Ex 7E

(1)(i) $$\frac{4}{(1-3x)(1-x)^2} \equiv \frac{A}{1-3x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

$$4 \equiv A(1-x)^2 + B(1-3x)(1-x) + C(1-3x)$$

$x=1$

$4 = -2C$

$C = -2$

$x = \frac{1}{3}$ $\quad 4 = A\left(\frac{2}{3}\right)^2$

$4 = \frac{4}{9}A$

$A = 9$

$\therefore \quad 4 \equiv 9(1-x)^2 + B(1-3x)(1-x) - 2(1-3x)$

$4 \equiv A(1-2x+x^2) + 3(1-4x+3x^2) + C(1-3x)$

equating coefficients:

x^2: $A + 3B = 0$ — (1)

x^1: $-2A - 4B - 3C = 0$ — (2)

x^0: $A + B + C = 4$ — (3)

from (1) $A = -3B$ — (4)

in (2) $-2(-3B) - 4B - 3C = 0$

$6B - 4B - 3C = 0$

$3C = 2B$

$C = \frac{2B}{3}$ — (5)

in (3) $-3B + B + \frac{2B}{3} = 4 \quad \Rightarrow B = -3$

in (4) $A = +9$

in (5) $C = -2$

$\therefore$ equiv to $\frac{9}{1-3x} - \frac{3}{1-x} - \frac{2}{(1-x)^2}$

(11) $\frac{4+2x}{(2x-1)(x^2+1)} \equiv \frac{A}{(2x-1)} + \frac{Bx+C}{(x^2+1)}$

$4+2x \equiv A(x^2+1) + (Bx+C)(2x-1)$

$\equiv A(x^2+1) + 2Bx^2 - Bx + 2Cx - C$

Compare coeficients

x^2: $A + 2B = 0$ — (1)

x^1: $-B + 2C = 2$ — (2)

x^0: $A - C = 4$ — (3)

from (1) $A = -2B$ — (4)

from (2) $2C = 2 + B$

$C = \frac{2+B}{2}$ — (5)

in (3) $-2B - \left(\frac{2+B}{2}\right) = 4$

x2

$-4B - 2 - B = 8$

$-5B = 10$

$B = -2$

in (4) $A = 4$

in (5) $C = 0$

$\therefore$ equiv to $\frac{4}{2x-1} + \frac{-2}{x^2+1}$

$\Rightarrow \frac{4}{2x-1} - \frac{2}{x^2+1}$

(iii) $\dfrac{5-2x}{(x-1)^2(x+2)} \equiv \dfrac{A}{(x-1)^2} + \dfrac{B}{(x-1)} + \dfrac{C}{(x+2)}$

$5-2x \equiv A(x+2) + B(x-1)(x+2) + C(x-1)^2$

$\equiv A(x+2) + B(x^2+x-2) + C(x^2-2x+1)$

Compare Coefficients

x^2: $B + C = 0$ — (1)

x^1: $A + B - 2C = -2$ — (2)

x^0: $2A - 2B + C = 5$ — (3)

from (1) $B = -C$ — (4)

in (2) $A - C - 2C = -2$

$A = 3C - 2$ — (5)

in (3) $2(3C-2) - 2(-C) + C = 5$

$6C - 4 + 2C + C = 5$

$9C = 9$

$C = 1$

in (4) $B = -1$

in (5) $A = 3(1) - 2 = 1$

∴ equiv $\dfrac{1}{(x-1)^2} - \dfrac{1}{(x-1)} + \dfrac{1}{(x+2)}$

(IV) $\frac{2x+1}{(x-2)(x^2+4)} \equiv \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$

$2x+1 \equiv A(x^2+4) + (Bx+C)(x-2)$

$\equiv Ax^2+4A + Bx^2 - 2Bx + Cx - 2C$

Compare Coefficients

x^2: $A+B = 0$ — (1)

x^1: $-2B + C = 2$ — (2)

x^0: $4A - 2C = 1$ — (3)

from (1) $A = -B$ — (4)

sub (4) from (2) $C = 2 + 2B$ — (5)

in (3) $-4B - 2(2+2B) = 1$

$-4B - 4 - 4B = 1$

$-8B = 5$

$B = -\frac{5}{8}$

in (4) $A = +\frac{5}{8}$

in (5) $C = 2 + 2\left(\frac{-5}{8}\right) = 2\frac{1}{4} = \frac{13}{4}$

$\therefore$ equiv $\frac{5}{8(x-2)} + \frac{-\frac{5}{8}x + \frac{13}{4}}{x^2+4}$ $\Rightarrow \frac{6}{8} - \frac{5x}{8}$

$\frac{5}{8(x-2)} + \frac{6-5x}{8(x^2+4)}$

(V) $\dfrac{2x^2+x+4}{(2x^2-3)(x+2)} \equiv \dfrac{Bx+C}{2x^2-3} + \dfrac{A}{x+2}$

$2x^2+x+4 \equiv (Bx+C)(x+2) + A(2x^2-3)$

$\equiv Bx^2+2Bx+Cx+2C+2Ax^2-3A$

Compare coefficients

x^2: $B+2A = 2$ — (1)

x^1: $2B+C = 1$ — (2)

x^0: $2C-3A = 4$ — (3)

from (1) $B = 2-2A$ — (4)

in (2) $2(2-2A)+C = 1$

$4-4A+C = 1$

$C = 4A-3$ — (5)

in (3) $2(4A-3)-3A = 4$

$8A-6-3A = 4$

$5A = 10$

$A = 2$

in (4) $B = -2$

in (5) $C = 5$

∴ equiv $\dfrac{5-2x}{2x^2-3} + \dfrac{2}{x+2}$

(VI) $\frac{x^2-1}{x^2(2x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x+1}$

$x^2-1 \equiv A(2x+1) + Bx(2x+1) + Cx^2$

$\equiv 2Ax + A + 2Bx^2 + Bx + Cx^2$

Compare coefficients

x^2: $2B + C = 1$ — ①

x^1: $2A + B = 0$ — ②

x^0: $A = -1$ — ③

③ in ② $-2 + B = 0$

$B = 2$

in ① $4 + C = 1$

$C = -3$

∴ equiv $\frac{2}{x} - \frac{1}{x^2} - \frac{3}{2x+1}$

(VII) $\frac{x^2+3}{x(3x^2-1)} \equiv \frac{A}{x} + \frac{Bx+C}{3x^2-1}$

$x^2+3 \equiv A(3x^2-1) + (Bx+C)x$

$\equiv 3Ax^2 - A + Bx^2 + Cx$

(vii) compare coefficients

x^2: $3A + B = 1$

x^1: $C = 0$

x^0: $-A = 3$

$A = -3$, $B = 10$, $C = 0$

$\therefore$ equiv $\frac{10x}{3x^2-1}$ $\frac{-3}{x}$

(viii) $\frac{2x^2+x+2}{(2x^2+1)(x+1)} \equiv \frac{Bx+C}{2x^2+1} + \frac{A}{x+1}$

$2x^2+x+2 \equiv (Bx+C)(x+1) + A(2x^2+1)$

$\equiv Bx^2 + Bx + Cx + C + 2Ax^2 + A$

Compare coefficients:

x^2: $B + 2A = 2$ — (1)

x^1: $B + C = 1$ — (2)

x^0: $C + A = 2$ — (3)

from (1) $B = 2 - 2A$ — (4)

in (2) $2 - 2A + C = 1$

$C = 2A - 1$ — (5)

in (3) $2A - 1 + A = 2$

$A = 1$

in (4) $B = 0$, in (5) $C = 1$ $\therefore$ equiv $\frac{1}{2x^2+1} + \frac{1}{x+1}$

(IX) $\dfrac{4x^2-3}{x(2x-1)^2} \equiv \dfrac{A}{x} + \dfrac{B}{(2x-1)} + \dfrac{C}{(2x-1)^2}$

$4x^2-3 \equiv A(2x-1)^2 + Bx(2x-1) + Cx$

$\equiv A(4x^2-4x+1) + 2Bx^2 - Bx + Cx$

Compare Coefficients

x^2: $4A + 2B = 4 \Rightarrow 2A + B = 2$ — (1)

x^1: $-4A - B + C = 0$ — (2)

x^0 $A = -3$

sub (1) $-6 + B = 2$

$B = 8$

sub (2) $12 - 8 + C = 0$

$C = -4$

$\therefore$ equiv $\dfrac{8}{2x-1} - \dfrac{4}{(2x-1)^2} - \dfrac{3}{x}$

(2) $\frac{x^2+2x+7}{(2x+3)(x^2+4)} \equiv \frac{A}{2x+3} + \frac{Bx+C}{x^2+4}$

$x^2+2x+7 \equiv A(x^2+4) + (Bx+C)(2x+3)$

$\equiv Ax^2+4A+2Bx^2+3Bx+2Cx+3C$

Compare coefficients

x^2: $A+2B=1$ — (1)

x^1: $3B+2C=2$ — (2)

x^0: $4A+3C=7$ — (3)

from (1) $A = 1-2B$ — (4)

from (2) $2C = 2-3B$

$C = \frac{2-3B}{2}$ — (5)

in (3) $4(1-2B) + 3\left(\frac{2-3B}{2}\right) = 7$

x2 $8(1-2B) + 3(2-3B) = 14$

$8-16B+6-9B = 14$

$-25B = 0$

$B = 0$

in (4) $A = 1$

in (5) $C = 1$

(5) $\frac{x^2 - 4x + 23}{(x-5)(x^2+3)} \equiv \frac{A}{x-5} + \frac{Bx+C}{(x^2+3)}$

$x^2 - 4x + 23 \equiv A(x^2+3) + (Bx+C)(x-5)$

$\equiv Ax^2 + 3A + Bx^2 - 5Bx + Cx - 5C$

Comparing Coefficients

x^2: $A + B = 1$ — (1)

x^1: $-5B + C = -4$ — (2)

x^0: $3A - 5C = 23$ — (3)

from (1) $A = 1 - B$ — (4)

from (2) $C = 5B - 4$ — (5)

in (3) $3(1-B) - 5(5B-4) = 23$

$3 - 3B - 25B + 20 = 23$

$-28B = 0$

$B = 0$

in (4) $A = 1$

in (5) $C = -4$