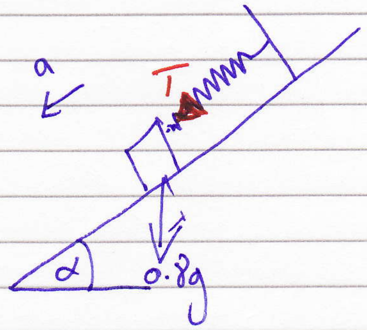
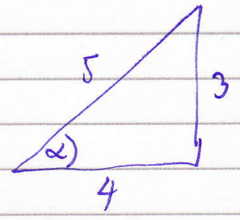
M3 - June 05

Q1





$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

Thrust: Hookes Law $\quad T = \frac{20 \times 0.4}{2} = 4N$

NZL: $\quad T + 0.8g\sin\alpha = 0.8a$

$4 + 0.8g \times \frac{3}{5} = 0.8a$

$a = 10.9\ ms^{-2}$

M3 June 05

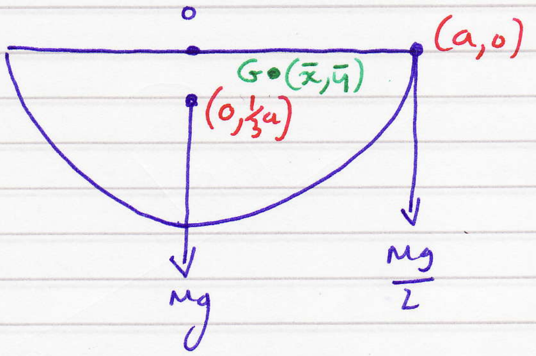
Q4(a) $(\pi a^2 + 2\pi a^2) M\bar{x} = \pi a^2 M(0) + 2\pi a^2 M\left(\frac{1}{2}a\right)$

area of disc — Surface Area of hemisphere

$3\cancel{\pi}\cancel{a^2}\cancel{M}\bar{x} = \cancel{\pi}a^{\cancel{3}}\cancel{M}$

$\bar{x} = \frac{1}{3}a$ As required

(b) Let position of new centre of mass, $G(\bar{x}, \bar{y})$

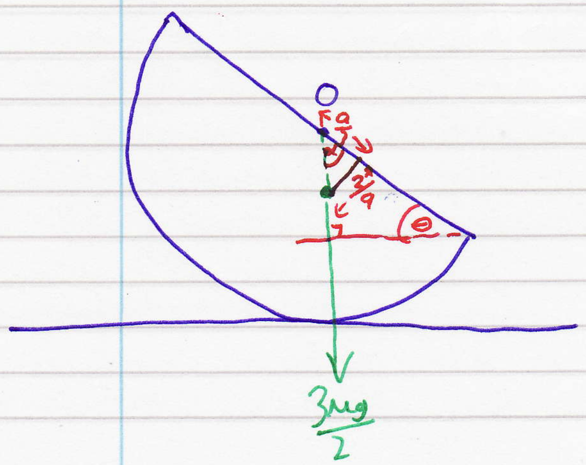


$\left(\cancel{Mg} + \frac{\cancel{Mg}}{2}\right)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \cancel{Mg}\begin{pmatrix}0\\ \frac{1}{3}a\end{pmatrix} + \frac{\cancel{Mg}}{2}\begin{pmatrix}a\\ 0\end{pmatrix}$

$\frac{3}{2}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}a/2\\ a/3\end{pmatrix}$

$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}a/3\\ 2a/9\end{pmatrix}$

Now when in equilibrium on horizontal plane, G is directly above point of contact:



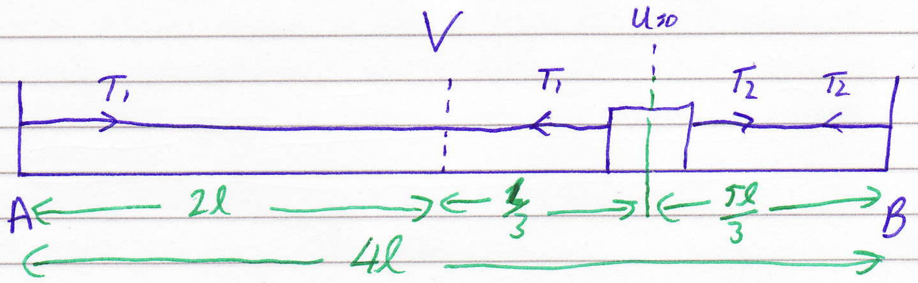
$\tan\alpha = \frac{2/9}{1/3}$

$\alpha = 33.69$

Now $\theta = 90 - \alpha = 56.3^\circ$

M3 - June 05

Q3 (a)



At point of release, $EPE = \frac{4mg}{2(l)}\left(2l+\frac{l}{3}-l\right)^2 + \frac{4mg}{2(l)}\left(\frac{5l}{3}-l\right)^2$

$$EPE = \frac{2mg}{l}\left(\frac{4l}{3}\right)^2 + \frac{2mg}{l}\left(\frac{2}{3}l\right)^2$$

$$EPE = \frac{32mgl}{9} + \frac{8mgl}{9} = \frac{40mgl}{9} \quad \cancel{=\frac{8mgl}{3}}$$

At midpoint strings at same length $\therefore$ EPE = equal + opposite $= 2\times\frac{4mg}{2l}(l)^2$

Table is smooth $\therefore$ no wd against friction $= 4mgl$

$\therefore$ Gain in KE = loss in EPE

$\cancel{\frac{1}{2}mv^2 = \frac{8mgl}{3}}$ $\quad \frac{1}{2}mv^2 = \frac{40}{9}mgl - 4mgl$

$\cancel{v^2 = \frac{16}{3}gl}$ $\quad \frac{1}{2}v^2 = \frac{4}{9}gl$

$$v^2 = \frac{8}{9}gl$$

$$v = \frac{1}{3}\sqrt{8gl}$$

(b) At midpoint, $T_1 = T_2$ $\therefore$ particle is in equilibrium and $a=0$, hence maximum speed.