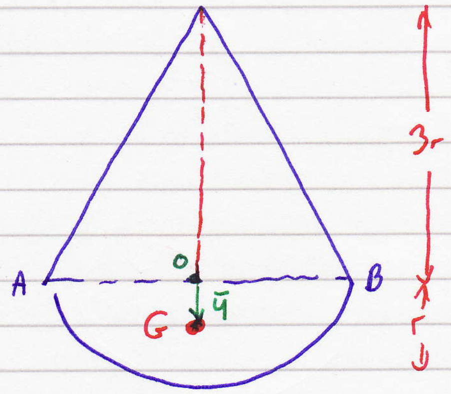
M3 – Jan 05

Q2 (a)



If c.o.m of toy is G $\downarrow$ is poistive

$$(m+M)\bar{y} = m\left(\frac{-1}{4}(3r)\right) + M\left(\frac{3r}{8}\right)$$

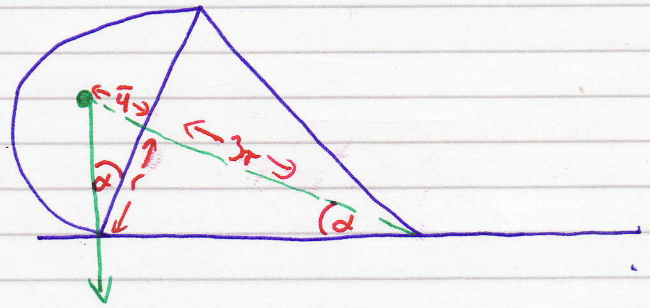
$$(m+M)\bar{y} = -\frac{3}{4}mr + \frac{3}{8}Mr$$

$$(m+M)\bar{y} = \frac{3}{8}Mr - \frac{3}{4}mr$$

$$(m+M)\bar{y} = \frac{3Mr - 6mr}{8}$$

$$\bar{y} = \frac{3r(M-2m)}{8(m+M)}$$ As required

(b) At limiting equilibrium:



$\tan\alpha = \frac{r}{3r}$ (Δ in cone)

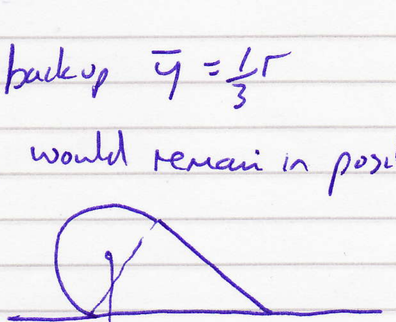
$\tan\alpha = \frac{1}{3}$

$\therefore$ $\frac{\bar{y}}{r} = \tan\alpha$ (Δ in hemisphere)

$\bar{y} = r\tan\alpha = \frac{1}{3}r$

So at point of tipping back up $\bar{y} = \frac{1}{3}r$

If $\bar{y} < \frac{1}{3}r$, toy would remain in position:



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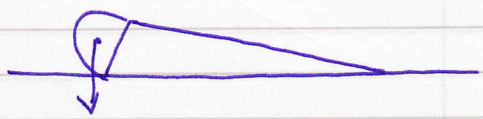
Q2(b) So if $\bar{y} > \frac{l}{3}$, no equilibrium



$$\therefore \frac{3\cancel{l}(M-2m)}{8(m+M)} > \frac{1\cancel{l}}{3}$$

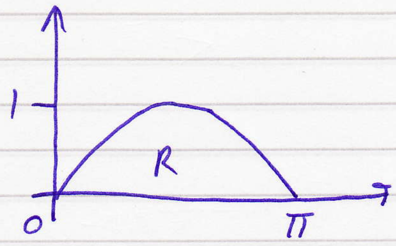
$$9(M-2m) > 8(m+M)$$

$$9M - 18m > 8m + 8M$$

$$M > 26m$$ As required.

JAN05

Q3(a)



By symmetry, $\bar{x} = \frac{\pi}{2}$

Now $M\bar{y} = \frac{1}{2}\int_0^{\pi} \rho y^2 \, dx$

where $M = \int_0^{\pi} \rho y \, dx$

$M = \rho \int_0^{\pi} \sin x \, dx$

$M = \rho \left[-\cos x\right]_0^{\pi}$

$\cancel{M = \rho(\pi(-1)}$

$M = \rho\left[(-\cos\pi) - (-\cos 0)\right]$

$M = \rho\left[-(-1) + 1\right]$

$M = 2\rho$

$\therefore 2\rho\bar{y} = \frac{1\rho}{2}\int_0^{\pi} (\sin x)^2 \, dx$

but $\cos 2x = 1 - 2\sin^2 x$

$\therefore \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$\therefore 4\bar{y} = \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$

$8\bar{y} = \int_0^{\pi} 1 - \cos 2x \, dx$

$8\bar{y} = \left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$

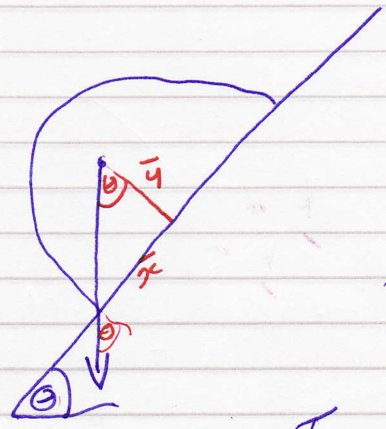
$8\bar{y} = (\pi - 0) - (0 - 0)$

$8\bar{y} = \pi$

$\bar{y} = \frac{\pi}{8}$ As required.

JAN 07

Q3(b)



At point of toppling

$$\tan\theta = \frac{\bar{x}}{\bar{y}}$$

$$\tan\theta = \frac{\pi/2}{\pi/8}$$

$$\tan\theta = 4$$

$$\theta = 75.96^\circ$$

MB JANOT

Q5 $a = -\dfrac{3}{\sqrt{t+4}}$ ms^{-2}

$a = \dfrac{dv}{dt}$

$\dfrac{dv}{dt} = -\dfrac{3}{\sqrt{t+4}}$ M1

$V = \int -3(t+4)^{-1/2}\, dx$

$V = -3 \div \dfrac{2}{\cancel{3}} (t+4)^{1/2} + c$

$V = -\cancel{6}(t+4)^{1/2} + c$ M1 A1

when $t=0$ $V=18$

$18 = -\cancel{6}(4)^{1/2} + c$ M1

$18 = -\cancel{12} + c \implies c = 30$ A1

$V = 30 - 6\sqrt{t+4}$ As required.

(b) P comes to rest when $V=0$ $30 - 6\sqrt{t+4} = 0$

$\sqrt{t+4} = 5$

$t+4 = 25$ M1A1

$t = 21$ sec

Now $V = \dfrac{dx}{dt} = 30 - 6\sqrt{t+4}$

$x = \int 30 - 6(t+4)^{1/2}\, dt$ M1

$x = 30t - 6.\dfrac{2}{3}(t+4)^{3/2} + c$

$x = 30t - 4(t+4)^{3/2} + c$ A1

when $t=0$, $x=0$

$0 = 0 - 4(4)^{3/2} + c$ $c = 32$ M1

A1

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Q5(b) contd $x = 30t - 4(t+4)^{3/2} + 32$

when $t = 21$ $x = 30(21) - 4(21+4)^{3/2} + 32$ M1

$x = 630 - 500 + 32$

$x = 162$ metres from O. A1

$\frac{M}{12}$