1. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

   (a) Sketch a diagram to show the distribution of heights represented by this information.

   (3)

   (b) Show that $\mu = 154 + 1.6449\sigma$.

   (3)

   (c) Obtain a second equation and hence find the value of μ and the value of σ.

   (4)

   A woman is chosen at random from the population.

   (d) Find the probability that she is taller than 160 cm.

   (3)

2. The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.

(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.

(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.

(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.

(2)

3. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim N(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3, of D.

(3)

(c) Write down the lower quartile, Q_1, of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)

4. The random variable X has a normal distribution with mean 30 and standard deviation 5.

   (a) Find $P(X < 39)$.

   (2)

   (b) Find the value of d such that $P(X < d) = 0.1151$

   (4)

   (c) Find the value of e such that $P(X > e) = 0.1151$

   (2)

   (d) Find $P(d < X < e)$.

   (2)