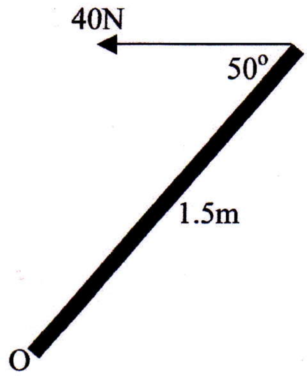
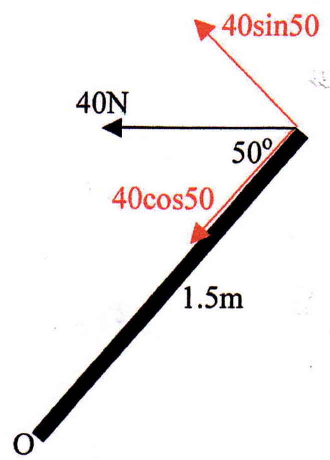
The moment of a force which acts at an angle

Consider a force of 40N acting on a rod as shown below:



The force can be split into components parallel and perpendicular to the rod and these can be used to calculate the moment of the force about the point marked O.

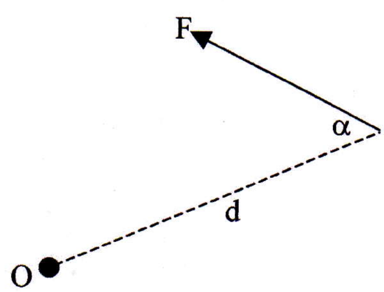


The moment of the component which is parallel to the rod is zero because it acts through O. Hence the moment of the force about the point O is found using the perpendicular component of the force only.

ie moment about O (anti-clockwise +ve) = $1.5 \times 40\sin 50 = 46.0$ NM

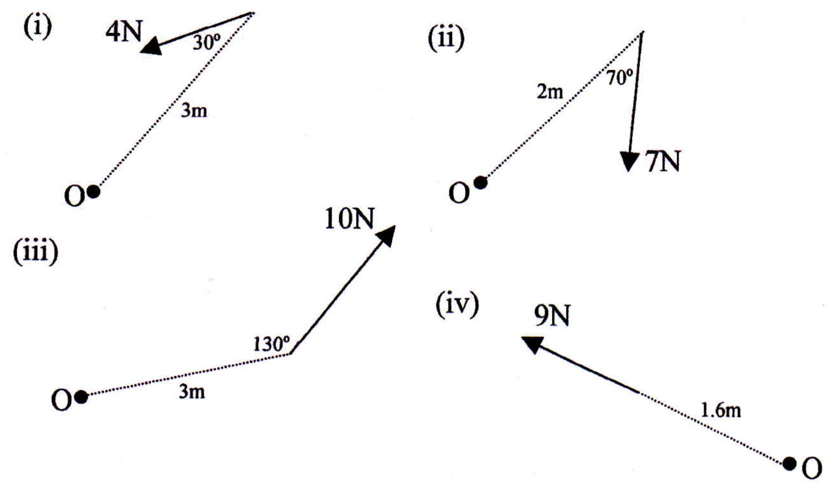
In general, if a force F is acting at an angle α about a pivot, O, a distance d metres away, the moment so produced is from the perpendicular component of the force only.

Moment* = *d* x *F*sin*α

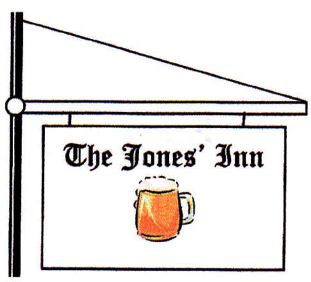


Examples

1. Find the moment about O of each of the forces illustrated below:

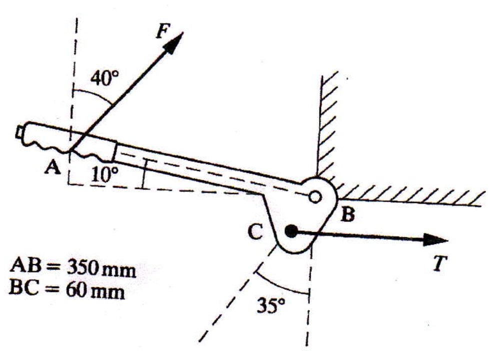


2. A sign outside a pub is attached to a light rod of length 1 metre which is freely hinged to the wall and supported in a vertical plane by a light string as in the diagram. The sign is assumed to be a uniform rectangle of mass 10kg. The angle of the string to the horizontal is 25°.



(i) Draw a diagram showing all the forces which are acting on the rod.
(ii) Find the tension in the string.
(iii) Find the magnitude and direction of the reaction force of the hinge on the sign.

3. The diagram shows a car's handbrake. The force F is exerted by the hand in operating the brake, and this creates a tension T in the brake cable. The handbrake is freely pivoted at point B.
   a. Draw a diagram showing all the forces acting on the handbrake.
   b. What is the required magnitude of force F if the tension in the brake cable is 1000N?
   c. A child applies the handbrake with a force of 10N. What is the tension in the brake cable?



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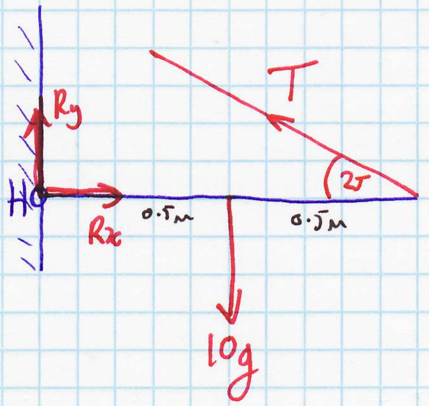
Eg1 (i) $\circlearrowleft_O \quad 3 \times 4 \sin 30 = 6\ Nm$

(ii) $\circlearrowleft_O \quad -2 \times 7 \sin 70 = -13.2\ Nm$

(iii) $\circlearrowleft_O \quad 3 \times 10 \sin 50 = 23.0\ Nm$

(iv) $\circlearrowleft_O \quad 0\ Nm$

Eg2 (i)



(ii) $\circlearrowleft_H: \quad 1 \times T \sin 25 - 0.5 \times 10g = 0$

$$T = \frac{5g}{\sin 25} = 116\ N.$$

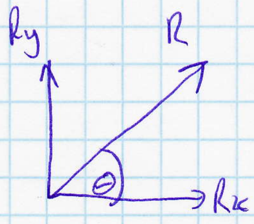
(iii) If in equilibrium then $\Sigma F_x = 0$, $\Sigma F_y = 0$

$$R_x - T \cos 25 = 0$$

$$R_x = 116 \cos 25 = 105 N$$

$$R_y + T \sin 25 - 10g = 0$$

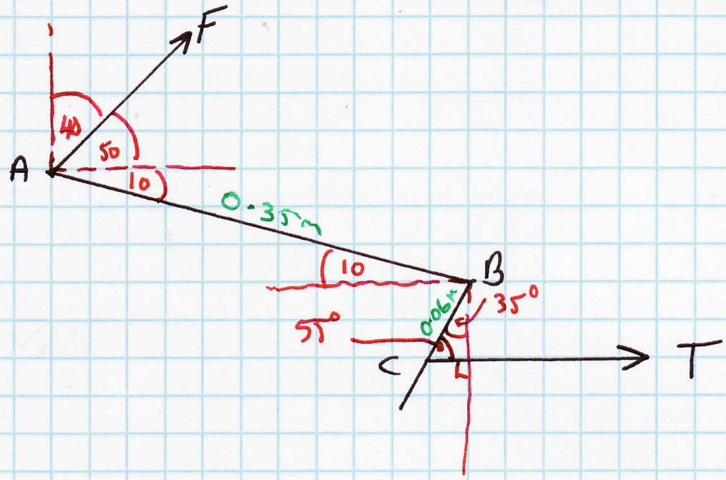
$$R_y = 10g - 5g = 5g = 49 N$$



$$R = \sqrt{105^2 + 49^2} = 115.9 N$$

$$\theta = \tan^{-1}\left(\frac{49}{105}\right) = 25^{\circ}$$

Eg3 (a)



(b) $\circlearrowright_B$: $0.06\,T\sin 55 - 0.35\,F\sin 60 = 0$

when $T = 1000$ $\quad F = \dfrac{0.06 \times 1000 \times \sin 55}{0.35 \times \sin 60} = 162$ N

(c) when $F = 10$ $\quad T = \dfrac{0.35 \times 10 \times \sin 60}{0.06 \times \sin 55} = 61.7$ N

Ladder Problems

The situation of a ladder resting against a wall, with the foot of the ladder on the ground, gives rise to a variety of problems. The wall may be rough or smooth, as also may be the ground.

It should be remembered that where the ladder rests against a smooth surface, there will only be a normal reaction, R, at that point.

When the surfaces in contact are rough, there is also a frictional force, F, which acts parallel to the surfaces in contact, and in a direction opposite to that in which the ladder would move.

Example 1

A uniform ladder AB, of mass 10kg and length 4m, rests with its upper end A against a smooth vertical wall and end B on smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall keeps the ladder in equilibrium inclined at 40° to the horizontal. The vertical plane containing the ladder and the string is at right angles to the wall. Find the tension T in the string and the normal reactions at the points A and B.

Rough contact at the foot of a ladder

If the ladder rests on ground which is rough, then there will be a frictional force F acting on the ladder at this point. The effect of this force is similar to that of the tension in the string in the above example. The maximum value of this frictional force depends on the roughness of the contact between the ladder and the ground.

Example 2

A ladder AB of mass 8kg and length 6m rests in equilibrium at an angle of 50° to the horizontal with its upper end A against a smooth vertical wall and its lower end B on rough horizontal ground, coefficient of friction μ. Given that the ladder is on the point of sliding down the wall, find the reaction forces at A and B, the frictional force and the value of μ if the centre of gravity G of the ladder is 2m from B.

Climbing a ladder

Whether or not it is safe to ascend to the top of a ladder will depend upon the magnitude of the frictional force which acts on the foot of the ladder. This will depend upon the roughness of the ground on which the ladder rests.

If the ladder is found to be in limiting equilibrium when a person is part way up a ladder, then any further ascent will cause the ladder to slip.

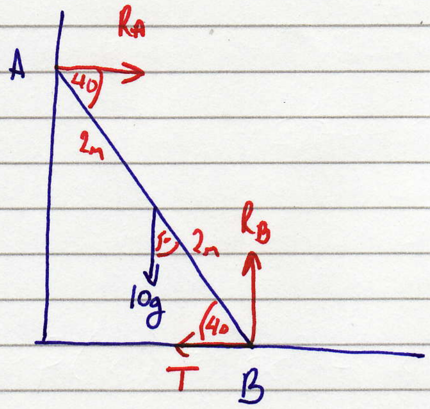
To determine how far a ladder may be ascended, consider the situation when the climber is at a distance d up the ladder and the ladder is in limiting equilibrium.

Example 3

A uniform ladder of mass 30kg and length 5m rests against a smooth vertical wall with its lower end on rough ground, coefficient of friction 0.4. The ladder is inclined at 60° to the horizontal. Find out how far a man of mass 80kg can ascend the ladder without slipping.

Exercise 5B All again!

Eg1



In equilibrium
$\therefore \sum F_x = 0$
$\sum F_y = 0$
$\sum G = 0$

$\sum F_x: \quad R_A - T = 0 \quad -(1)$
$\sum F_y: \quad R_B - 10g = 0 \quad -(2)$

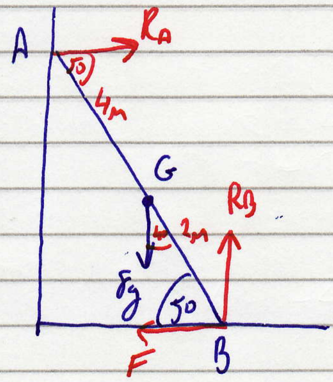
$\sum G_B: \quad 2 \times 10g \sin 50 - 4 R_A \sin 40 = 0 \quad -(3)$

$$R_A = \cancel{\frac{20g \sin 50}{4}} = \frac{20g \sin 50}{4 \sin 40} = 58.4N$$

$\therefore$ from (1) $T = 58.4N$

From (2) $R_B = 10g = 98N$

Eg2



$\sum F_x: \quad R_A - F = 0 \quad -(1)$
$\sum F_y: \quad R_B - 8g = 0 \quad -(2)$
$\mu = \frac{F_{max}}{R_B} \quad -(3)$

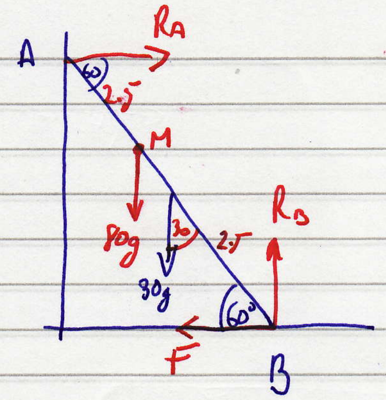
$\sum G_B: \quad 2 \times 8g \sin 40 - 6 \times R_A \sin 50 = 0$

$$R_A = \frac{16g \sin 40}{6 \sin 50} = 21.9N$$

From (1) $F_{max} = 21.9$
From (2) $R_B = 8g = 78.4N$

$$\therefore \mu = \frac{21.9}{78.4} = 0.28$$

Q3



$MB = d$ Ladder on point of slipping $\therefore F = \mu R_B$

$\Sigma f_x: \quad R_A - F = 0$

$\Sigma f_y: \quad R_B - 80g - 30g = 0 \quad R_B = 110g = 1078N$

$\therefore \quad F = 0.4 \times 1078 = 431.2N$

$= R_A$

$\Sigma G_B: \quad 2.5 \times 30g \sin 30 + 80g \sin 30 \times d - 5 \times R_A \sin 60 = 0$

$367.5 + 392d - 5 \times 431.2 \times \sin 60 = 0$

$$d = \frac{2156 \sin 60 - 367.5}{392}$$

$d = 3.83$ metres

$\therefore$ Man can climb 3.83 metres. Any further & ladder will ~~spid~~ slip.