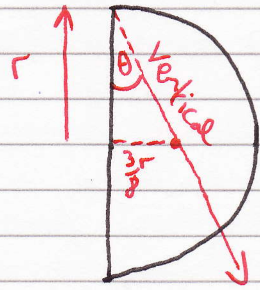
Ex5C

(1)

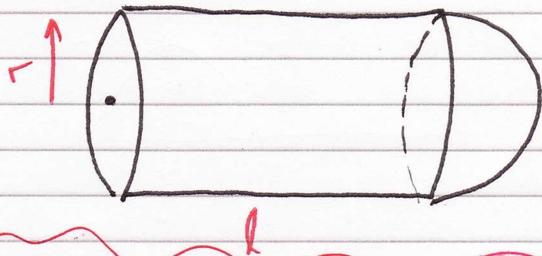


$\tan\theta = \dfrac{\frac{3r}{8}}{r} = \dfrac{3}{8}$

$\theta = 20.6^\circ$ = angle to vertical

$\therefore$ Angle to horizontal $= 69.4^\circ$

(2).



Volume of sphere $= \frac{4\pi r^3}{3}$

$\left(\pi r^2 l + \frac{2}{3}\pi r^3\right)\rho\,\bar{x} = \left(\pi r^2 l\right)\rho\left(\frac{l}{2}\right) + \left(\frac{2}{3}\pi r^3\right)\rho\left(l + \frac{1}{2}r\right)$

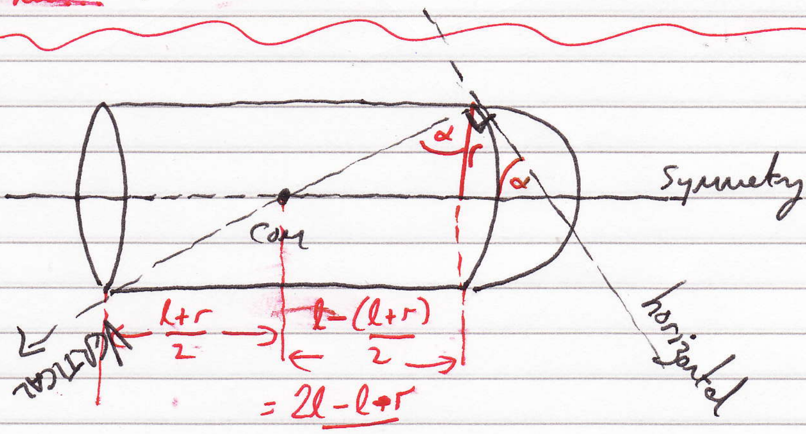
$r^2\left(l + \frac{2r}{3}\right)\bar{x} = r^2\frac{l^2}{2} + \frac{2}{3}r^2 l + \frac{1}{2}r^{2}$

$\left(l + \frac{2r}{3}\right)\bar{x} = \frac{l^2}{2} + \frac{2}{3}rl + \frac{1}{2}r^2$

rhs needs to factorise to $\left(l+\frac{2}{3}r\right)\left(\frac{l}{2}+\frac{r}{2}\right)$ but won't !??

Then

SEE OVERLEAF



$= \frac{2l-l+r}{2}$

$= \frac{l-r}{2}$

$\tan\alpha = \dfrac{\frac{l-r}{2}}{r} = \dfrac{l-r}{2r}$ As required.

(2) <u>First part</u> hollow cylinder, so use <u>area</u> not volume

Area of cylinder $= 2\pi r l$

Area of hemisphere $= 2\pi r^2$

$\therefore$

$$(2\pi r l + 2\pi r^2)\rho \bar{x} = (2\pi r l)\rho\left(\frac{l}{2}\right) + (2\pi r^2)\rho\left(l + \frac{r}{2}\right)$$

$$r(l+r)\bar{x} = \frac{rl^2}{2} + r^2 l + \frac{r^3}{2}$$

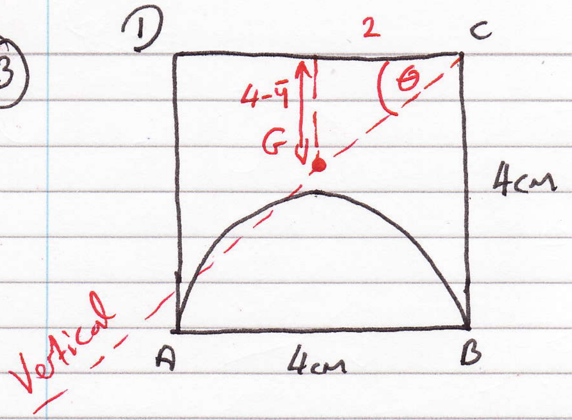
$$(l+r)\bar{x} = \frac{l^2}{2} + rl + \frac{r^2}{2}$$

$$(l+r)\bar{x} = \frac{1}{2}(l^2 + 2rl + r^2)$$

$$(l+r)\bar{x} = \frac{1}{2}(l+r)^2$$

$$\bar{x} = \frac{1}{2}(l+r). \quad \text{As required.}$$

3



$$\left(16 - \frac{4\pi}{2}\right)\rho\bar{y} = 16(2)\rho - \frac{4\pi}{2}\rho\left(\frac{2\times 2\times \sin\frac{\pi}{2}}{3\frac{\pi}{2}}\right)$$

$$(16 - 2\pi)\bar{y} = 32 - \frac{16}{3}$$

$$(16-2\pi)\bar{y} = \frac{96-16}{3}$$

$$2(8-\pi)\bar{y} = \frac{80}{3}$$

$$\bar{y} = \frac{40}{3(8-\pi)}$$ As required.

$$\tan\theta = \frac{4-\bar{y}}{2}$$

$$= \left(4 - \frac{40}{3(8-\pi)}\right)\times\frac{1}{2}$$

$$= 2 - \frac{20}{3(8-\pi)}$$

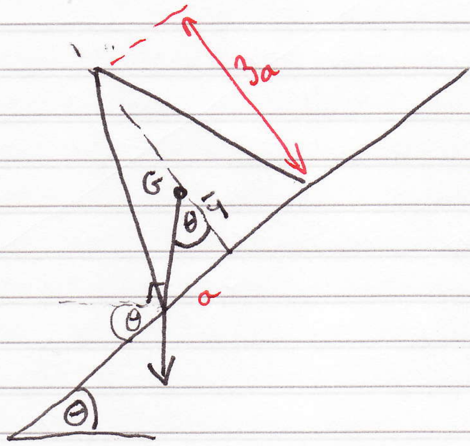
$$= \frac{6(8-\pi)-20}{3(8-\pi)}$$

$$= \frac{48-6\pi-20}{3(8-\pi)}$$

$$= \frac{28-6\pi}{3(8-\pi)}$$

$$\tan\theta = \frac{2(14-3\pi)}{3(8-\pi)}$$ As required.

4 contd

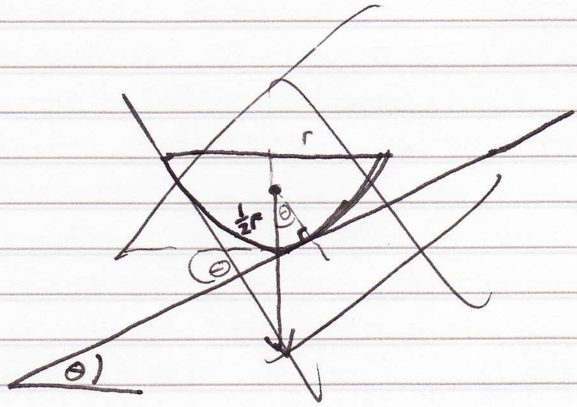


$$\bar{y} = \frac{3}{4} \times 3a = \frac{3a}{4}$$

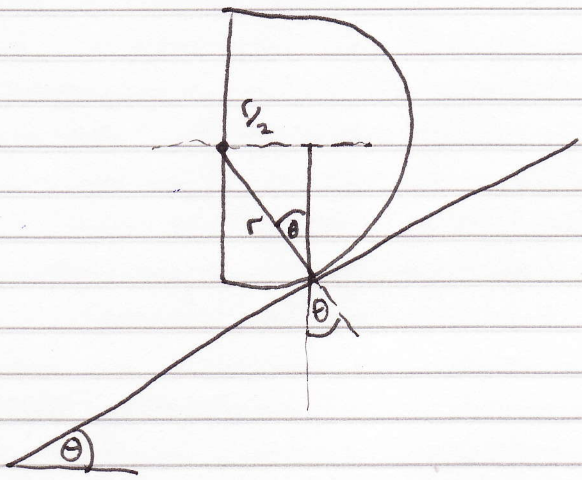
$$\tan\theta = \frac{a}{\frac{3a}{4}} = \frac{4}{3}$$

$$\theta = 53.1^\circ$$

5



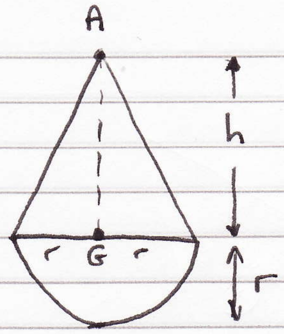
5



$$\sin\theta = \frac{r/2}{r} = \frac{1}{2}$$

$$\theta = 30^\circ.$$

(6)



Position of com from A:

$$\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)\rho \bar{y} = \frac{1}{3}\pi r^2 h\left(\frac{3}{4}h\right)\rho + \frac{2}{3}\pi r^3\left(h + \frac{3}{8}r\right)\rho$$

$$(r^2 h + 2r^3)\bar{y} = \frac{3}{4}r^2h^2 + 2r^3h + \frac{3}{4}r^4$$

$\div r^2$

$$(h+2r)\bar{y} = \frac{3}{4}h^2 + 2rh + \frac{3}{4}r^2$$

$$\bar{y} = \frac{1}{4}\left(\frac{1}{h+2r}\right)(3h^2 + 8rh + 3r^2)$$ not necessary

For solid to rest in equilib with curved surface in contact, $\bar{y} = h$

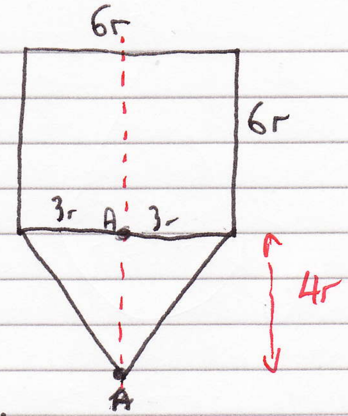
$$\therefore (h+2r)h = \frac{3}{4}h^2 + 2rh + \frac{3}{4}r^2$$

$$h^2 + 2rh = \frac{3}{4}h^2 + 2rh + \frac{3}{4}r^2$$

$$\frac{1}{4}h^2 = \frac{3}{4}r^2$$

$$h = r\sqrt{3}$$

(7)



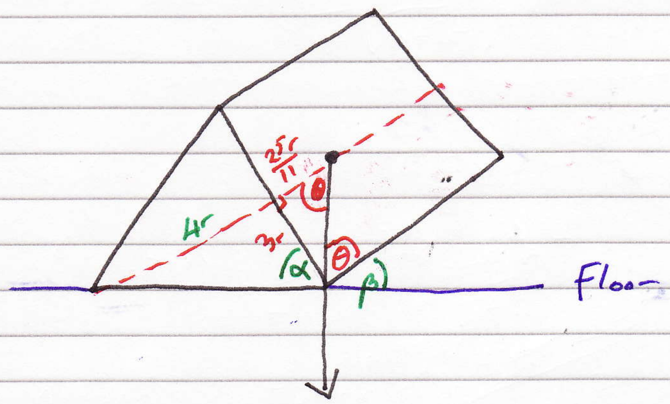
Distance from A:

$$\left(\frac{1}{3}\pi(3r)^2(4r) + \pi(3r)^2(6r)\right)\rho\bar{y} = \left(\frac{1}{3}\pi(3r)^2(4r)\right)\rho\left(-\frac{1}{4}(4r)\right) + \left(\pi(3r)^2(6r)\right)\rho(3r)$$

$$(12\pi r^3 + 54\pi r^3)\bar{y} = -12\pi r^4 + 162\pi r^4$$

$$66\bar{y} = 150r$$

$$\bar{y} = \frac{150r}{66} = \frac{25r}{11}$$ above join of surfaces.



Toy on point of ~~slipping~~ toppling $\tan\theta = \frac{3r}{\frac{25r}{11}} = \frac{33}{25}$

$\theta = 52.85°$

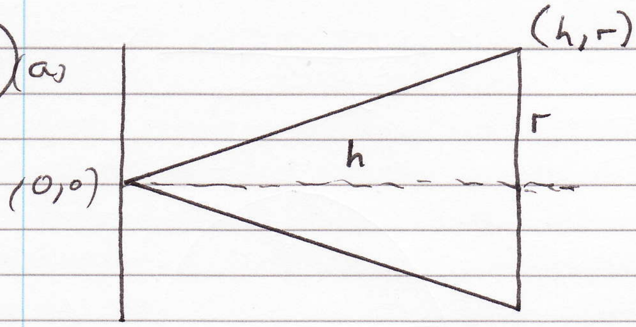
So for toy not to topple $\theta \leq 52.85°$

but $\tan\alpha = \frac{4r}{3r}$ $\alpha = 53.1°$ ~~[illegible]~~ and $\beta = 180 - 53.1 - 90 = 36.9°$

Now in this case, ~~[illegible]~~ $\theta = 90 - 36.9 = 53.1°$

So $\theta > 52.85°$, toy will topple.

8 (a)



$$\frac{y-0}{r-0} = \frac{x-0}{h-0} \Rightarrow \frac{y}{r} = \frac{x}{h} \Rightarrow y = \frac{r}{h}x$$

$$M = \frac{1}{3}\pi r^2 h \rho$$

$$M\bar{x} = \int_0^h \pi \rho x y^2 \, dx$$

$$M\bar{x} = \rho\pi \int_0^h x \frac{r^2}{h^2} x^2 \, dx$$

$$M\bar{x} = \frac{\pi \rho r^2}{h^2} \int_0^h x^3 \, dx$$

$$M\bar{x} = \frac{\pi \rho r^2}{h^2} \left[\frac{x^4}{4} \right]_0^h$$

$$M\bar{x} = \frac{\pi \rho r^2}{h^2} \cdot \frac{h^{4\,2}}{4}$$

$$M\bar{x} = \pi \rho \frac{r^2 h^2}{4}$$

Now

$$\frac{1}{3}\pi r^2 h \rho \, \bar{y} = \pi \rho \frac{r^2 h^2}{4}$$

$$\bar{y} = \frac{3}{4}h$$

∴ CoM is $\frac{3h}{4}$ in from vertex or $\frac{1h}{4}$ in from base.

8(b) $\left(\frac{1}{3}\pi a^2 \lambda h + \frac{1}{3}\pi a^2 h\right)\rho \bar{x} = \left(\frac{1}{3}\pi a^2 \lambda h\right)\rho\left(\frac{3}{4}\lambda h\right) + \left(\frac{1}{3}\pi a^2 h\right)\rho\left(\lambda h + \frac{1}{4}h\right)$

$\frac{1}{3}\pi a^2 h(\lambda + 1)\bar{x} = \frac{1}{4}\pi a^2 \lambda^2 h^2 + \frac{1}{3}\pi a^2 h^2 \lambda + \frac{1}{12}\pi a^2 h^2$

×3 $(\lambda+1)\bar{x} = \frac{3\lambda^2 h}{4} + h\lambda + \frac{1}{4}h$

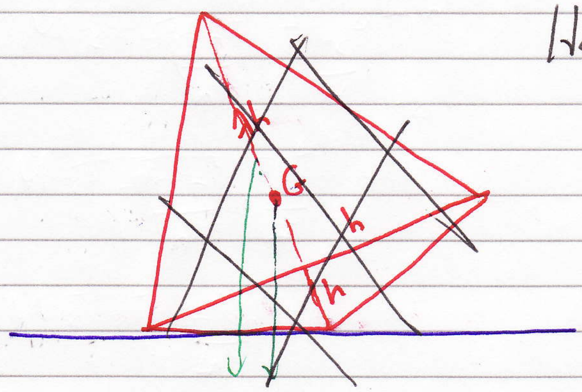
$\frac{4(\lambda+1)\bar{x}}{h} = 3\lambda^2 + 4\lambda + 1$

$\frac{4(\lambda+1)\bar{x}}{h} = (3\lambda+1)(\lambda+1)$

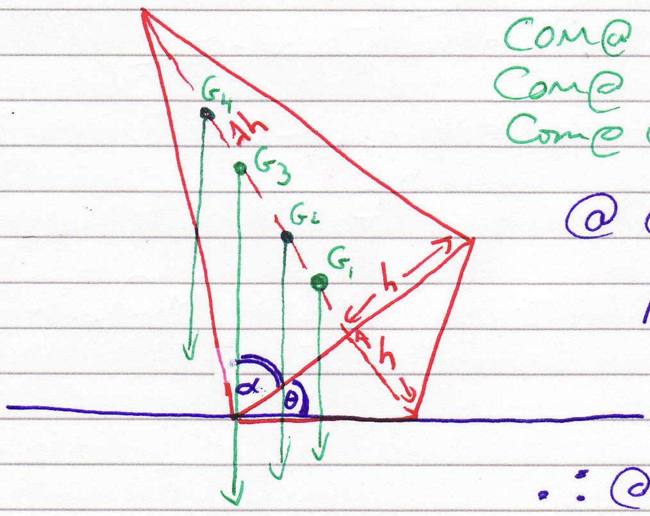
$\bar{x} = \frac{h}{4}\frac{(3\lambda+1)(\lambda+1)}{(\lambda+1)}$

$\bar{x} = \frac{1}{4}h(3\lambda+1)$ As required.

(c)



Hard to draw!



COM @ G_1 or G_2 = Stable
Com @ G_3 = about to topple
Com @ G_4 = topple

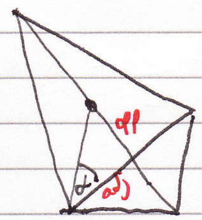
@ G_3 $\alpha + \theta = 90°$

Now $\tan\theta = \frac{h}{h} = 1$

$\theta = 45°$

∴ @ G_3 $\alpha = 45°$

8(c) contd $\therefore$ for toppling not to occur $\alpha \leq 45°$



so $\tan \alpha \leq 45$

$$\frac{opp}{adj} \leq 1$$

Now $opp = \lambda h - \frac{1}{4}h(3\lambda+1)$

$$= \lambda h - \frac{3\lambda h}{4} + \frac{1}{4}h$$

$$= \frac{1\lambda h}{4} - \frac{1}{4}h$$

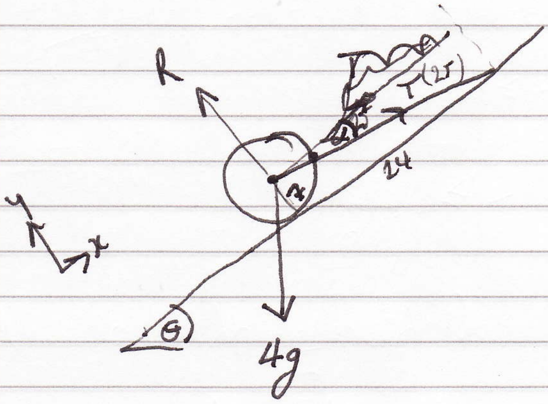
Now $\frac{opp}{adj} = \frac{h\left(\frac{1}{4}\lambda - \frac{1}{4}\right)}{h} \leq 1$

$$\frac{1}{4}\lambda - \frac{1}{4} \leq 1$$

$$\lambda - 1 \leq 4$$

$\lambda \leq 5$ As required.

(9)



(a) Σf_x: $T\cos\alpha - 4g\sin\theta = 0$

$T \times \frac{24}{25} = 4 \times 9.8 \times \frac{3}{5}$

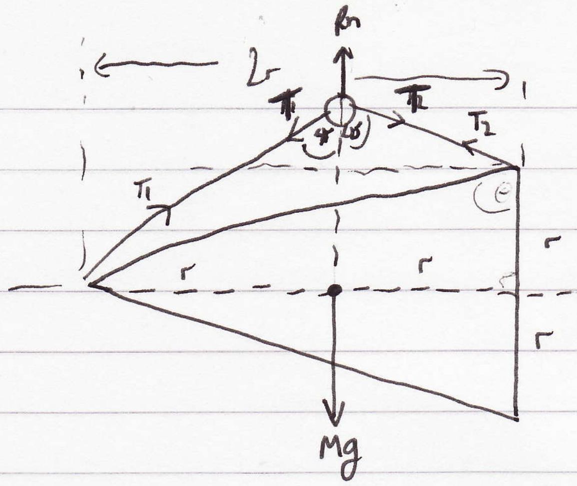
$T = 24.5N$

(b) Σf_y $R - T\sin\alpha - 4g\cos\theta = 0$

$R = 24.5 \times \frac{7}{25} + 4 \times 9.8 \times \frac{4}{5}$

$R = 6.86 + 31.36 = 38.22N.$

(10)

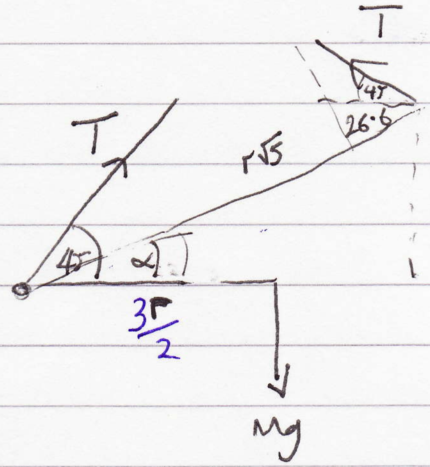


Com $\frac{1}{4} \times 2r = \frac{r}{2}$

Smooth peg ∴ tensions equal through string

Rn equal + opposite to Mg

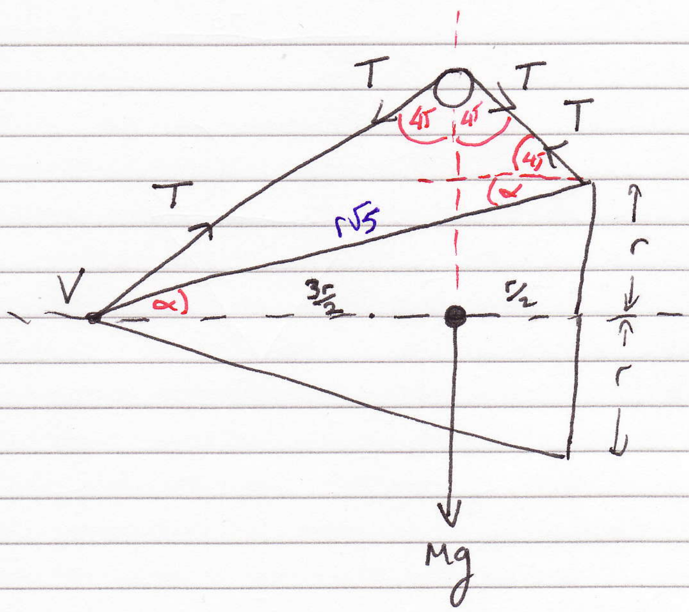
∴ T's must be equal either side ∴ $\theta = 45$.



$\tan\alpha = \frac{r}{2r} = \frac{1}{2} = 26\cdot6^\circ$

$\frac{3r}{2} \times Mg = r\sqrt{5} \times T \sin 71\cdot6$

$T = \frac{3Mg}{2\sin 71\cdot6\,\sqrt{5}} = \frac{Mg}{\sqrt{2}}$



Com $\frac{1}{4} \times 2r = \frac{1}{2}r$

$\tan\alpha = \frac{r}{2r}$

$\alpha = 26.6°$

$\circlearrowleft V: \quad \frac{3r}{2}Mg = r\sqrt{5}\,T\sin(45 + 26.6)$

$T = \frac{3Mg}{2\sqrt{5}\sin 71.6°} = \frac{Mg}{\sqrt{2}}$