Half Term Test – Practise Paper

(1) (a) $x^2-6x-4 = (x-3)^2-13$

(b) $x^2+3x+5 = \left(x+\frac{3}{2}\right)^2+p$

$\frac{9}{4}+p=5$

$p = 5-\frac{9}{4} = \frac{11}{4}$

$= \left(x+\frac{3}{2}\right)^2+\frac{11}{4}$

(2) $2x^2-8x+7=0$

$\div 2$

$x^2-4x+\frac{7}{2}=0$

$(x-2)^2+p=0$

$4+p=\frac{7}{2}$

$p=\frac{7}{2}-4=-\frac{1}{2}$

$(x-2)^2-\frac{1}{2}=0$

$(x-2)^2=\frac{1}{2}$

$x-2=\pm\sqrt{\frac{1}{2}}$

$x = 2\pm\frac{1}{\sqrt{2}}$

either $x = 2+\frac{1}{\sqrt{2}} = 2.71$

or $x = 2-\frac{1}{\sqrt{2}} = 1.29$

(3) $x+y=9$ — (1)

$x^2-3xy+2y^2=0$ — (2)

From (1) $y=9-x$ — (3)

in (2) $x^2-3x(9-x)+2(9-x)^2=0$

$x^2-27x+3x^2+2(81-18x+x^2)=0$

$x^2-27x+3x^2+162-36x+2x^2=0$

$6x^2-63x+162=0$

$x=\frac{-(-63)\pm\sqrt{(-63)^2-4\times6\times162}}{2\times6}$

$x=\frac{63\pm\sqrt{3969-3888}}{12}$

$x=\frac{63\pm\sqrt{81}}{12}$

③ contd either $x = \frac{63+\sqrt{81}}{12} = \frac{63+9}{12} = \frac{72}{12} = 6$

or $x = \frac{63-\sqrt{81}}{12} = \frac{63-9}{12} = \frac{54}{12} = 4.5$

Now sub in ③ when $x=6$, $y = 9-6 = 3$

when $x=4.5$, $y = 9-4.5 = 4.5$

$\therefore$ line + curve intersect @ $(6,3)$ and $(4.5, 4.5)$

④ $81^{-\frac{1}{2}} \times 27^{\frac{2}{3}} = \frac{1}{\sqrt{81}} \times (\sqrt[3]{27})^2 = \frac{1}{9} \times 9 = 1$

⑤ (a) $f(0) = 0^2 - 3(0) - 10 = -10$

(b) $g(5) = 2(5) - 1 = 9$

(c) $2x - 1 = 9$

$2x = 10$

$x = 5$

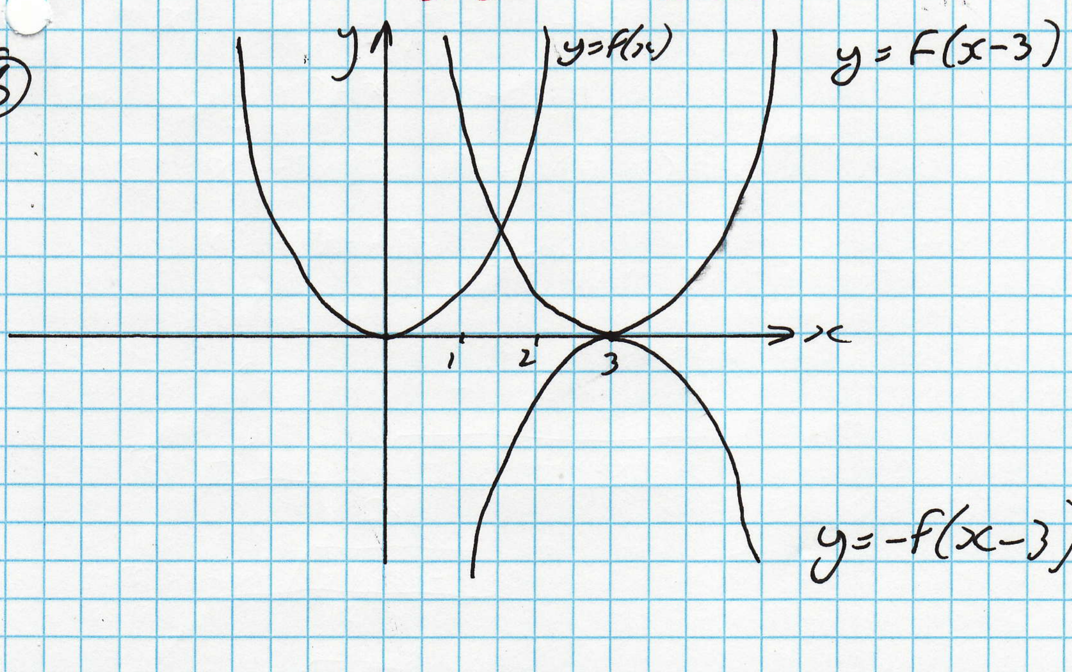
(d) $gf(x) = 2(x^2-3x-10) - 1 = 2x^2 - 6x - 20 - 1 = 2x^2 - 6x - 21$

(e) $f(x) = 0$ $x^2 - 3x - 10 = 0$

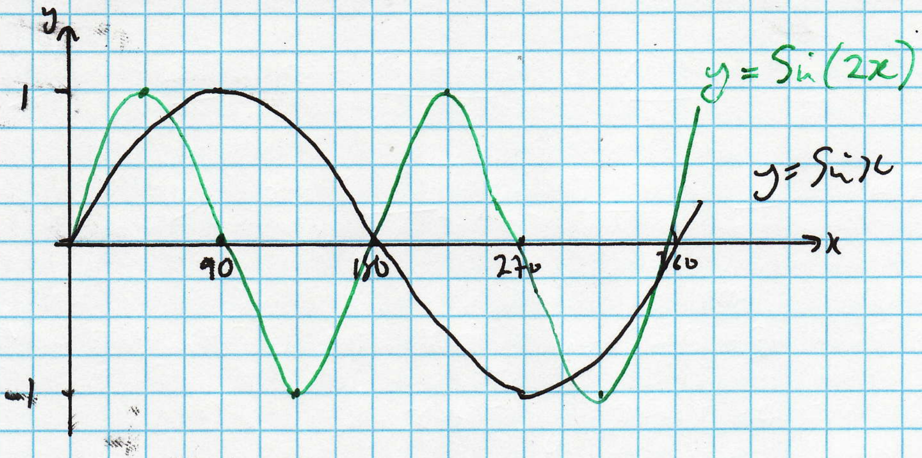
$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$

⑥



(7)



(8) $f(x) = x^3 + 2x^2 - 9x - 18$

$f(1) = 1^3 + 2(1)^2 - 9(1) - 18 = 1 + 2 - 9 - 18 \neq 0$

$f(2) = 2^3 + 2(2)^2 - 9(2) - 18 = 8 + 8 - 18 - 18 \neq 0$

$f(-2) = (-2)^3 + 2(-2)^2 - 9(-2) - 18 = -8 + 8 + 18 - 18 = 0$

$\therefore$ $x+2$ is a factor

$$
\begin{array}{r|l}
 & x^2 \qquad\quad -9 \\
\hline
x+2 & x^3 + 2x^2 - 9x - 18 \\
 & \underline{x^3 + 2x^2} \\
 & \qquad\quad -9x - 18 \\
 & \qquad\quad \underline{-9x - 18} \\
 & \qquad\quad \cdot \quad \cdot
\end{array}
$$

$\therefore f(x) = (x+2)(x^2-9) = (x+2)(x+3)(x-3)$

Now if $f(x) = 0$ $(x+2)(x+3)(x-3) = 0$

either $x+2=0$ or $x+3=0$ or $x-3=0$

$x=-2$ $\quad$ $x=-3$ $\quad$ $x=3$

(9) (a) $f(-1) = 12$

$2(-1)^3 + p(-1)^2 + q(-1) + 6 = 12$

$-2 + p - q + 6 = 12$

$p - q = 8$ —(1)

$f(1) = -6$

$2(1)^3 + p(1)^2 + q(1) + 6 = -6$

$2 + p + q + 6 = -6$

$p + q = -14$ —(2)

(1) + (2) $2p = -6$

$p = -3$ $\quad$ in (1) $-3 - q = 8$

$q = -3 - 8 = -11$

(9) (a) $\therefore \quad f(x) = 2x^3 - 3x^2 - 11x + 6$

(b) $f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^3 - 3(\tfrac{1}{2})^2 - 11(\tfrac{1}{2}) + 6$

$= \frac{2}{8} - \frac{3}{4} - \frac{11}{2} + 6$

$= 0$

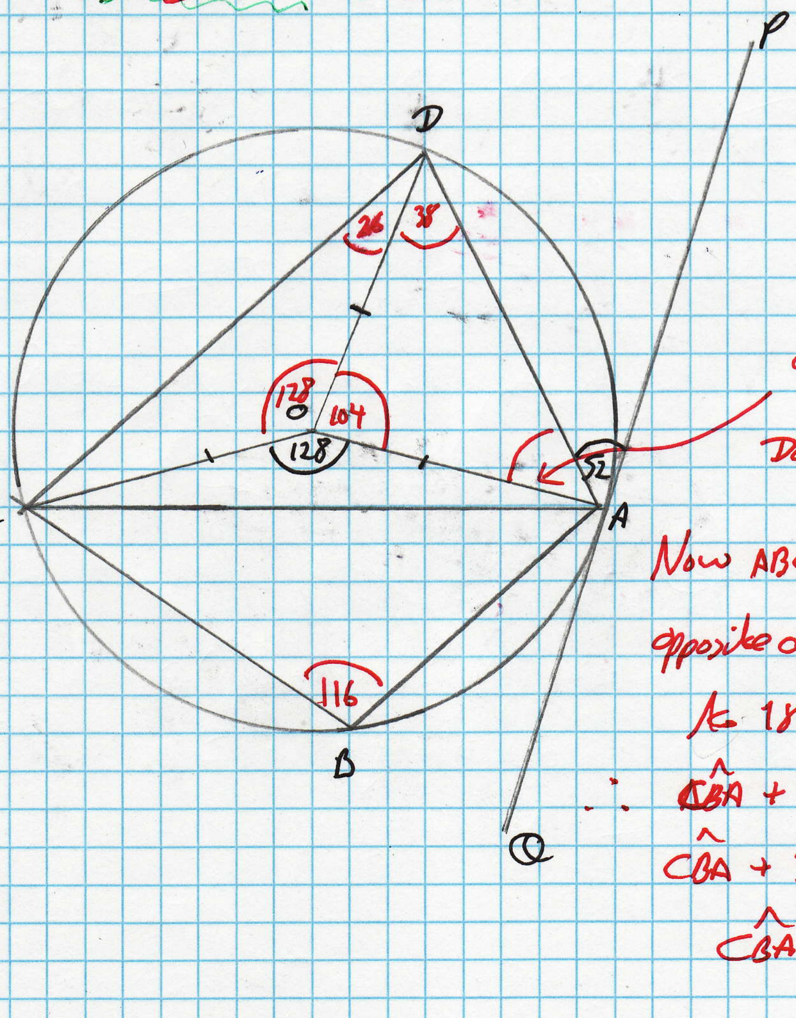
$\therefore \; x = \frac{1}{2}$ is a solution $2x-1$ is a factor

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
2x-1 & 2x^3 - 3x^2 - 11x + 6 \\
 & \underline{2x^3 - x^2} \\
 & \quad -2x^2 - 11x \\
 & \quad \underline{-2x^2 + x} \\
 & \qquad -12x + 6 \\
 & \qquad \underline{-12x + 6} \\
 & \qquad\quad \cdot
\end{array}
$$

$\therefore \; f(x) = (2x-1)(x^2 - x - 6)$

$= (2x-1)(x-3)(x+2)$

(10)



$90 - 52 = 38$

$D\hat{O}A = 180 - 38 - 38 = 104^\circ$

Now ABCD = cyclic quadrilateral

opposite angles in cyclic quad add up to 180°

$\therefore \; C\hat{B}A + C\hat{D}A = 180$

$C\hat{B}A + 38 + 26 = 180$

$C\hat{B}A = 116^\circ$