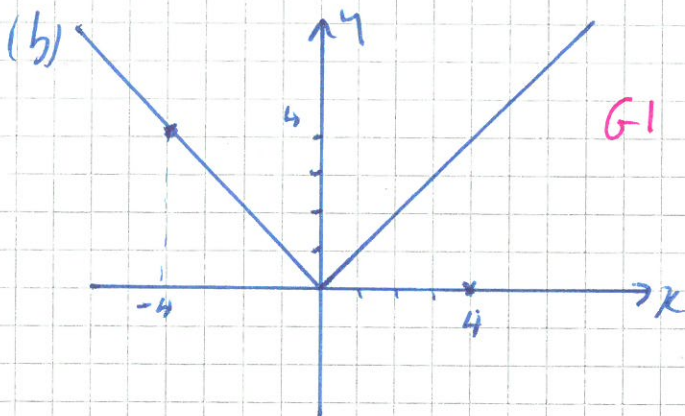
$x > 3$  A1

$3x-4 < -5$

$3x < -1$

$x < -\frac{1}{3}$  A1

$\therefore x > 3$  or  $x < -\frac{1}{3}$



$y = a|x+b|$

$|x|$  moved to  $(4,0)$

$\therefore b = -4$  B1

graph has been inverted

$\therefore a < 0$

y coord =  $-2 \times 4 = -8$

$\therefore a = -2$  B1

So  $y = -2|x-4|$

C3 May 13 @ 9

$$y = -\frac{2}{3}f(x+4)$$

multiplies y coords by  $-\frac{2}{3}$

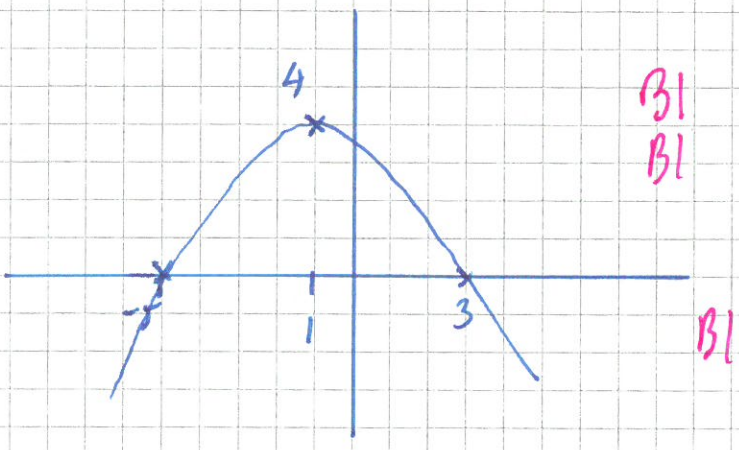
takes 4 off x-coords

$$(-1, 0) \rightarrow (-5, 0) \rightarrow (-5, 0)$$

$$(1, -6) \rightarrow (-1, -6) \rightarrow (-1, 4)$$

$$(7, 0) \xrightarrow{-4 \text{ off } x} (3, 0) \xrightarrow{4 \times \frac{2}{3}} (3, 0)$$

So



C3 Jan 13 Q9

(b)  $h \circ h(x) = x$

$$h^{-1}(x)$$

$$F(x) = \ln x$$

$$F^{-1}(e^x) = e^x$$

$$F F^{-1}(x) = \ln(e^x) = x$$

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$$F(x) = x - 2$$

$$F^{-1}(x) = x + 2$$

$$F F^{-1}(x) = (x + 2) - 2 = x$$

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$$F(x) = 2x$$

$$F^{-1}(x) = \frac{x}{2}$$

$$F F^{-1}(x) = 2\left(\frac{x}{2}\right) = x$$

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So if  $h \circ h(x) = x$

then  $h(x) = h^{-1}(x)$