Upper & Lower Bounds

Discrete Measurements

Number of people, shoe size, money, cookies in a jar, ..

$10 \div 2 = 5$

90 ± 5 trolls

There were an estimated 90 trolls in the field, to the nearest 10.

What was the lowest possible number of trolls ? 85

What was the highest possible number of trolls ? 94

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Nearest £1

$1 \div 2 = 0.5$

£10 ± 50p

This pile of coins is worth £10 to the nearest £1.

What is the lowest possible value of the coins ? 9.50

What is the highest possible value of the coins ? 10.49

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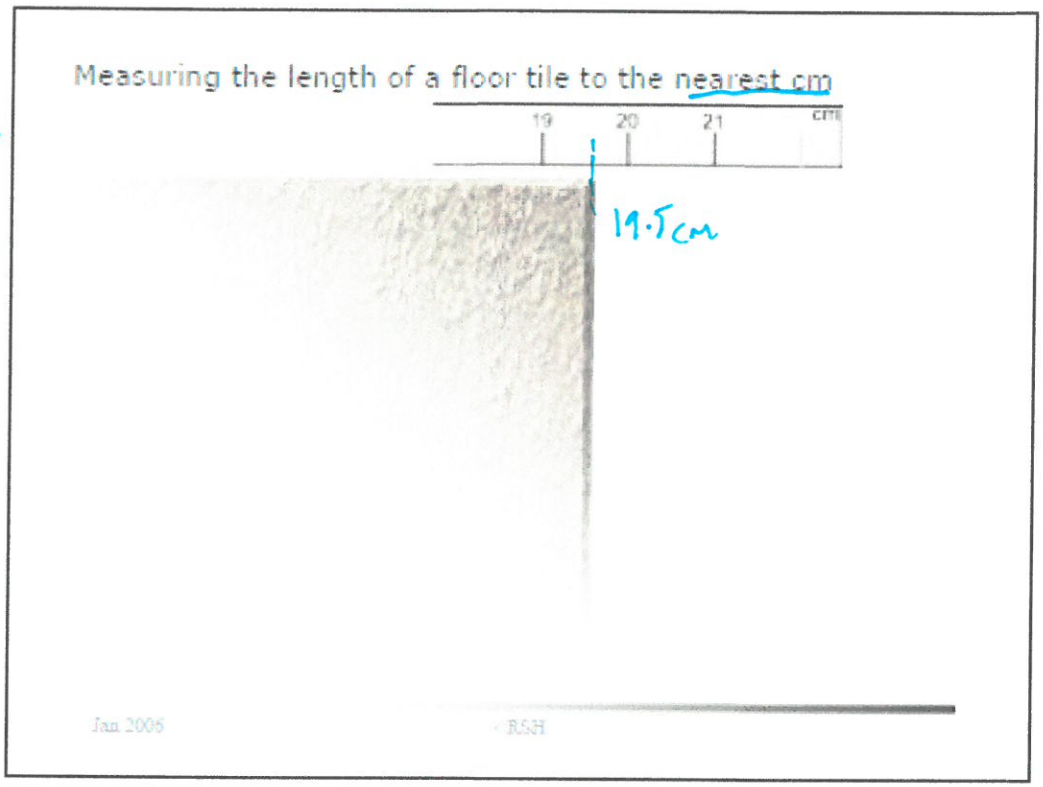
Continuous Measurements

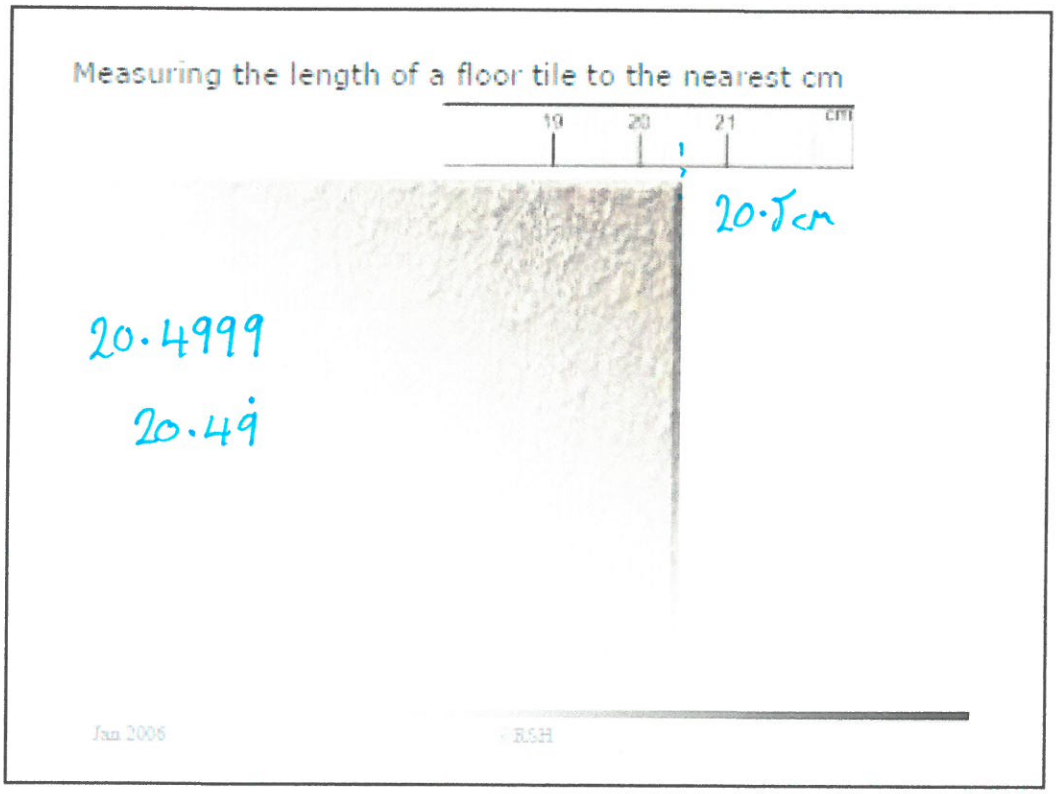
Height, Weight, Temperature, ...

nearest CM

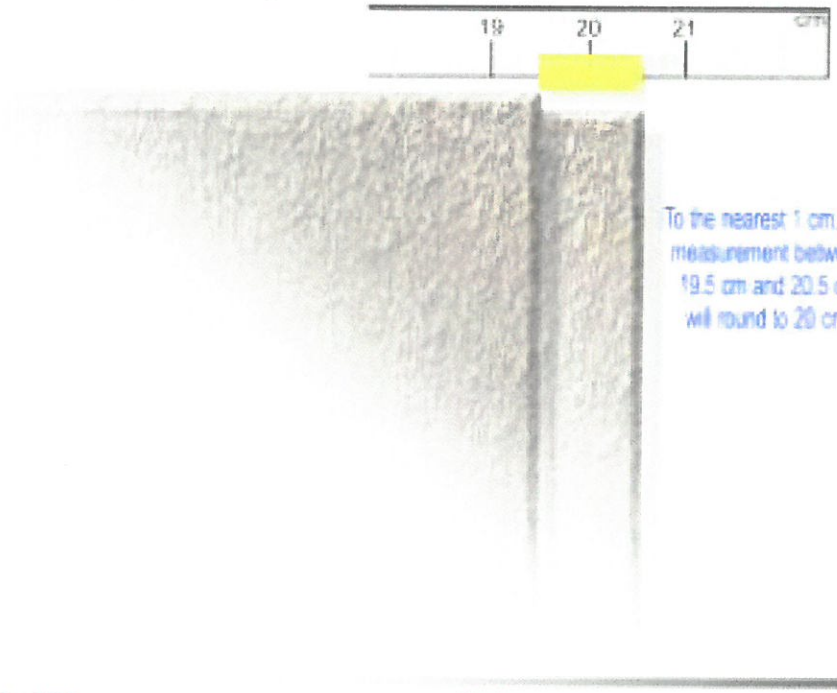
1cm ÷ 2 = 0.5

20 ± 0.5





Measuring the length of a floor tile to the nearest cm



Jake Girdlenurger caught a large trout.
He weighed and found it to be 14 kg measured to the nearest kg

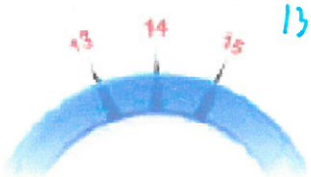
What is the least possible weight ? 13.5

What is the largest possible weight ? 14.5

He wants to sell the trout to a fishmonger.

What weight would Jake like to use ? 14.5

What weight would the fishmonger use ? 13.5



14kg to nearest kg

1kg ÷ 2 = 0.5kg

14 ± 0.5

least = 14 − 0.5 = 13.5 kg

largest = 14 + 0.5 = 14.5 kg

Exam Questions

Q1 A rectangular piece of cardboard measures 14 cm by 9 cm, each measurement being correct to the nearest cm.

9 ± 0.5

14 ± 0.5

a) Write down the least possible values of the length and the width of the rectangle. [1]

b) Write down the greatest possible values of the length and the width of the rectangle. [1]

boundaries = $1 \div 2 = \pm 0.5$ cm

(a) least width = $9 - 0.5 = 8.5$ cm

least length = $14 - 0.5 = 13.5$ cm

(b) greatest width = $9 + 0.5 = 9.5$ cm

greatest length = $14 + 0.5 = 14.5$ cm

Q1

c) Write down the least and greatest possible values of the perimeter of the rectangle. [2]

d) Write down the least and greatest possible values of the area of the rectangle. [2]

(c) for largest possible perimeter, need Max values

$14.5 + 14.5 + 9.5 + 9.5 = 48$ cm

for smallest perimeter, need Min values

$13.5 + 13.5 + 8.5 + 8.5 = 44$ cm.

(d) Max Area = $14.5 \times 9.5 = 137.75$ cm^2

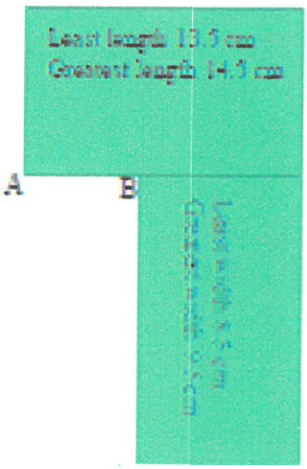
Min Area = $13.5 \times 8.5 = 114.75$ cm^2

Q1 e) **Four of these pieces of cardboard are placed, in a row, with their shorter sides joined. Calculate the least and greatest possible values of the length of the four pieces of cardboard. [3]**

Least length = 4 x Min length = 4 x 13.5 = 54 cm

Max length = 4 x Max length = 4 x 14.5 = 58 cm

Q1 f) **Two pieces of cardboard are placed as shown in the diagram. Calculate the least and greatest possible values of the length of the AB. [3]**



AB = length of tile − width of tile

for Min_{AB} = Min length − Max width

= 13.5 − 9.5

= 4cm

for MAX AB = Max length − Min Width

= 14.5 − 8.5

= 6cm

Q2 A rectangular piece of cardboard have lengths of 28 cm and widths of 16 cm, each measurement being correct to the nearest cm.

a) Write down the least possible values of the length and the width of the rectangle. [1]

error boundaries = 1cm ÷ 2 = ±0.5cm

Min length = 28 − 0.5 = 27.5 cm

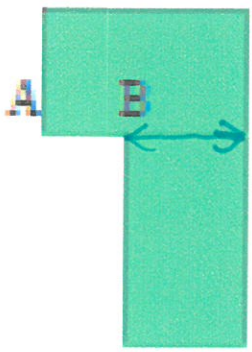
Min width = 16 − 0.5 = 15.5 cm

b) Four of these pieces of cardboard are placed in a row, with their shorter sides joined. Calculate the least and greatest possible values of the length of the four pieces of cardboard. [3]

Min length = 4 × 27.5 = 110 cm

Max length = 4 × 28.5 = 114 cm

c) Two pieces of cardboard are placed as shown in the diagram. Calculate the least and greatest possible values of the length AB. [3]



Max AB = Max length − Min Width

= 28.5 − 15.5

= 13cm

Min AB = Min length − Max Width

= 27.5 − 16.5

= 11cm

Q3 The length of a desk is measured as 180 cm, correct to the nearest cm.

a) Write down the least and greatest values of the length of the desk. [2]

error bound = 1cm ÷ 2 = ±0.5cm

Min length = 180 − 0.5 = 179.5cm

Max length = 180 + 0.5 = 180.5cm

b) Three of these desks are laid end to end along their lengths. What is the least value that the total length of the three desks can be ? [1]

Min length = 3 × 179.5 = 538.5cm

c) The distance between two walls is measured as 3 metres correct to the nearest centimetre. 300cm

i. Write down, in centimetres, the least and greatest values of the distance between the two walls. [1]

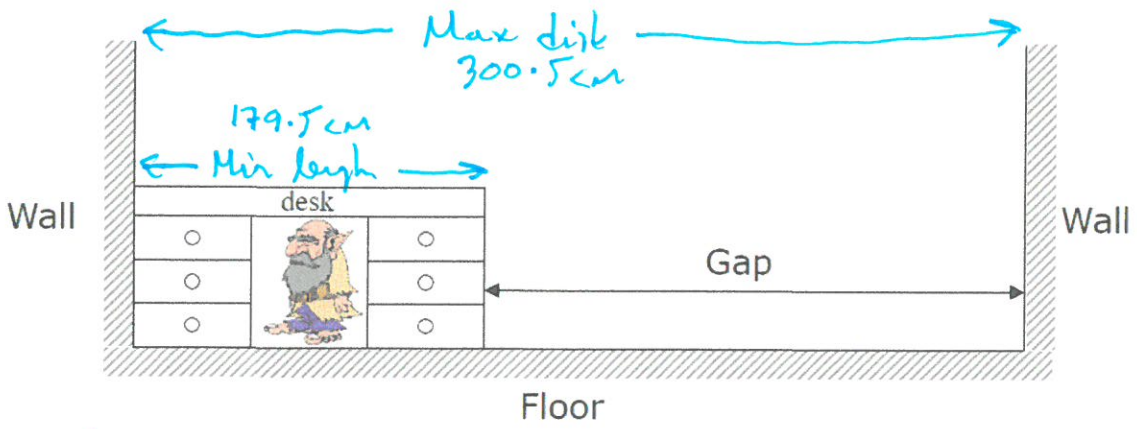
error bounds = 1 ÷ 2 = ±0.5cm

Min distance = 300 − 0.5 = 299.5cm

Max distance = 300 + 0.5 = 300.5cm

ii. One desk is placed lengthwise between two walls and in contact with the left hand wall, as shown in the diagram.

What is the greatest possible length of the gap between the desk and the right hand wall? [2]



MAX GAP = 300.5 − 179.5 = 121cm

Q4 A lump pf plasticine has a mass of 500 g, correct to the nearest 10 g. A piece of the plasticine is removed and found to have a mass of 310 g, correct to the nearest 10 g.

Find the greatest possible value of the mass of the remaining lump of plasticine. [2]

error bounds : $10 \div 2 = \pm 5g$

Max Mass of plasticine : $500 + 5 = 505g$

Min Mass of removed piece : $310 - 5 = 305g$

So Max Mass remaining : $505 - 305$

$= 200g$

Q5 A rectangle's measurements are given as 30 cm by 20 cm, correct to the nearest cm.

Find the least possible length of the perimeter of the rectangle. [2]

error bounds : $1cm \div 2 = \pm 0.5cm$

for Min perimeter : Min length and Min width

$= 19.5 + 19.5 + 29.5 + 29.5$

$= 98cm$