The Remainder Theorem

Consider what happens if we divide the function $f(x) = x^3 + x^2 - x + 3$ by $x - 1$:

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x - 1 & x^3 + x^2 - x + 3 \\
 & \underline{x^3 - x^2} \\
 & \quad 2x^2 - x \\
 & \quad \underline{2x^2 - 2x} \\
 & \qquad\quad x + 3 \\
 & \qquad\quad \underline{x - 1} \\
 & \qquad\qquad 4
\end{array}
$$

Remainder 4

Now lets evaluate f(1)

$$f(1) = 1^3 + 1^2 - 1 + 3$$
$$= 4$$

It follows that, in general if a polynomial f(x) is divided by (x – a) until the remainder no longer contains x, the remainder is f(a).

This is known as the ***remainder theorem***, of which the special case where f(a) = 0 is the factor theorem we have already met.

Examples

1. Find the remainder when $2x^3 + 3x - 4$ is divided by $x - 3$.

2. Find a if $(x - 3)$ is a factor of $x^3 + ax^2 + 11x - 6$

3. When $2x^3 + x^2 + bx + 58$ is divided by $x + 3$ the remainder is 4. Find the value of b.

4. When divided by $x + 1$, the polynomial $ax^3 + bx^2 - 13x + 6$ leaves a remainder of 18. Given that $2x - 1$ is a factor of the polynomial, find the values of the constants a and b.

EXAMPLES

(1) $2x^3 + 3x - 4 \div x - 3$

$f(3) = 2(3)^3 + 3(3) - 4$
$= 54 + 9 - 4$
$= 59$

$\therefore$ remainder 59

(2) If $x-3$ is a factor, then $f(3) = 0$

$(3)^3 + a(3)^2 + 11(3) - 6 = 0$

$27 + 9a + 33 - 6 = 0$

$9a = -54$

$a = \frac{-54}{9} = -6$

(3) $f(-3) = 4$

$2(-3)^3 + (3)^2 + b(-3) + 58 = 4$

$-54 + 9 - 3b + 58 = 4$

$3b = 9$

$b = 3$

(4) $f(-1) = 18$

$a(-1)^3 + b(-1)^2 - 13(-1) + 6 = 18$

$-a + b + 13 + 6 = 18$

$b - a = -1$

$a - b = 1 \quad — (1).$

Eg4 (contd) $F\left(\frac{1}{2}\right)=0$

$$a\left(\frac{1}{2}\right)^3+b\left(\frac{1}{2}\right)^2-13\left(\frac{1}{2}\right)+6=0$$

$$\frac{a}{8}+\frac{b}{4}-\frac{13}{2}+6=0$$

$\times 8$

$$a+2b-4\times 13+8\times 6=0$$

$$a+2b-52+48=0$$

$$a+2b=4 \quad — \quad 2$$

From 1 $\quad a=1+b \quad — \quad 3$

in 2 $\quad 1+b+2b=4$

$$3b=3$$
$$b=1$$

in 3 $\quad a=2.$