Algebraic Fractions

Simplifying Fractions

As with numerical fractions, to simplify an algebraic fraction you look for a factor common to the denominator and numerators and cancel by it, this will often require you to factorise the numerator and denominator in order to do so.

Eg1 Simplify $\frac{x^2-1}{x^2+4x+3}$ $= \frac{(x-1)(x+1)}{(x+1)(x+3)} = \frac{x-1}{x+3}$

Eg2 Simplify $\frac{1-x^2}{4x(7+x)} \times \frac{6x^2}{3+x-2x^2}$ $= \frac{(1-x)(1+x)}{2\cancel{4}x(7+x)} \times \frac{3\cancel{6}x^2}{(3-2x)(\cancel{1}+x)} = \frac{3x^2(1-x)}{2(7+x)(3-2x)}$

Eg3 Simplify $\frac{3n-9}{n} \div \frac{(n^2-9)}{1}$ $= \frac{3(n-3)}{n} \times \frac{1}{n^2-9} = \frac{3(n-3)}{n} \times \frac{1}{(n+3)(n-3)} = \frac{3}{n(n+3)}$

Adding and Subtracting Fractions

Again the rules for algebraic fractions are the same for numerical ones – you write them as equivalent fractions with the lowest common denominator.

Eg4 Express as a single fraction: $\frac{3}{2x+5} + \frac{x-7}{4x^2+10x}$

$2x$ $\frac{3}{2x(2x+5)} + \frac{x-7}{2x(2x+5)}$

$\frac{6x + x - 7}{2x(2x+5)}$

$\frac{7x-7}{2x(2x+5)} = \frac{7(x-1)}{2x(2x+5)}$

Eg5 Express as a single fraction: $\frac{3}{4x-5} - \frac{2}{6-2x}$

$\frac{3(6-2x) - 2(4x-5)}{(4x-5)(6-2x)}$

$\frac{18-6x-8x+10}{(4x-5)(6-2x)} = \frac{28-14x}{(4x-5)(6-2x)} = \frac{14(2-x)}{(4x-5)(6-2x)}$

Exercise 7B Pg 168 Primes

Equations containing algebraic fractions

Often easiest to multiply the whole equation by the lowest common denominator.

Eg6 Find the values of x for which $\frac{x-2}{x-3} - \frac{x+2}{x+3} = \frac{4}{9}$

$\times 9(x-3)(x+3)$

$9(x-3)(x+3) \times \frac{(x-2)}{(x-3)} - 9(x-3)\frac{(x+3)}{(x+3)}(x+2) = 9(x-3)(x+3) \times \frac{4}{9}$

$9(x+3)(x-2) - 9(x-3)(x+2) = 4(x-3)(x+3)$

$9(x^2+x-6) - 9(x^2-x-6) = 4(x^2-9)$

$9x^2+9x-54-9x^2+9x+54 = 4x^2-36$

$0 = 4x^2-18x-36$

$\div 2 \quad 2x^2-9x-18 = 0$

-36

$2x^2-12x+3x-18 = 0$

$2x(x-6)+3(x-6) = 0$

$(x-6)(2x+3) = 0$

either $x-6=0$

$x=6$

or $2x+3=0$

$x=-\frac{3}{2}$

Eg7 Some toys, all at the same price, were bought for £40. If the cost of each toy had been £1 more, then two fewer toys could have been bought. Find the number of toys that were bought.

let n° of toys to be bought = n

the cost per toy = £ $\frac{40}{n}$

If cost £1 more, cost per toy £ $\left(\frac{40}{n}+1\right)$

two fewer toys bought $(n-2)$ toys.

$\left(\frac{40}{n}+1\right)(n-2) = 40$

$\left(\frac{40+n}{n}\right)(n-2) = 40$

$\times n \quad (40+n)(n-2) = 40n$

$40n-80+n^2-2n = 40n$

$n^2-2n-80 = 0$

$(n-10)(n+8) = 0$

either $n-10=0$

$n=10$

or $n+8=0$

$n=-8$

In context of question, $n>0$

$\therefore$ 10 toys were bought.

Exercise 7C Pg 171 Evens