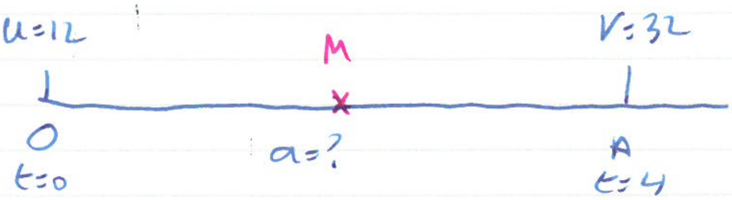
C3 - M1 Homework (16/03/16)

(1)(a)



Using $v = u + at$

$32 = 12 + 4a$ M1 A1

$\frac{32-12}{4} = a$

$a = \frac{20}{4} = 5\,ms^{-2}$ A1

(b) $s = ut + \frac{1}{2}at^2$

$s = (12 \times 4) + \frac{1}{2} \times 5 \times 4^2 = 48 + 40 = 88m$ M1 A1 A1

(c) $s = 44$, $u = 12$, $a = 5$, $v = ?$

Use $v^2 = u^2 + 2as$

$v^2 = 12^2 + 2 \times 5 \times 44$ M1 A1

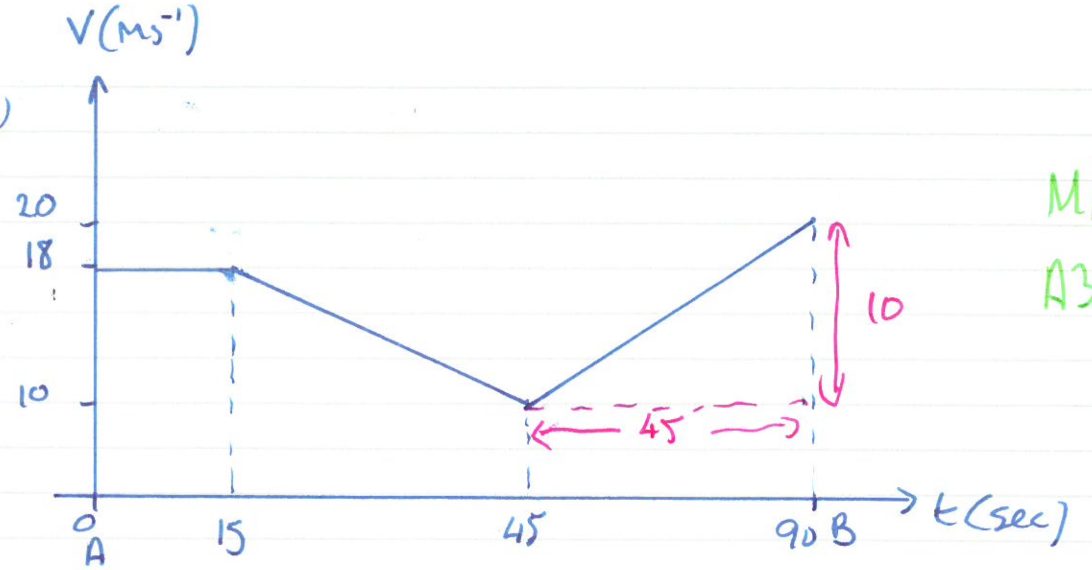
$v^2 = 144 + 440$

$v^2 = 584$

$v = \sqrt{584} = 24.2\,ms^{-1}$ A1

M/9

(2) (a)



M1

A3

(b) accel = gradient = $\frac{10}{45}$ = $\frac{2}{9}$ ms⁻² M1 A1

(c) dist = area beneath graph

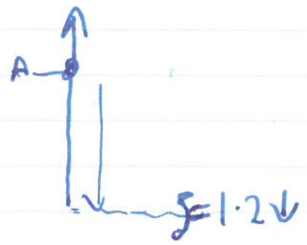
= $(18\times15) + \frac{1}{2}(18+10)\times30 + \frac{1}{2}(10+20)\times45$ M1 A1

= 270 + 420 + 675 A1

= 1365 m A1

M1 / 10

(3) $u = 15 \uparrow$



$a = 9.8 \downarrow$

(a) $u = 15\uparrow$, $s = 1.2\downarrow$, $a = 9.8\downarrow$, $t = ?$
$= -15\downarrow$

$$s = ut + \frac{1}{2}at^2$$

$$1.2 = -15t + \frac{1}{2} \times 9.8t^2$$ M1 A1

$$4.9t^2 - 15t - 1.2 = 0$$

$$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4 \times 4.9 \times -1.2}}{2 \times 4.9}$$ M1

$$t = \frac{15 \pm \sqrt{225 + 23.52}}{9.8}$$

$$t = \frac{15 \pm 15.76}{9.8}$$

$$t = \frac{15 + 15.76}{9.8} = 3.1 \text{ sec}$$ A1

(b) the time would be the same as these calculations are independant of weight

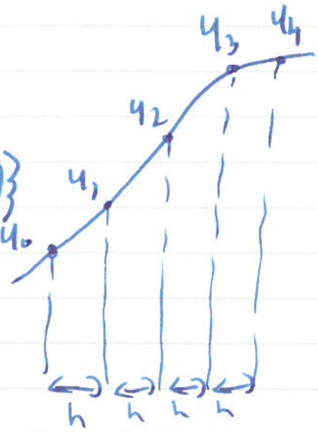
E1

M/5

④ $$\int_1^2 \frac{1}{2+e^x}\,dx$$

$$\left[\ \int_a^b y\,dx = \frac{h}{3}\left\{(y_0+y_4)+4(y_1+y_3)+2(y_2)\right\}\right.$$

$$h = \frac{b-a}{n}$$



$$h = \frac{2-1}{4} = \frac{1}{4}$$

$$\cancel{y_0 = \frac{1}{2+e^0} = \frac{1}{3}}$$

$$\cancel{y_0 = \frac{1}{2+e^1} = 0.2119415576}$$

$$\cancel{y_1 = \frac{1}{2+e^2} = 0.1065069784}$$

$$\cancel{y_2 = \frac{1}{2+e^3} = 0.04522850074}$$

$$\cancel{y_3 = \frac{1}{2+e^4} =}$$

$x_0 = 1 \quad y_0 = \dfrac{1}{2+e^1} = 0.2119415576$

$x_1 = 1.25 \quad y_1 = \dfrac{1}{2+e^{1.25}} = 0.1821379844$

$x_2 = 1.5 \quad y_2 = \dfrac{1}{2+e^{1.5}} = 0.154280773$

B2

$x_3 = 1.75 \quad y_3 = \dfrac{1}{2+e^{1.75}} = 0.1284556721$

$x_4 = 2 \quad y_4 = \dfrac{1}{2+e^2} = 0.1065069789$

$$\therefore \int_1^2 \frac{1}{2+e^x}\,dx \approx \frac{1}{12}\left\{(0.211941557+0.1065069789)+4(0.1821379844+0.1284556721)+2(0.154280773)\right\}$$

M1

≈ 0.156 to 3dp.

A1

4/4

(5) $x^3 + 5x^4y - 2y^3 + 7 = 0$

$\frac{d}{dx}(x^3) = 3x^2$ B1

$\frac{d}{dx}(5x^4y)$ let $u = 5x^4$ $V = y$

$\frac{du}{dx} = 20x^3$ $\frac{dv}{dx} = 1.\frac{dy}{dx}$

$= 5x^4 . 1\frac{dy}{dx} + 20x^3y$ B1

$\frac{d}{dx}(-2y^3) = -6y^2\frac{dy}{dx}$ B1

$\frac{d}{dx}(+7) = 0$

$\therefore$ $3x^2 + 5x^4\frac{dy}{dx} + 20x^3y - 6y^2\frac{dy}{dx} + 0 = 0$

$3x^2 + 20x^3y = 6y^2\frac{dy}{dx} - 5x^4\frac{dy}{dx}$

$\frac{dy}{dx}[6y^2 - 5x^4] = 3x^2 + 20x^3y$

$\frac{dy}{dx} = \frac{3x^2 + 20x^3y}{6y^2 - 5x^4}$ B1

(b)(i) $x = t^3 - 5$ $y = t^4 + 7t^5$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

So $\frac{dx}{dt} = 3t^2$ B1, $\frac{dy}{dt} = 4t^3 + 35t^4$ B1

(iii) when $x = 3$

$t^3 - 5 = 3$

$t^3 = 8$ M1

$t = 2$

$\therefore \frac{d^2y}{dx^2} = \frac{4 + 140}{9 \times 4} = \frac{144}{36} = 4$ A1

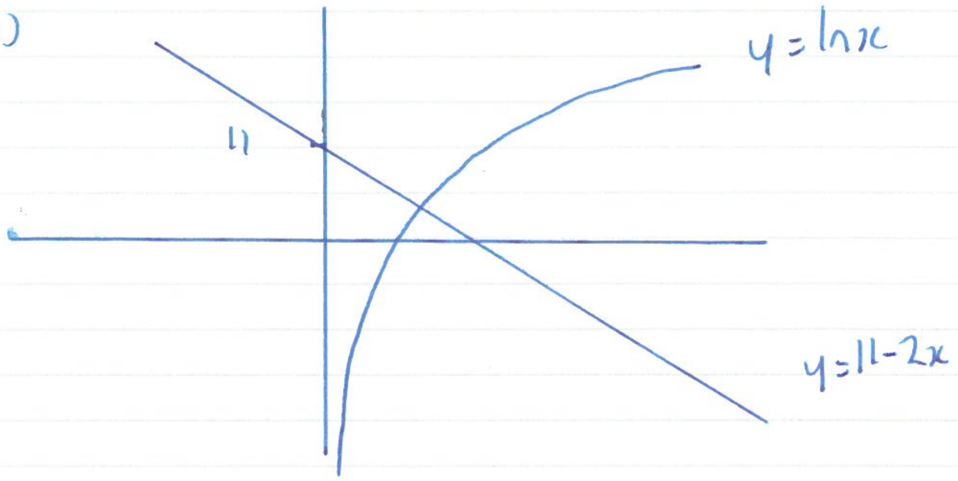
hence $\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2} = \frac{\cancel{t^2}(4t + 35t^2)}{3\cancel{t^2}} = \frac{4t + 35t^2}{3}$ M1 A1

(ii) $\frac{d^2y}{dx^2} = \frac{d}{\cancel{dt}}\left(\frac{dy}{dx}\right) \times \frac{\cancel{dt}}{dx} = \frac{d^2y}{dx^2} = \frac{4 + 70t}{3} \times \frac{1}{3t^2} = \frac{4 + 70t}{9t^2}$

B1 M1 A1

M/13

6 (a)



B1

B1

graphs only intersect in one place, so only one root. B1

(b) $x_0 = 4.7$

$x_1 = \dfrac{11 - \ln 4.7}{2} = 4.726218746$ B1

$x_2 = \dfrac{11 - \ln(4.726...)}{2} = 4.723437268$

$x_3 = \dfrac{11 - \ln(4.7234...)}{2} = 4.723731615$

$x_4 = \dfrac{11 - \ln(4.7237...)}{2} = 4.723700458$

So $x_4 = 4.72370$ to 5dp B1

$F(4.723695) = \ln(4.723695) + 2(4.723695) - 11 = -1.87 \times 10^{-5}$ < 0

$F(4.723705) = \ln(4.723705) + 2(4.723705) - 11 = 3.45 \times 10^{-6}$ M1 > 0 A1

Change of sign ∴ $\alpha = 4.72370$ to 5dp. A1

$\frac{M}{8}$

5 (a) (i) $y = \sqrt{5x^2 - 3x}$

$y = (5x^2 - 3x)^{1/2}$

Use chain rule : let $u = 5x^2 - 3x$

$\frac{du}{dx} = 10x - 3$

Now $y = u^{1/2}$

$\frac{dy}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2}(5x^2 - 3x)^{-1/2}$ M1

So $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{2}(5x^2 - 3x)^{-1/2} \times (10x - 3)$ A1

(ii) $y = \sin^{-1}(7x)$

Chain rule let $u = 7x$

$\frac{du}{dx} = 7$

$y = \sin^{-1} u$

$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-(7x)^2}} = \frac{1}{\sqrt{1-49x^2}}$ M1

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{7}{\sqrt{1-49x^2}}$ A1

(iii) $y = e^{3x} \ln x$

product rule let $u = e^{3x}$ $V = \ln x$

$\frac{du}{dx} = 3e^{3x}$ $\frac{dv}{dx} = 1/x$ M1

A1

Now $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = \frac{e^{3x}}{x} + 3e^{3x}\ln x$ A1

(5) b) $y = \frac{\cos x}{\sin x}$ $\frac{u}{v}$

Quotient Rule : $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

let $u = \cos x$ $v = \sin x$

$\frac{du}{dx} = -\sin x$ $\frac{dv}{dx} = \cos x$

So $\frac{dy}{dx} = \frac{\sin x . -\sin x - \cos x \cos x}{\sin^2 x}$ M1 A1

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x}$

Now $\sin^2 x + \cos^2 x \equiv 1$

$= -\frac{1}{\sin^2 x}$

$\frac{1}{\sin x} = \operatorname{cosec} x$

$\therefore \frac{dy}{dx} = -\operatorname{cosec}^2 x$ As required A1

M/10