C3/M1 Homework (Due 16/03/16) (M/57)

M1 Jan 13 Q1

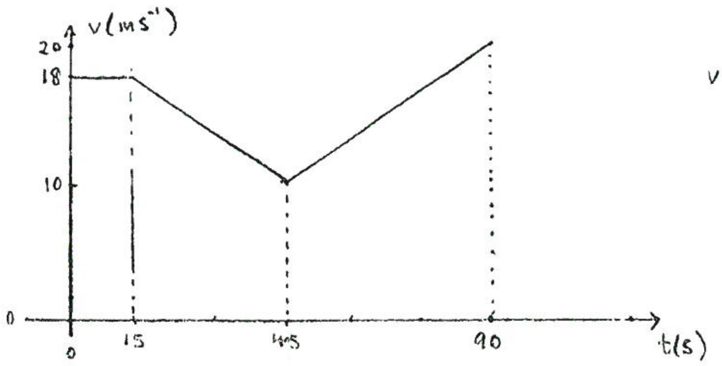
Q	Solution	Mark	Notes
1(a).	Using $v = u + at$ with $u=12$, $v=32$, $t=4$	M1	o.e.
	$32 = 12 + 4a$	A1	
	$a = \underline{5 \text{ ms}^{-2}}$	A1	cao
1(b)	Using $s = ut + 0.5at^2$, $u=12, t=4$, $a=5$	M1	
	$s = 12 \times 4 + 0.5 \times 5 \times 4^2$	A1	
	$s = \underline{88 \text{ m}}$	A1	cao
	OR		
	Using $v^2 = u^2 + 2as$, $u=12$, $v=32$, $a=5$	M1	
	$32^2 = 12^2 + 2 \times 5s$	A1	
	$s = \underline{88 \text{ m}}$	A1	cao
	OR		
	Using $s = 0.5(u + v)t$, $u=12$, $v=32$, $t=4$	M1	
	$s = 0.5(12 + 32) \times 4$	A1	
	$s = \underline{88\text{m}}$	A1	cao
1(c)	Using $v^2 = u^2 + 2as$, $u=12$, $a=5$, $s=44$	M1	ft answer in (b) for s
	$v^2 = 12^2 + 2 \times 5 \times 44$	A1	ft (b)
	$v = \underline{24.2 \text{ ms}^{-1}}$	A1	ft (b)

(M/9)

M1 Jun 05 Q2

2.

(a)



v-t graph M1

A3

(b) $a = \frac{20-10}{45}$ M1

$= \frac{2}{9}\ ms^{-2}$ A1

(c) Distance AB $= (18 \times 15) + \frac{1}{2} \times 30(18+10) + \frac{1}{2} \times 45(10+20)$ M1 A1

$= 270 + 420 + 675$ B1

$= 1365\ m.$ cao A1

~~(M/8)~~ M/10

M1 Jan 13 Q3

Q	Solution	Mark	Notes
3(a)	$s = ut + 0.5at^2$, $s=(\pm)1.2$, $a=(\pm)9.8$, $u=15$	M1	complete method
	$-1.2 = 15t + 0.5 \times (-9.8)t^2$	A1	
	$4.9t^2 - 15t - 1.2 = 0$		
	Use of correct formula to solve quad eq	m1	
	$t = 3.139$		
	$t =$ 3.1 s (to one d. p.)	A1	
3(b)	For the model used, the time taken for the particle to reach the ground is independent of the weight of the particle. I would expect the time to be the same as that in (a).	E1	no reason given gets E0

(M/5)

C3 Jan 13 Q1

1.

1	0·211941557		
1·25	0·182137984		
1·5	0·154280773		
1·75	0·128955672		
2	0·106506978	(5 values correct)	B2

(**If B2** not awarded, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·25$ — M1

$I \approx \frac{0·25}{3} \times \{0·211941557 + 0·106506978 + 4(0·182137984 + 0·128955672) + 2(0·154280773)\}$

$I \approx 1·871384705 \times 0·25 \div 3$

$I \approx 0·155948725$

$I \approx 0·156$ — (f.t. one slip) — A1

Note: Answer only with no working earns 0 marks

(M/4)

C3 Jan 13 Q3

3. (*a*)

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ — B1

$\frac{d}{dx}(5x^4y) = 5x^4\frac{dy}{dx} + 20x^3y$ — B1

$\frac{d}{dx}(x^3) = 3x^2, \ \frac{d}{dx}(7) = 0$ — B1

$\frac{dy}{dx} = \frac{20x^3y + 3x^2}{6y^2 - 5x^4}$ (o.e.) — (c.a.o.) B1

(*b*) (i) candidate's x-derivative $= 3t^2$ — B1

candidate's y-derivative $= 4t^3 + 35t^4$ — B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ — M1

$\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2}$ — (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{4 + 70t}{3}$ (o.e.) — B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

(f.t. candidate's expression for $\frac{dy}{dx}$) — M1

$\frac{d^2y}{dx^2} = \frac{4 + 70t}{9t^2}$ (o.e.) — A1

(iii) An attempt to solve $t^3 - 5 = 3$ and substitution of answer in candidate's expression for $\frac{d^2y}{dx^2}$ — M1

$\frac{d^2y}{dx^2} = 4$ — (c.a.o.) A1

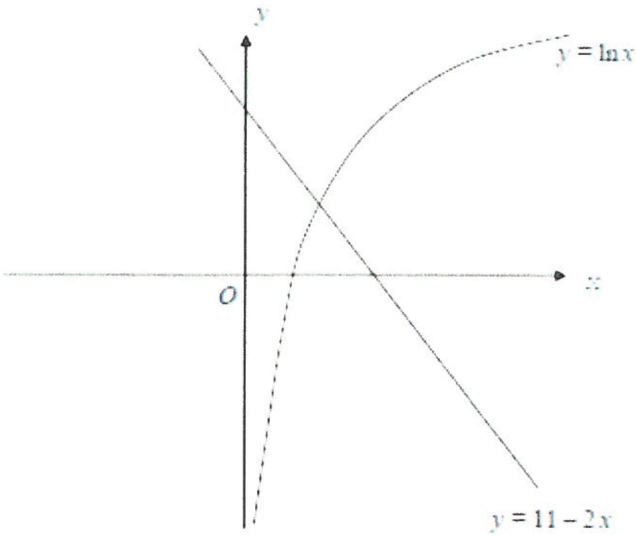
(M/13)

5. (a)	(i)	$\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times f(x)$	$(f(x) \neq 1)$	M1
		$\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times (10x - 3)$		A1
	(ii)	$\frac{dy}{dx} = \frac{\pm 7}{\sqrt{(1-(7x)^2)}}$ or $\frac{1}{\sqrt{(1-(7x)^2)}}$ or $\frac{7}{\sqrt{(1-7x^2)}}$		M1
		$\frac{dy}{dx} = \frac{7}{\sqrt{(1-49x^2)}}$		A1
	(iii)	$\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$		M1
		$\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$	(either $f(x) = 1/x$ or $g(x) = 3e^{3x}$)	A1
		$\frac{dy}{dx} = \frac{e^{3x}}{x} + 3e^{3x} \ln x$	(all correct)	A1
(b)		$\frac{d}{dx}(\cot x) = \frac{\sin x \times m \sin x - \cos x \times k \cos x}{\sin^2 x}$	$(m = 1, -1, k = 1, -1)$	M1
		$\frac{d}{dx}(\cot x) = \frac{\sin x \times (-\sin x) - \cos x \times (\cos x)}{\sin^2 x}$		A1
		$\frac{d}{dx}(\cot x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$		
		$\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\mathrm{cosec}^2 x$	(convincing)	A1

(M/10)

C3 Jan 13 Q4

4. (*a*)



Correct shape for $y = \ln x$, including the fact that the y-axis is an asymptote at $-\infty$	B1
A straight line with positive intercept and negative gradient intersecting once with $y = \ln x$ in the first quadrant.	B1
Equation has one root (c.a.o.)	B1

(*b*)

$x_0 = 4{\cdot}7$	
$x_1 = 4{\cdot}726218746$ (x_1 correct, at least 5 places after the point)	B1
$x_2 = 4{\cdot}723437268$	
$x_3 = 4{\cdot}723731615$	
$x_4 = 4{\cdot}723700458 = 4{\cdot}72370$ (x_4 correct to 5 decimal places)	B1
Let $h(x) = \ln x + 2x - 11$	
An attempt to check values or signs of $h(x)$ at $x = 4{\cdot}723695$, $x = 4{\cdot}723705$	M1
$h(4{\cdot}723695) = -1{\cdot}87 \times 10^{-5} < 0$, $h(4{\cdot}723705) = 3{\cdot}45 \times 10^{-6} > 0$	A1
Change of sign $\Rightarrow \alpha = 4{\cdot}72370$ correct to five decimal places	A1

(M/8)