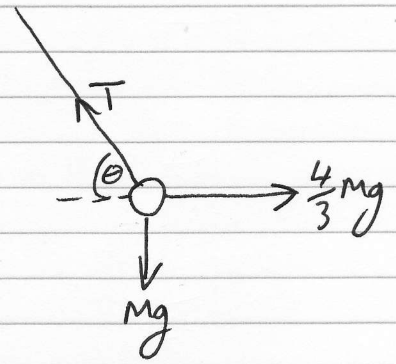
Q2

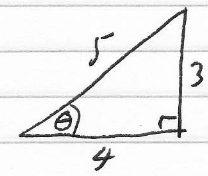


(a) HL $T = \frac{3Mgx}{a}$ — ①

ΣF_y: $T\sin\theta = Mg$ — ②

ΣF_x: $\frac{4}{3}Mg - T\cos\theta = 0$

$T\cos\theta = \frac{4}{3}Mg$ — ③



② ÷ ③ $\tan\theta = \frac{3}{4}$

$\therefore \sin\theta = \frac{3}{5}$

n ② $T \times \frac{3}{5} = Mg$

$T = \frac{5}{3}Mg$

(b) n ① $\frac{5}{3}\cancel{Mg} = \frac{3\cancel{Mg}x}{a}$

$x = \frac{5a}{9}$

Now energy $= \frac{\lambda x^2}{2a} = \frac{3Mg}{2a}\left(\frac{5a}{9}\right)^2 = \frac{25}{54}Mga$

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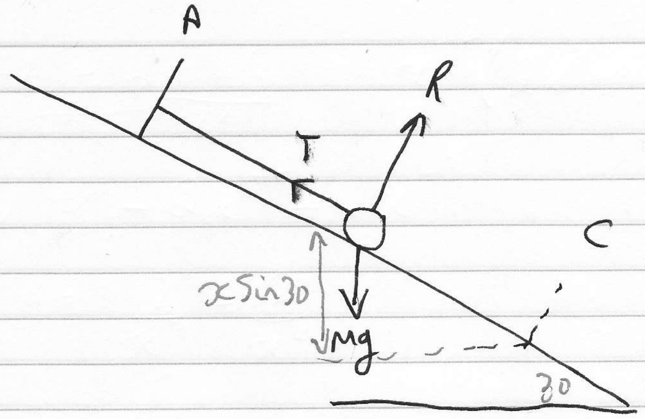

(a) $BC = \text{extension} = x$ No change in KE

@C loss in pe = gain in Epe

$$Mg\,x\sin 30 = \frac{\lambda x^2}{2a}$$

$$\frac{\cancel{Mg}\cancel{x}}{\cancel{2}} = \frac{6\cancel{Mg}\,x^{\cancel{2}}}{\cancel{2}a}$$

$$x = \frac{a}{6}$$

$$\therefore AC = \frac{a}{6} + a = \frac{7a}{6}$$

(b) Max speed when accel $= 0$

N2L $Mg\sin 30 - T = 0$

$$T = \frac{Mg}{2}$$

$\therefore$ Max speed when Tension in string $= \frac{Mg}{2}$

Now using Hookes Law $T = \frac{\lambda x}{l}$

$$\frac{Mg}{2} = \frac{6Mg\,x}{a}$$

$$x = \frac{a}{12}$$

$\therefore$ Max speed occurs after sliding $\frac{a}{12}$ from B.

5 (b) contd

loss in PE = gain in KE + gain in EPE

$$x \sin 30 \, mg = \frac{1}{2}mv^2 + \frac{\lambda x^2}{2a}$$

but $x = \frac{a}{12}$

$$\frac{a}{12} \times \frac{1}{2} \times \cancel{m}g = \frac{1}{2}\cancel{m}v^2 + \frac{6\cancel{m}g\left(\frac{a}{12}\right)^2}{2a}$$

$$\frac{ag}{24} = \frac{1}{2}v^2 + \frac{3g}{\cancel{a}} \times \frac{a^{\cancel{2}}}{144}$$

$$\frac{ag}{24} = \frac{1}{2}v^2 + \frac{ag}{48}$$

$$\frac{1}{2}v^2 = \frac{ag}{48}$$

$$v^2 = \frac{ag}{24}$$

$$v = \sqrt{\frac{ag}{24}}$$

6 (a) $M\bar{x} = \rho\pi \int y^2 x \, dx$

where $M = \rho\pi \int y^2 \, dx$

$$M = \rho\pi \int_0^2 (4-x^2)^2 \, dx$$

$$M = \rho\pi \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$M = \rho\pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2$$

$$M = \rho\pi\left[32 - \frac{64}{3} + \frac{32}{5}\right] = \cancel{\frac{32}{3}\rho\pi} \; \frac{256}{15}\rho\pi$$

Now

$$\frac{256}{15}\cancel{\frac{32}{3}}\cancel{\rho\pi}\,\bar{x} = \cancel{\rho\pi}\int_0^2 (4-x^2)^2 x \, dx$$

$$\frac{256}{15}\cancel{\frac{32}{3}}\,\bar{x} = \int_0^2 16x - 8x^3 + x^5 \, dx$$

$$\frac{256}{15}\cancel{\frac{32}{3}}\,\bar{x} = \left[8x^2 - 2x^4 + \frac{x^6}{6}\right]_0^2$$

$$\frac{256}{15}\cancel{\frac{32}{3}}\,\bar{x} = 32 - 32 + \frac{32}{3}$$

$$\bar{x} = \frac{32}{3} \times \frac{15}{256} = \frac{5}{8} \quad \text{As required.}$$

(b) $$\left(\pi \times 4^2 \times l + \frac{256\pi}{15}\right)\rho\,\bar{x} = \left(\pi \times 4^2 \times l\right)\rho\left(\frac{l}{2}\right) + \frac{256\pi}{15}\rho\left(l + \frac{5}{8}\right)$$

Now $\bar{x} = l$

$$\left(16\pi l + \frac{256\pi}{15}\right) l = 8\pi l^2 + \frac{256\pi l}{15} + \frac{32\pi}{3}$$

$\cancel{16\pi l^2} + \frac{256\pi l}{15} = 8\pi l$

$$\cancel{16\pi l^2 + \frac{256\pi l}{15} = 8\pi l + \frac{256\pi l}{15} + \frac{32\pi}{3}}$$

$$\cancel{16\pi l^2 - 8\pi l - \frac{32\pi}{3} = 0} \qquad l = 1.1\,\text{m}$$

6)b) $16\cancel{\pi}l^2 + \frac{25\cancel{6}}{\cancel{\sqrt{5}}}\cancel{\pi}l = 8\cancel{\pi}l^2 + \frac{25\cancel{6}}{\cancel{\sqrt{5}}}\cancel{\pi}l + \frac{32}{3}\cancel{\pi}$

$8l^2 = \frac{32}{3}$

$l^2 = \frac{32}{24}$

$l^2 = \frac{4}{3}$

$l = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$