C3 Composite Functions (1)

9. The function f has domain $(-\infty, \infty)$ and is defined by $f(x) = e^x$.

The function g has domain $(2, \infty)$ and is defined by $g(x) = \ln(x^2 - 4)$.

(a) State the domain of fg. [1]

(b) Solve the equation $fg(x) = 5$. [4]

10. The functions f and g have domains $(0, \infty)$ and $(5, \infty)$ respectively, and are defined by

$$f(x) = x^2 + 1,$$
$$g(x) = 2x - 3.$$

(a) Write down the ranges of f and g. [2]

(b) Give the reason why $gf(1)$ cannot be formed. [1]

(c) Solve the equation $fg(x) = 3x^2 - 6x + 17$. [4]

9. The functions f and g have domains $(0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = e^{4x}.$$

Find and simplify an expression for

(a) $fg(x)$,

(b) $gf(x)$.

8. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively, and are defined by

$$f(x) = e^x,$$

$$g(x) = x^2 + 1.$$

(a) Find the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$, simplifying your expression as much as possible. [2]

(c) Write down the domain and range of gf. [2]

(d) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = gf(x)$ indicating where the graphs meet the y-axis. [5]

C3 Composite Functions (2)

8. The function f has domain $x \geqslant 1$ and is defined by

(5)

$$f(x) = x - \frac{1}{x}.$$

(a) Show that $f'(x)$ is always positive. Deduce the least value of $f(x)$. [3]

(b) Find the range of f. [1]

(c) The function g has domain $[0, \infty)$ and is defined by

$$g(x) = 3x^2 + 2.$$

Solve the equation

$$gf(x) = \frac{3}{x^2} + 8.$$ [4]