Exercise (Leave Q4 'till last – it's the worst one!)

1. Find the remainder when f(x) is divided by g(x)
   a. $f(x) = x^3 - 2x^2 - 3x + 1$, $g(x) = x + 2$
   b. $f(x) = 3x^3 - 8x^2 - 5x + 2$, $g(x) = x - 4$
   c. $f(x) = 4x^3 + 6x^2 + 3x + 2$, $g(x) = 2x + 3$
2. Find a if $(x + 2)$ is a factor of $x^3 + 6x^2 + ax + 6$
3. When $x^3 + 2x^2 + ax + 7$ is divided by $x - 2$ the remainder is 3. Find a.
4. Both $x + 2$ and $2x - 1$ are factors of $2x^3 + ax^2 + bx + 6$. Find a, b and the third factor.
5. When $x^3 + ax^2 - x + 14$ is divided by $x - 3$ the remainder is 11. Find a. What is the remainder when the resulting expression is divided by $x + 2$?
6. When divided by $x - 1$ the polynomial $ax^3 + x^2 + bx - 4$ leaves a remainder of -6. Given that $x - 2$ is a factor of the polynomial, find the values of the constants a and b.

Numerical Answers (As usual, I may have made some slips – if in doubt check with Sam ☺)

1. (a) –9 (b) 46 (c) –5/2
2. a = 9 11
3. a = -10
4. a = -3, b = -11, 3rd factor is x – 3
5. a = -3, remainder –4
6. a = 1, b = -4

Exercice 1

(1) a) $f(-2) = (-2)^3 - 2(-2)^2 - 3(-2) + 1$

$= -8 - 8 + 6 + 1$

$= \underline{-9}$

b) $f(4) = 3(4)^3 - 8(4)^2 - 5(4) + 2$

$= 192 - 128 - 20 + 2$

$= \underline{46}$

c) $f\left(\frac{-3}{2}\right) = 4\left(\frac{-3}{2}\right)^3 + 6\left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 2$

$= 4\left(-\frac{27}{8}\right) + 6\left(\frac{9}{4}\right) - \frac{9}{2} + 2$

$= -\frac{108}{8} + \frac{54}{4} - \frac{9}{2} + 2$

$= \underline{-\frac{5}{2}}$

(2) $f(-2) = 0 \quad (-2)^3 + 6(-2)^2 + a(-2) + 6 = 0$

$-8 + 24 - 2a + 6 = 0$

$2a = 22$

$\underline{a = +11}$

(3) $f(2) = 3 \quad (2)^3 + 2(2)^2 + a(2) + 7 = 3$

$8 + 8 + 2a + 7 = 3$

$2a = 3 - 23$

$\underline{a = -10}$

(4) $F(-2) = 0 \quad 2(-2)^3 + a(-2)^2 + b(-2) + 6 = 0$

$-16 + 4a - 2b + 6 = 0$

$4a - 2b = 10$

$\div 2 \quad 2a - b = 5 \quad —(1)$

$F(\frac{1}{2}) = 0 \quad 2(\frac{1}{2})^3 + a(\frac{1}{2})^2 + b(\frac{1}{2}) + 6 = 0$

$\frac{2}{8} + \frac{a}{4} + \frac{b}{2} + 6 = 0$

$\times 8$

$2 + 2a + 4b + 48 = 0$

$2a + 4b = -50$

$\div 2 \quad a + 2b = -25 \quad —(2)$

From (1) $b = 2a - 5 \quad —(3)$

in (2) $a + 2(2a - 5) = -25$

$a + 4a - 10 = -25$

$5a = -15$

$a = -3$

in (3) $b = 2(-3) - 5 = -11$

$\therefore F(x) = 2x^3 - 3x^2 - 11x + 6$

$= (x+2)(2x-1)(Ax+B)$

$= (2x^2 + 3x - 2)(Ax+B)$

Now

$$2x^2+3x-2 \;\overline{)\; 2x^3 - 3x^2 - 11x + 6}$$

$x - 3$

$2x^3 + 3x^2 - 2x$

$-6x^2 - 9x + 6$

$-6x^2 - 9x + 6$

$\therefore$ 3rd Factor is $(x-3)$

(5) $f(3) = 11 \quad (3)^3 + a(3)^2 - (3) + 14 = 11$

$27 + 9a - 3 + 14 = 11$

$9a = -27$

$a = -3$

$\therefore f(x) = x^3 - 3x^2 - x + 14$

$f(-2) = (-2)^3 - 3(-2)^2 - (-2) + 14$

$= -8 - 12 + 2 + 14$

$= -4$.

(6) $f(1) = -6 \quad a(1)^3 + (1)^2 + b(1) - 4 = -6$

$a + 1 + b - 4 = -6$

$a + b = -3 \quad -①$

$f(2) = 0 \quad a(2)^3 + (2)^2 + b(2) - 4 = 0$

$8a + 4 + 2b - 4 = 0$

$8a + 2b = 0$

$b = -4a$

sub ①: $a - 4a = -3$

$-3a = -3$

$a = 1$

$b = -4$