M3 - Jan 07

Q1 (b) $a = \frac{1}{12}(30-x)$

$\cancel{dx}$ if $a = f(x)$, $\frac{V^2}{2} = \int f(x)\,dx + c$

$\therefore \frac{V^2}{2} = \int \frac{1}{12}(30-x)\,dx$

$6V^2 = \int 30 - x\,dx$

$6V^2 = 30x - \frac{x^2}{2} + c$

when $x = 30$, $V = 10$

$6 \times 10^2 = 30(30) - \frac{(30)^2}{2} + c$

$600 = 450 + c$

$\therefore c = 150$

So $6V^2 = 30x - \frac{x^2}{2} + 150$

$\div 6$

$V^2 = 25 + 5x - \frac{x^2}{12}$

(a) Max speed occurs when $a = 0$ $\therefore x = 30$.

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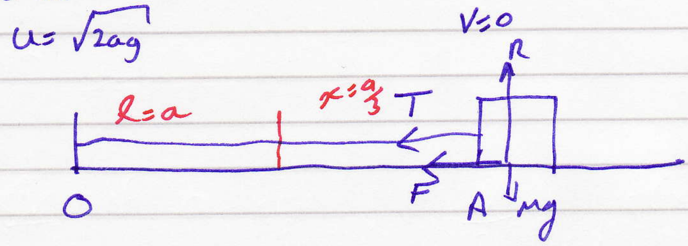
Q3



(a) $EPE = \dfrac{\lambda x^2}{2l} = \dfrac{3.6mg\left(\frac{a}{3}\right)^2}{2a} = \underline{0.2mga}$

(b) ~~Gain in EPE = loss in KE + wd v's Friction~~

$$\cancel{0.2mga = \frac{1}{2}m\left(\sqrt{2ag}\right)^2 + \frac{4}{3}a \times (\mu R)}$$

$$\cancel{0.2mga = mga + \frac{4}{3}a \times \mu mg}$$

$$\cancel{0.2 - 1 = \frac{4}{3}\mu}$$

(b) loss in KE = Gain in EPE + wd v's Friction

$$\frac{1}{2}m\left(\sqrt{2ag}\right)^2 = 0.2mga + \frac{4}{3}a \times (\mu R) \qquad \text{but } R = mg$$

$$mga = 0.2mga + \frac{4}{3}a\mu mg$$

$$mga - 0.2mga = \frac{4}{3}a\mu mg$$

$$0.8\cancel{mga} = \frac{4}{3}\cancel{a}\mu \cancel{mg}$$

$$\mu = \frac{0.8 \times 3}{4} = \underline{0.6}$$

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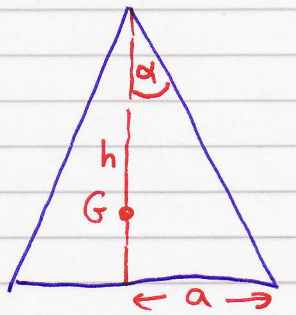
Q2

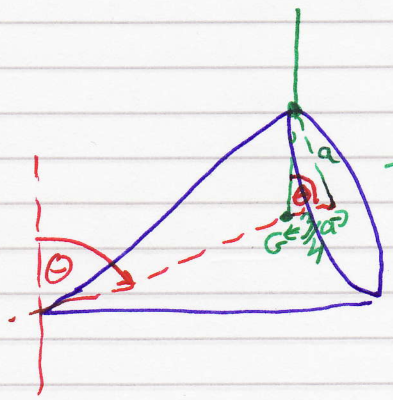


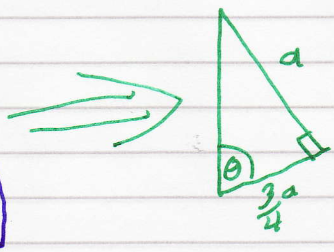
height : $\tan\alpha = \frac{a}{h}$

$\frac{1}{3} = \frac{a}{h}$

$h = 3a$

Now $G = \frac{1}{4}h = \frac{3a}{4}$





$\tan\theta = \frac{a}{\frac{3a}{4}} = \frac{4}{3}$

$\theta = 53.1°$

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Q6(a) $y = \frac{1}{2x^2} = \frac{1}{2}x^{-2}$

$$M\bar{x} = \rho\pi \int y^2 x \, dx$$

where $M = \rho\pi \int y^2 \, dx$

$$y^2 = \left(\frac{1}{2x^2}\right)^2 = \frac{1}{4x^4} = \frac{1}{4}x^{-4}$$

$$\therefore M = \frac{\rho\pi}{4} \int_1^2 x^{-4} \, dx$$

$$= \frac{\rho\pi}{4} \left[\frac{x^{-3}}{-3} \right]_1^2$$

$$= \frac{\rho\pi}{12} \left[-\frac{1}{x^3} \right]_1^2$$

$$= \frac{\rho\pi}{12} \left(-\frac{1}{8} + 1 \right)$$

$$\therefore \text{Mass} = \frac{7\rho\pi}{96}$$

Now $\frac{7\cancel{\rho\pi}}{96}\bar{x} = \cancel{\rho\pi} \int_1^2 \frac{1}{4} x^{-4} . x \, dx$

$$\frac{7}{96}\bar{x} = \frac{1}{4} \int_1^2 x^{-3} \, dx$$

$$\frac{7}{24}\bar{x} = \left[\frac{x^{-2}}{-2} \right]_1^2$$

$$\frac{7}{24}\bar{x} = \frac{1}{2} \left[-\frac{1}{x^2} \right]_1^2$$

$$\frac{7}{12}\bar{x} = \left(-\frac{1}{4} + 1 \right)$$

$\bar{x} = \frac{3}{4} \times \frac{12}{7} = \frac{9}{7}$ ie $1\frac{2}{7}$ m from origin $= \frac{2}{7}$ m from large plane face.

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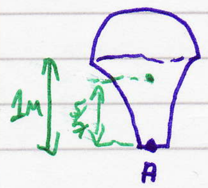
Q6 (b) Volume of S $= \pi \int_1^2 y^2 \, dx = \frac{7\pi}{96}$

(Already worked out $\int_1^2 y^2$ in A)

Volume of H $= \frac{2}{3}\pi\left(\frac{1}{2}\right)^3 = \frac{\pi}{12}$



origin of COM @ A

$$\left(\frac{7\pi}{96} + \frac{\pi}{12}\right)\cancel{\rho}\,\bar{y} = \frac{7\pi}{96}\cancel{\rho}\left(\frac{5}{7}\right) + \frac{\pi}{12}\cancel{\rho}\left(1 + \frac{3}{8}\left(\frac{1}{2}\right)\right)$$

$$\frac{5}{32}\cancel{\pi}\,\bar{y} = \frac{5}{96}\cancel{\pi} + \frac{19}{192}\cancel{\pi}$$

$$\frac{5}{32}\bar{y} = \frac{29}{192}$$

$$\bar{y} = \frac{29}{30}$$

$\therefore$ COM of T is $\frac{29}{30}$ metres above A.