

# NOVEMBER 06 PAPER 2 - SOLUTIONS

3

Examiner  
only  
Arholwr  
yn unig

1. Hassan, Elin and Richard buy 19, 25 and 16 tickets respectively in a raffle. They agree to share any prize money they win in the ratio of the number of tickets they have bought. How much does each one get when they win £540?

$$19 + 25 + 16 = 60 \quad 540 \div 60 = 9$$

$$\text{Hassan gets } 19 \times 9 = £171$$

$$\text{Elin gets } 25 \times 9 = £225$$

$$\text{Richard gets } 16 \times 9 = £144$$

[3]

2. Solve the following equation.

$$2x - 15 = 4(5 - 2x)$$

$$2x - 15 = 20 - 8x$$

$$2x + 8x = 20 + 15$$

$$10x = 35$$

$$x = \frac{35}{10} = 3.5$$

[3]

4

Examiner  
only  
Arholwr  
yn unig

3. The heights of 210 leylandii trees were measured. The table shows a grouped frequency distribution of the results.

Height ( $x$ cm)	Number of trees
$40 \leq x < 80$	6
$80 \leq x < 120$	30
$120 \leq x < 160$	114
$160 \leq x < 200$	42
$200 \leq x < 240$	18

Find an estimate for the mean height of the trees.

$$60 \times 6 = 360$$

$$100 \times 30 = 3000$$

$$140 \times 114 = 15960$$

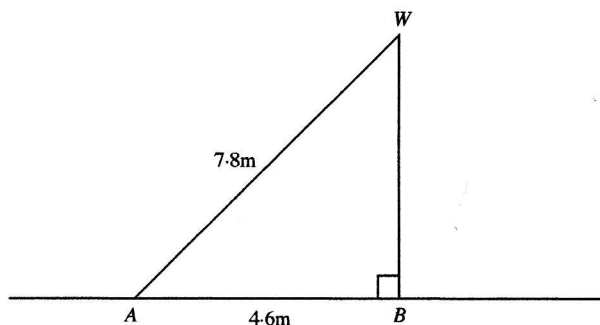
$$180 \times 42 = 7560$$

$$220 \times 18 = 3960$$

$$\text{Mean} = \frac{30840}{210} = 146.9 \text{ cm}$$

[4]

4. A ladder which is 7.8m long is resting against a vertical wall at  $W$ . The foot of the ladder is at  $A$ , which is at a horizontal distance of 4.6m from the base of the vertical wall at  $B$ . Calculate the length of  $WB$ .



$$WB^2 = 7.8^2 - 4.6^2$$

$$WB^2 = 39.68$$

$$WB = \sqrt{39.68} = 6.3m$$

[3]

5. Find, to the nearest penny, the compound interest when £500 is invested for 3 years at 4% per annum.

$$\text{Amount in account} = (1.04)^3 \times 500 = £562.43$$

$$\therefore \text{interest earned} = £62.43$$

[4]

6. A cuboid of copper measures 5.3 cm by 6.7 cm by 13.4 cm. The density of copper is 8.96 g/cm<sup>3</sup>. Calculate the mass, in kg, of the cuboid.

$$\text{Volume} = 5.3 \times 6.7 \times 13.4 = 475.834 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$= 8.96 \times 475.834$$

$$= 4263.5 \text{ g}$$

$$= 4.26 \text{ kg}$$

[4]

7. An electricity bill came to £103.95 inclusive of V.A.T. at 5%. What was the cost of the electricity before V.A.T. was added?

$$1.05 \times x = 103.95$$

$$x = \frac{103.95}{1.05} = \underline{\underline{99}}$$

[3]

8. A solution to the equation

$$x^3 - 7x + 1 = 0$$

lies between 2.5 and 2.6.

Use the method of trial and improvement to find this solution correct to 2 decimal places.

$$x = 2.55 \quad -0.269 \quad \text{too small}$$

$$x = 2.57 \quad -0.054 \quad \text{" "}$$

$$x = 2.58 \quad +0.11 \quad \text{too big}$$

$\therefore$  solution lies between  $x = 2.57$  &  $x = 2.58$

$$\text{Test } x = 2.575 \quad 0.0488 \quad \text{too big}$$

$$\therefore x = 2.57 \text{ to 2dp.}$$

[4]

9. (a) Write each of the following numbers in standard form.

(i) 0.0083

$$0.0083 = 8.3 \times 10^{-3}$$

[1]

(ii) 7500000000

$$7500000000 = 7.5 \times 10^9$$

[1]

- (b) Find, in standard form, the value of

$$(2.66 \times 10^9) \div (7.6 \times 10^{-3}).$$

$$3.5 \times 10^{11}$$

[2]

10. (a) Rearrange the inequality  $3 - 3n < 9 - 5n$  into the form  $n < \text{some number}$ .

$$5n - 3n < 9 - 3$$

$$2n < 6$$

$$n < 3$$

[2]

- (b) Given that  $n$  also satisfies the inequality  $3n > -6$ , write down all the integer values of  $n$  that satisfy both inequalities.

$$3n > -6$$

$$n > -\frac{6}{3}$$

$$n > -2$$

$$\therefore -2 < n < 3$$

[2]

$$\text{So } n \text{ can be } -1, 0, 1, 2$$

11. Solve the following equation.

$$\frac{4x+3}{6} + \frac{x}{3} = 2$$

$$\times 6 \quad 1 \quad 2$$

$$\frac{6(4x+3)}{6} + \frac{6 \times x}{3} = 6 \times 2$$

$$4x+3 + 2x = 12$$

$$6x = 12 - 3$$

$$6x = 9$$

$$x = \frac{9}{6} = \frac{3}{2}$$

[4]

12. (a) In the diagram
- $PQ$
- represents a vertical pole. When the sun is at an angle of elevation of
- $36^\circ$
- the pole casts a shadow,
- $PR$
- , of length 15.8 m on horizontal ground. Calculate the height of the pole
- $PQ$
- .

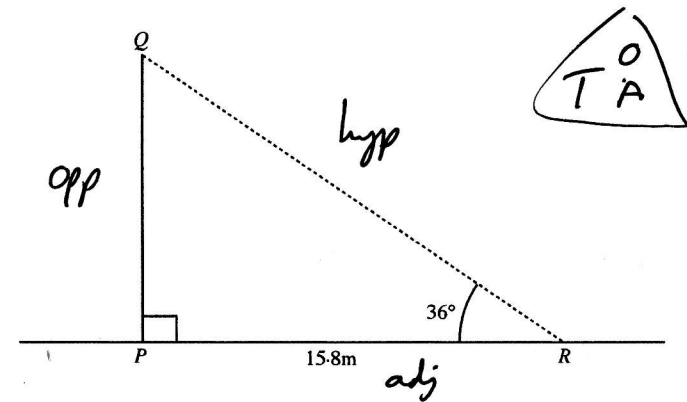


Diagram is not drawn to scale.

$$\text{opp} = \tan 36^\circ \times \text{adj}$$

$$PQ = \tan 36^\circ \times 15.8$$

$$PQ = 11.5 \text{ m}$$

[3]

- (b) In the diagram  $AB$  represents the arm of a crane which is 35.4 m long. The point  $C$  is 24.3 m horizontally from  $A$  and vertically below  $B$ . Calculate the angle of elevation,  $x$ , of the arm of the crane.

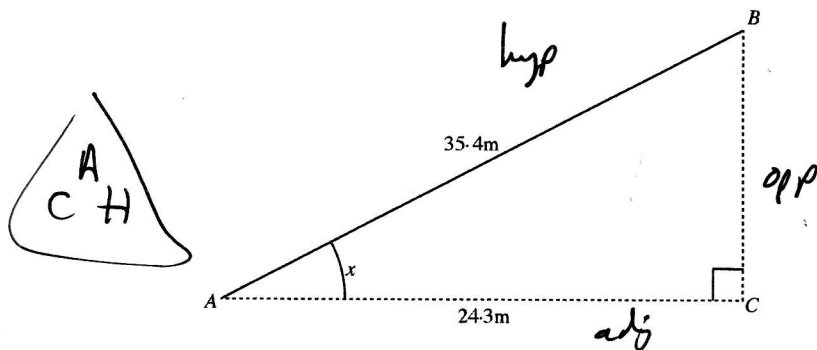


Diagram is not drawn to scale.

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\cos x = \frac{24.3}{35.4}$$

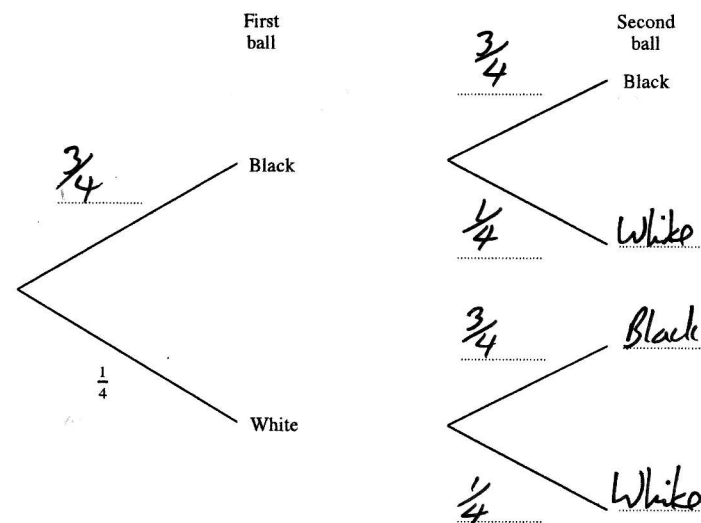
$$x = \cos^{-1}\left(\frac{24.3}{35.4}\right) = 46.7^\circ$$

[3]

13. A bag contains only black balls and white balls. The probability that a ball drawn at random from the bag is coloured white is  $\frac{1}{4}$ .

Two balls are drawn from the bag in the following way. The first ball is drawn at random from the bag and its colour is noted. This ball is then replaced in the bag and a second ball is drawn at random from the bag and its colour is also noted.

- (a) Complete the following tree diagram to show all the possible outcomes and their probabilities when two balls are drawn from the bag in this way.



[2]

- (b) Calculate the probability that the 2 balls are the same colour.

$$P(\text{2 the same}) = P(BB) \text{ or } P(WW)$$

$$= \left(\frac{3}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$$

$$= \frac{9}{16} + \frac{1}{16} = \frac{10}{16}$$

[2]

14.

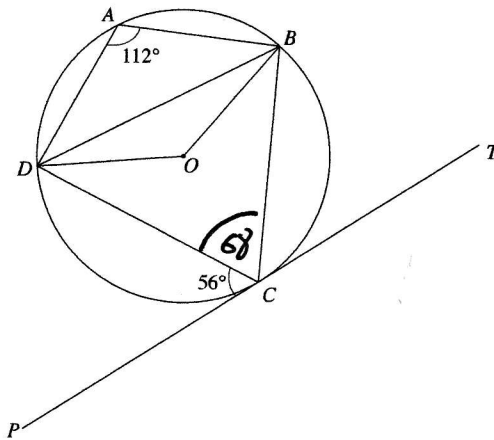


Diagram not drawn to scale.

Four points A, B, C and D lie on the circumference of the circle centre O.

The tangent TP touches the circle at C.

Given that  $\hat{DCP} = 56^\circ$  and  $\hat{DAB} = 112^\circ$ , find **each** of the following angles, giving reasons for your answers.

(a)  $\hat{DBC} = 56^\circ$  (Alternate Segment Theorem)

[2]

(b)  $\hat{BOD} = 136^\circ$  (Opp angles in cyclic quad add up to  $180^\circ$ )  
 $\hat{BOD} = 2 \times 68 = 136^\circ$  (angle at centre is twice angle at circumference)

[2]

15. Use the formula method to solve the equation  $2x^2 + 19x + 13 = 0$ , giving your solutions correct to two decimal places.

$$a=2 \quad b=19 \quad c=13$$

$$x = \frac{-19 \pm \sqrt{19^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$x = \frac{-19 \pm \sqrt{361 - 104}}{4}$$

$$x = \frac{-19 \pm \sqrt{257}}{4}$$

either  $x = \frac{-19 + \sqrt{257}}{4} = -0.74$   
 or  $x = \frac{-19 - \sqrt{257}}{4} = -8.76$

[3]

16. Make  $e$  the subject of the following formula.

$$f = \frac{e(7+g)}{3-2e}$$

$$f(3-2e) = e(7+g)$$

$$3f - 2ef = 7e + ge$$

$$3f = 7e + ge + 2ef$$

$$3f = e(7+g+f)$$

$$e = \frac{3f}{(7+g+f)}$$

[5]

17. Express 0.546 as a fraction.

$$\begin{aligned} x &= 0.546464646\dots \\ 10x &= 5.4646464\dots \quad - \textcircled{1} \\ 100x &= 54.6464646\dots \\ 1000x &= 546.4646464\dots \quad - \textcircled{2} \end{aligned}$$

$$\begin{aligned} \textcircled{2} - \textcircled{1} \quad 990x &= 541 \\ x &= \frac{541}{990} \end{aligned}$$

Examiner  
only  
Arholwr  
yn unig

[2]

18. (a) (i) Factorise  $36x^2 - 49y^2$ .

$$(6x + 7y)(6x - 7y)$$

[2]

(ii) Hence simplify  $\frac{36x^2 - 49y^2}{12x^2 + 14xy}$

$$\frac{(6x+7y)(6x-7y)}{2x(6x+7y)}$$

$$= \frac{6x-7y}{2x}$$

[2]

(b) Factorise the expression  $10x^2 + 19x + 6$  and hence solve the equation  $10x^2 + 19x + 6 = 0$ .

$$\begin{aligned} &\textcircled{60} \quad 15x, 4x \\ &10x^2 + 15x + 4x + 6 \\ &5x(2x+3) + 2(2x+3) \\ &(5x+2)(2x+3) = 0 \end{aligned}$$

$$\begin{aligned} \text{either } 5x+2 &= 0 & \text{or } 2x+3 &= 0 \\ x &= -\frac{2}{5} & x &= -\frac{3}{2} \end{aligned}$$

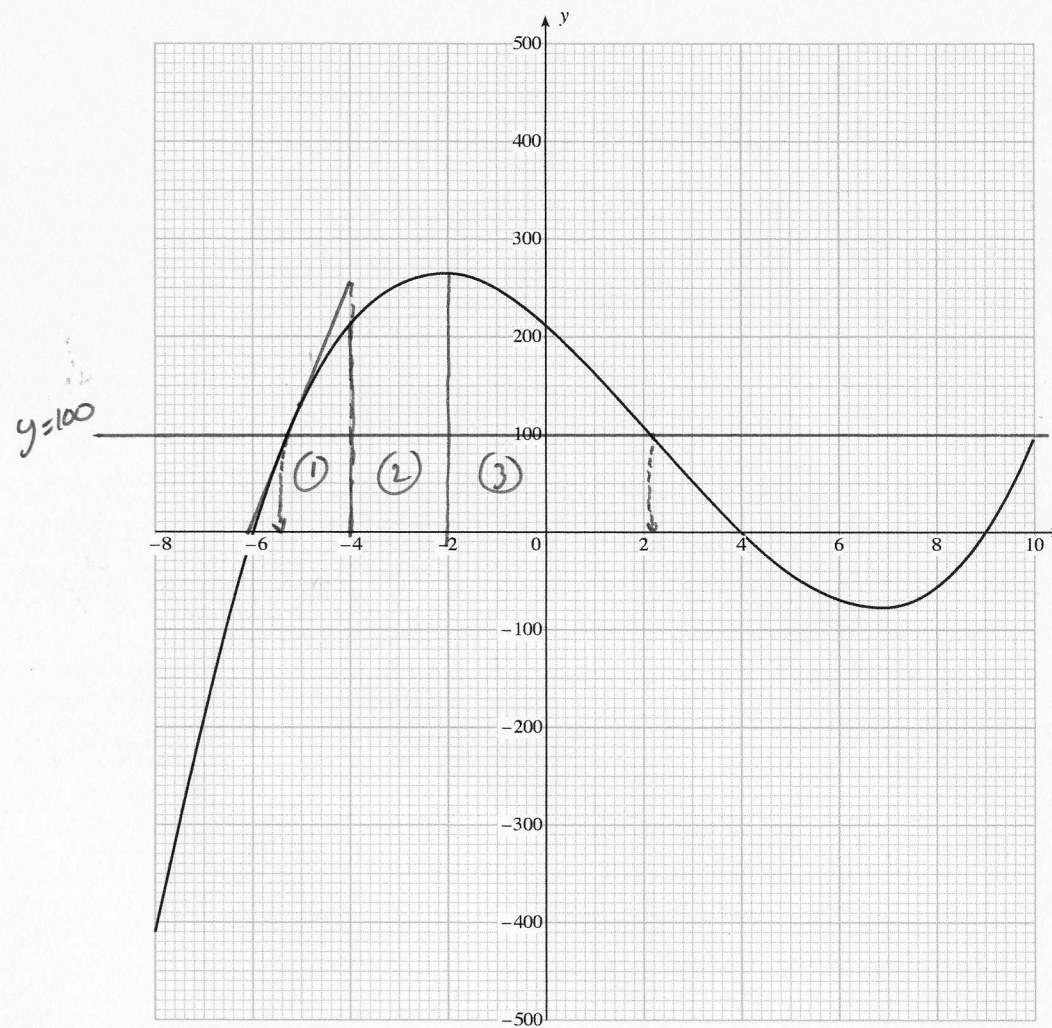
[3]

Examiner  
only  
Arholwr  
yn unig

**BLANK PAGE**

**Turn over.**

19. The graph of  $y = x^3 - 7x^2 - 42x + 216$ , for values of  $x$  between  $x = -8$  and  $x = 10$ , has been drawn below.



- (a) Use the graph to solve  $x^3 - 7x^2 - 42x + 216 = 0$ .

Where graph crosses  $x$  axis:

$$x = -6, x = 4 \text{ or } x = 9$$

[2]

- (b) By drawing an appropriate line on the graph, solve the equation  $x^3 - 7x^2 - 42x + 116 = 0$ .

Write as  $x^3 - 7x^2 - 42x + 216 - 100 = 0$

$$x^3 - 7x^2 - 42x + 216 = 100$$

So draw line  $y = 100$  & read off  $x$  coords where line & curve intersect.  $x = -5.4$  or  $x = 2.2$  [3]

- (c) Using the graph, estimate the gradient of the curve  $y = x^3 - 7x^2 - 42x + 216$  when  $x = -5$ .

gradient of tangent =  $\frac{250}{2} = 125$

[3]

- (d) Use the trapezium rule with 3 strips to estimate the area of the region enclosed by the curve and the  $x$ -axis between  $x = -6$  and  $x = 0$ .

Area of Trapezium ① =  $\frac{1}{2}(0 + 220) \times 2 = 220$

Area of Trapezium ② =  $\frac{1}{2}(220 + 260) \times 2 = 480$

Area of Trapezium ③ =  $\frac{1}{2}(260 + 210) \times 2 = 470$

$\therefore$  Total Area = 1170

[4]

20. (a) The marks scored by 10 pupils in a test were as follows.

63 35 57 92 25 77 64 58 63 42

Calculate the mean and standard deviation of the 10 test marks.

$$\Sigma x = 576 \quad \Sigma x^2 = 36654$$

$$\text{Mean} = \frac{576}{10} = 57.6$$

$$s = \sqrt{\frac{36654}{10} - \left(\frac{576}{10}\right)^2} = \sqrt{34764} = 18.6$$

[3]

- (b) The ages of the pupils have a mean of 15.8 years and a standard deviation of 0.36 years. State the mean and the standard deviation of the ages of these pupils in 4 years time. Give a reason for your answer.

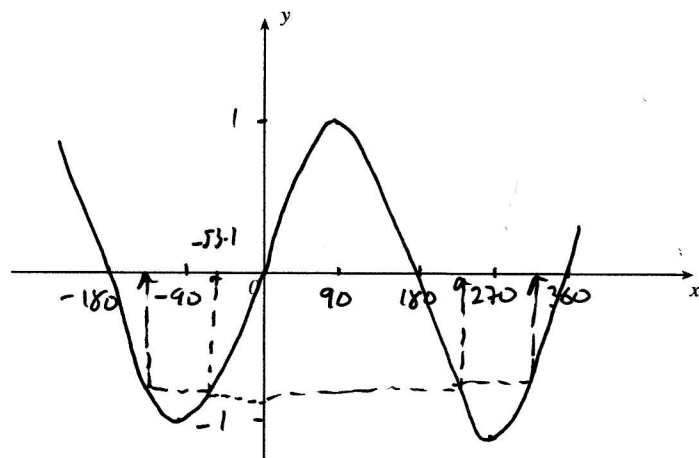
Mean age will have increased by 4 = 19.8 yrs

Standard Deviation unchanged because the spread will be the same = 0.36

[3]

21. (a) Using the axes below, sketch the graph of  $y = \sin x$  for values of  $x$  from  $-180^\circ$  to  $360^\circ$ .

[2]



- (b) Find all solutions of the following equation in the range  $-180^\circ$  to  $360^\circ$ .

From calc  $x = \sin^{-1}(0.8) = -53.1^\circ$   
 From symmetry  $x = -180 + 53.1 = -126.9^\circ$   
 $x = 180 + 53.1 = 233.1^\circ$   
 $x = 360 - 53.1 = 306.9^\circ$

[3]

22. The diagram shows two triangles  $ABC$  and  $ACD$  with the common side  $AC$ .

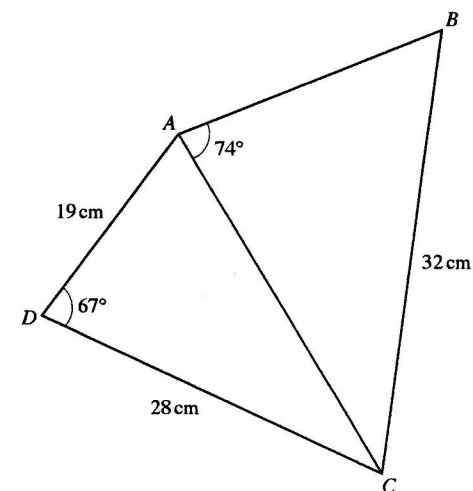
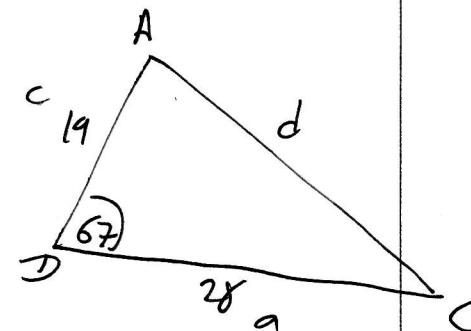
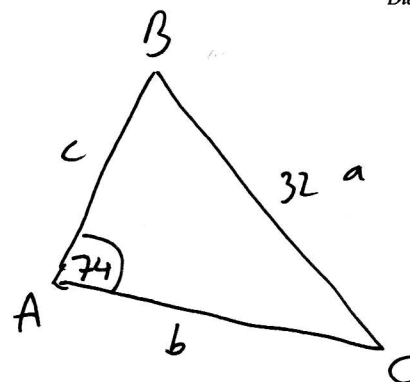


Diagram not drawn to scale.



The triangles  $ABC$  and  $ACD$  are such that  $BC = 32$  cm,  $AD = 19$  cm,  $CD = 28$  cm,  $\hat{BAC} = 74^\circ$  and  $\hat{ADC} = 67^\circ$ .

Find the size of  $\hat{ABC}$ .

From  $\triangle ADC$  using cosine rule to find  $AC$

$$d^2 = a^2 + c^2 - 2ac \cos D$$

$$d^2 = 28^2 + 19^2 - 2 \times 28 \times 19 \cos 67$$

$$d^2 = 1145 - 1064 \cos 67$$

$$d^2 = 1145 - 415.74$$

$$d^2 = 729.26$$

$$d = \sqrt{729.26} = 27.0 \text{ cm} = AC$$

Now from  $\triangle ABC$  using sine rule to find  $\hat{ABC}$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{27} = \frac{\sin 74}{32}$$

$$\sin B = \frac{\sin 74}{32} \times 27$$

[6]

$$\sin B = 0.811$$

$$B = \sin^{-1}(0.811)$$

$$B = 54.2^\circ$$