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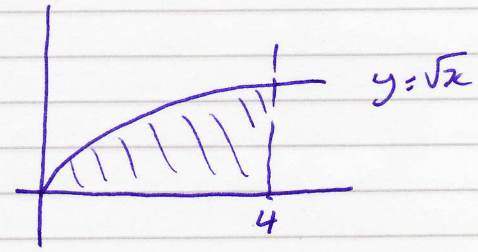
Q1
5



$$M\bar{x} = \rho\pi \int y^2 x \, dx$$

where $M = \rho\pi \int y^2 \, dx$

$$M = \rho\pi \int_0^4 (\sqrt{x})^2 \, dx$$

$$M = \rho\pi \int_0^4 x \, dx$$

$$M = \rho\pi \left[\frac{x^2}{2}\right]_0^4 = 8\rho\pi$$

$$\therefore \; 8\cancel{\rho\pi}\,\bar{x} = \cancel{\rho\pi} \int_0^4 (\sqrt{x})^2 x \, dx$$

$$8\bar{x} = \int_0^4 x^2 \, dx$$

$$8\bar{x} = \left[\frac{x^3}{3}\right]_0^4$$

$$8\bar{x} = \frac{64}{3}$$

$$\bar{x} = \frac{8}{3}$$

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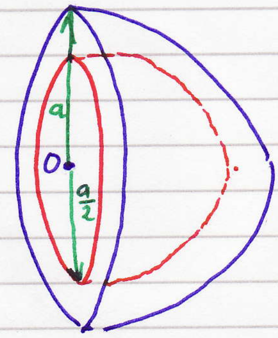
Q2 (a)



$$\left(\frac{2}{3}\cancel{\pi}a^3 - \frac{2}{3}\cancel{\pi}\left(\frac{a}{2}\right)^3\right)\cancel{\rho}\,\bar{x} = \frac{2}{3}\cancel{\pi}a^3\left(\frac{3}{8}a\right)\cancel{\rho} - \frac{2}{3}\cancel{\pi}\left(\frac{a}{2}\right)^3\left(\frac{3}{8}\left(\frac{a}{2}\right)\right)\cancel{\rho}$$

$$\frac{7}{12}\cancel{a^3}\,\bar{x} = \frac{1}{4}\cancel{a^4} - \frac{1}{64}\cancel{a^4}$$

$$\frac{7}{12}\bar{x} = \frac{15}{64}a$$

$$\bar{x} = \frac{15a}{64}\,\frac{12}{7} = \frac{45a}{112}$$ As required.

(b) $$(\cancel{M} + k\cancel{M})\left(\frac{17}{48}a\right)\cancel{\rho} = \cancel{M}\left(\frac{45}{112}a\right) + k\cancel{M}\left(\frac{3}{8}\left(\frac{a}{2}\right)\right)$$

$$\frac{17}{48}\cancel{a} + \frac{17}{48}\cancel{a}k = \frac{45}{112}\cancel{a} + \frac{3}{16}k\cancel{a}$$

$$\frac{17}{48}k - \frac{3}{16}k = \frac{45}{112} - \frac{17}{48}$$

$$\frac{1}{6}k = \frac{1}{21}$$

$$k = \frac{6}{21} = \frac{2}{7}$$

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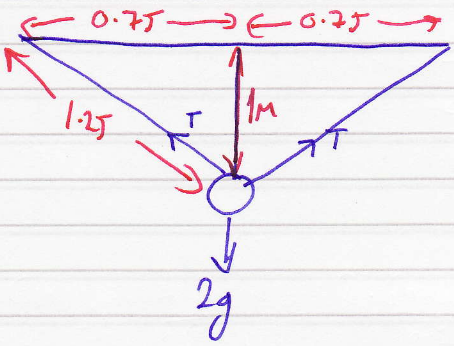
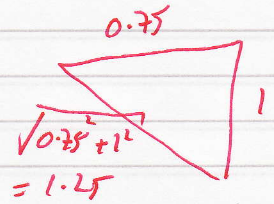
Q5 (a)





loss in PE = gain in KE + gain in EPE

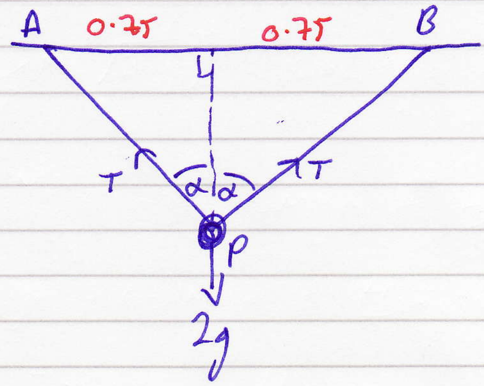
$$2g\times 1=\frac{1}{2}\times 2\times V^2+2\times\frac{49\times(1.25-0.75)^2}{2\times 0.75}$$

$$2g=V^2+\frac{49}{3}$$

$$V^2=\frac{49}{15}$$

$$V=1.80\ ms^{-1}$$

(b)



In equilibrium

$$2T\cos\alpha=2g$$

$$T\cos\alpha=g$$

Hookes Law $\quad T=\frac{49x}{0.75}$

Now $\frac{0.75}{AP}=\sin\alpha$

$\therefore AP=\frac{0.75}{\sin\alpha}$

$$x=\frac{0.75}{\sin\alpha}-0.75$$

$$\therefore\ \frac{g}{\cos\alpha}=\frac{49}{0.75}\left(\frac{0.75-0.75\sin\alpha}{\sin\alpha}\right)$$

$$\frac{g}{\cos\alpha}=\frac{49}{\sin\alpha}\left(1-\sin\alpha\right)$$

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Q8(b) contd

$$g\frac{\tan\alpha}{49} = 1 - \sin\alpha$$

$$\frac{\tan\alpha}{5} = 1 - \sin\alpha$$

$$\tan\alpha = 5 - 5\sin\alpha$$

$\therefore \tan\alpha + 5\sin\alpha = 5$ As required.

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Q6 (a) $V = 3t(t-4) \quad 0 \le t \le 5$

when $t=0 \quad V=0$
when $t=5 \quad V=15$

when $V=0$, $3t(t-4)=0 \quad t=0,\ t=4$

$V = 3t^2 - 12t$

$V_{MAX\ or\ min}$ when $\frac{dV}{dt}=0 \quad 6t-12=0$
$t=2$

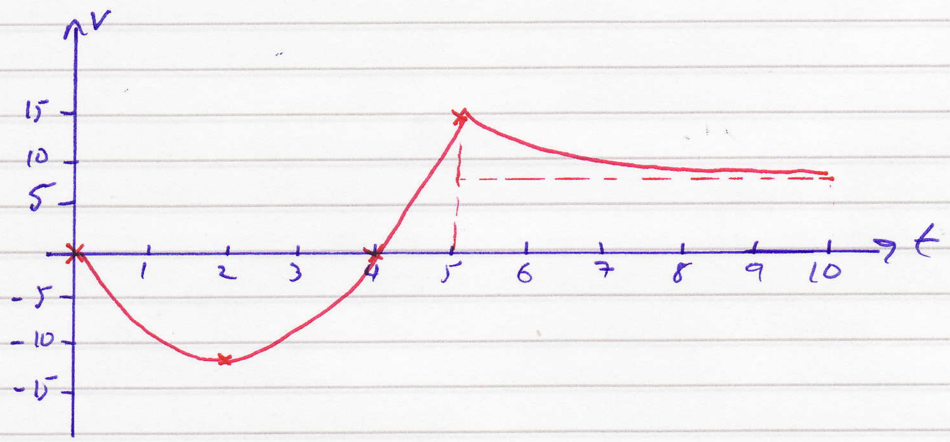
$\frac{d^2V}{dt^2} = 6 > 0 \therefore$ min

when $t=2 \quad V = 6(-2) = -12$

Now between $5 \le t \le 10 \quad V = \frac{75}{t}$

$t=5,\ V = \frac{75}{5} = 15$

$t=10 \quad V = 7.5$



(b) From graph positive acceleration = +ve gradient $2 < t < 5$

(c) Total dist travelled between $t=0$ & $t=5$ = Area under graph

$$\int_0^4 3t^2 - 12t\,dt + \int_4^5 3t^2 - 12t\,dt$$

$$[t^3 - 6t^2]_0^4 + [t^3 - 6t^2]_4^5$$

$$(4^3 - 6(4)^2 - 0) + [(5^3 - 6(5)^2) - (4^3 - 6(4)^2)] = 32 + 7 = 39 \text{ metres}$$

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Q6(d) Returns to O, when $x=0$

$$x = \int \frac{75}{t}\,dt$$

$$x = 75\ln t + c \quad — (1)$$

Need to know what x is when $t=5$.

For $0 \le t \le 5$

$$x = \int 3t^2 - 12t\,dt$$

$$x = t^3 - 6t^2 + c$$

when $x=0$, $t=0$ $\therefore c=0$

$$x = t^3 - 6t^2$$

when $t=5$ $\quad x = 5^3 - 6(5)^2 = -25$

$\therefore$ in (1) $\quad -25 = 75\ln 5 + c$

$$c = -25 - 75\ln 5$$

$$x = 75\ln t - 25 - 75\ln 5$$

Now when $x=0$

$$75\ln t = 25 + 75\ln 5$$

$$\ln t = \frac{25 + 75\ln 5}{25}$$

$$~~t=~~$$

$$t = e^{\frac{25+75\ln 5}{25}}$$

$$t = 6.98 \text{ secs}$$