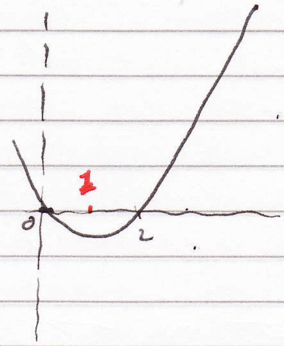
Ex 5B

(2)

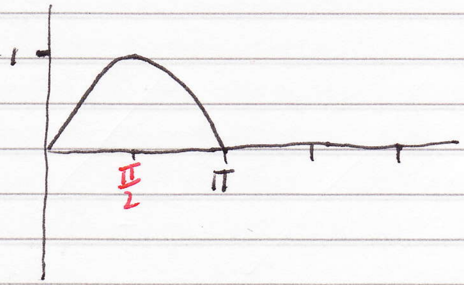


$y = x^2 - 2x$

$y = x(x-2)$

CoM @ $(1,0)$

(3)



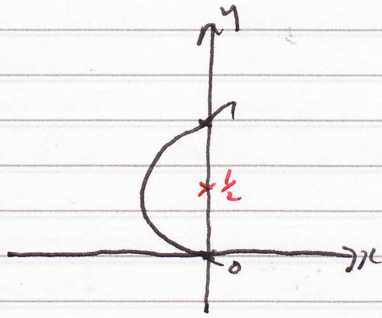
$y = \sin x$ CoM @ $\left(\frac{\pi}{2}, 0\right)$

(4)

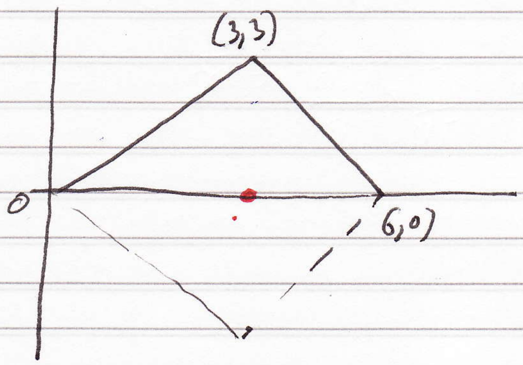
$y^2 - y = x$

$y(y-1) = x$

CoM @ $\left(0, \frac{1}{2}\right)$



(1)



CoM @ $(3,0)$.

Ex 5B

(5) $M = \rho\pi \int_0^1 y^2 \, dx$

$M = \rho\pi \int_0^1 (x^2)^2 \, dx = \rho\pi \int_0^1 x^4 \, dx = \rho\pi \left[\frac{x^5}{5}\right]_0^1 = \frac{\rho\pi}{5}$

Now $M\bar{x} = \int_0^1 \rho\pi y^2 x \, dx = \rho\pi \int_0^1 x^5 \, dx = \rho\pi \left[\frac{x^6}{6}\right]_0^1 = \frac{\rho\pi}{6}$

$\therefore \frac{\rho\pi}{5}\bar{x} = \frac{\rho\pi}{6}$

$\bar{x} = \frac{5}{6}$

$\therefore$ CoM @ $\left(\frac{5}{6}, 0\right)$

(6) $M = \rho\pi \int_0^1 y^2 \, dx$

$M = \rho\pi \int_0^1 (x^3)^2 \, dx = \rho\pi \int_0^1 x^6 \, dx = \rho\pi \left[\frac{x^7}{7}\right]_0^1 = \frac{\rho\pi}{7}$

$M\bar{x} = \rho\pi \int_0^1 y^2 x \, dx = \rho\pi \int_0^1 x^7 \, dx = \rho\pi \left[\frac{x^8}{8}\right]_0^1 = \frac{\rho\pi}{8}$

$\therefore \frac{\rho\pi}{7}\bar{x} = \frac{\rho\pi}{8}$

$\bar{x} = \frac{7}{8}$ $\therefore$ CoM @ $\left(\frac{7}{8}, 0\right)$

(7) $M = \rho\pi \int_0^1 (1+x^2)^2 \, dx = \rho\pi \int_0^1 1 + 2x^2 + x^4 \, dx$

$= \rho\pi \left[x + \frac{2}{3}x^3 + \frac{x^5}{5}\right]_0^1$

$= \cancel{\rho\pi} \frac{28\rho\pi}{15}$

$M\bar{x} = \rho\pi \int_0^1 (1+2x^2+x^4)x \, dx = \rho\pi \int_0^1 x + 2x^3 + x^5 \, dx$

$M\bar{x} = \rho\pi \left[\frac{x^2}{2} + \frac{2x^4}{4} + \frac{x^6}{6}\right]_0^1$

(7) contd

$$\frac{28\rho\pi\bar{x}}{15} = \frac{7\rho\pi}{6}$$

$$\bar{x} = \frac{5}{8} \quad \therefore \text{Com @ } \left(\frac{5}{8}, 0\right)$$

(8) $$M = \rho\pi\int_1^2 \left(\frac{1}{x}\right)^2 dx = \rho\pi\int_1^2 x^{-2}\,dx = \rho\pi\left[-\frac{1}{x}\right]_1^2$$

$$= \rho\pi\left(-\frac{1}{2} - -\frac{1}{1}\right)$$

$$= \frac{\rho\pi}{2}$$

$$M\bar{x} = \rho\pi\int_1^2 \frac{1}{x^2}.x\,dx = \rho\pi\int_1^2 \frac{1}{x}\,dx = \rho\pi\left[\ln|x|\right]_1^2 = \rho\pi\ln 2$$

$$\frac{\rho\pi}{2}\bar{x} = \rho\pi\ln 2$$

$$\bar{x} = 2\ln 2 \quad \therefore \text{com @ } (2\ln 2, 0)$$

(10) $$M = \rho\pi\int_0^1 (e^x)^2 dx = \rho\pi\int_0^1 e^{2x}\,dx = \rho\pi\left[\frac{1}{2}e^{2x}\right]_0^1 = \cancel{\frac{\rho\pi e^2}{2}}$$

$$= \frac{\rho\pi}{2}(e^2 - 1)$$

$$M\bar{x} = \rho\pi\int_0^1 xe^{2x}\,dx \qquad \text{let } u = x \qquad \frac{dv}{dx} = e^{2x}$$

$$\frac{du}{dx} = 1 \qquad v = \frac{1}{2}e^{2x}$$

$$M\bar{x} = \rho\pi\left[\frac{1}{2}xe^{2x} - \frac{1}{2}\int_0^1 e^{2x}\,dx\right]$$

$$M\bar{x} = \rho\pi\left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_0^1$$

$$M\bar{x} = \rho\pi\left[\frac{1}{2}e^2 - \frac{1}{4}e^2 - \left(0 - \frac{1}{4}\right)\right]$$

$$\cancel{\frac{\rho\pi}{2}e^2\bar{x} = \rho\pi\left[\frac{1}{4}e^2 + \frac{1}{4}\right]} \qquad \frac{\cancel{\rho}\pi}{2}(e^2 - 1)\bar{x} = \frac{\cancel{\rho}\pi}{4}(e^2 + 1)$$

$$\cancel{\bar{x} = \frac{1}{2} + \frac{1}{2}e^{-2}} \qquad \bar{x} = \frac{1}{2}\frac{(e^2+1)}{(e^2-1)}$$

$$\therefore \text{Com @ } \left(\frac{1}{2}\frac{(e^2+1)}{(e^2-1)}, 0\right)$$

(9) $M = \rho\pi \int_1^2 \left(\frac{1}{x^2}\right)^2 dx = \rho\pi \int_1^2 x^{-4} dx = \rho\pi \left[-\frac{1}{3x^3}\right]_1^2$

$= \rho\pi\left(-\frac{1}{24} - -\frac{1}{3}\right)$

$= \frac{7\rho\pi}{24}$

$M\bar{x} = \rho\pi \int_1^2 x^{-4}.x\, dx = \rho\pi \int_1^2 x^{-3} dx = \rho\pi\left[\frac{-1}{2x^2}\right]_1^2$

$= \rho\pi\left(-\frac{1}{8} - -\frac{1}{2}\right)$

$= \frac{3\rho\pi}{8}$

$\therefore \frac{7\rho\pi}{24}\bar{x} = \frac{3\rho\pi}{8}$

$\bar{x} = \frac{9}{7}$ $\therefore$ com @ $(1\frac{2}{7}, 0)$

(11) $M = \rho\pi \int_0^1 (e^{-x})^2 dx = \rho\pi \int_0^1 e^{-2x} dx = \rho\pi\left[-\frac{1}{2}e^{-2x}\right]_0^1$

$= \rho\pi\left(-\frac{1}{2}e^{-2} - -\frac{1}{2}\right)$

$= \frac{\rho\pi}{2}\left(1 - e^{-2}\right)$

$M\bar{x} = \rho\pi \int_0^1 x e^{-2x} dx$ let $u = x$ $\frac{dv}{dx} = e^{-2x}$

$\frac{du}{dx} = 1$ $v = -\frac{1}{2}e^{-2x}$

$M\bar{x} = \rho\pi\left[\frac{-1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x} dx\right]_0^1$

$M\bar{x} = \rho\pi\left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1$

$M\bar{x} = \rho\pi\left[-\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} - \left(0 - \frac{1}{4}\right)\right]$

$M\bar{x} = \rho\pi\left[\frac{1}{4} - \frac{3}{4}e^{-2}\right]$

(11) contd

$$\frac{\cancel{\rho\pi}}{2}(1-e^{-2})\bar{x} = \frac{1}{4}\cancel{\rho\pi}(1-3e^{-2})$$

$$\bar{x} = \frac{1}{2}\,\frac{1-3e^{-2}}{1-e^{-2}}$$

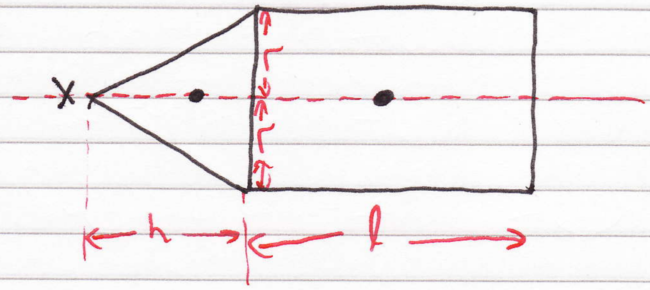
$$(1-e^{-2})\bar{x} = \frac{1}{2}(1-3e^{-2})$$

$$\left(1-\frac{1}{e^2}\right)\bar{x} = \frac{1}{2}\left(1-\frac{3}{e^2}\right)$$

$$\left(\frac{e^2-1}{\cancel{e^2}}\right)\bar{x} = \frac{1}{2}\left(\frac{e^2-3}{\cancel{e^2}}\right)$$

$$\bar{x} = \frac{1}{2}\,\frac{(e^2-3)}{(e^2-1)}$$

(12)



$$\left(\frac{1}{3}\cancel{\pi}\cancel{r^2}h + \cancel{\pi}\cancel{r^2}l\right)\cancel{\rho}\,\bar{x} = \frac{1}{3}\cancel{\pi}\cancel{r^2}h\cancel{\rho}\left(\frac{3h}{4}\right) + \cancel{\pi}\cancel{r^2}l\cancel{\rho}\left(h + \frac{1}{2}l\right)$$

$$\left(\frac{h+3l}{3}\right)\bar{x} = \frac{h^2}{4} + l\left(\frac{2h+l}{2}\right)$$

$$\left(\frac{h+3l}{3}\right)\bar{x} = \frac{h^2 + 2l(2h+l)}{4}$$

$$\left(\frac{h+3l}{3}\right)\bar{x} = \frac{h^2 + 4lh + 2l^2}{4}$$

$$\bar{x} = \frac{3(h^2+4lh+2l^2)}{4(h+3l)}$$

(13) for given situation in (12) but now $\bar{x} = h$

$$\therefore \quad h = \frac{3(h^2+4lh+2l^2)}{4(h+3l)}$$

$$4h^2 + \cancel{12}lh = 3h^2 + \cancel{12}lh + 6l^2$$

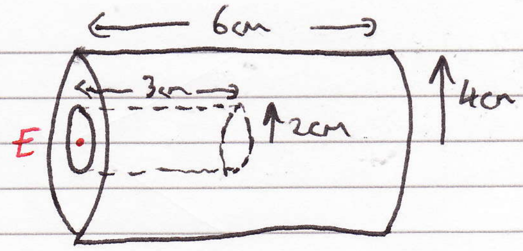
$$h^2 = 6l^2$$

$$\frac{h^2}{l^2} = 6$$

$$\frac{h}{l} = \frac{\sqrt{6}}{1}$$

$\therefore$ ratio of $h:l$ is $\sqrt{6}:1$

(14)

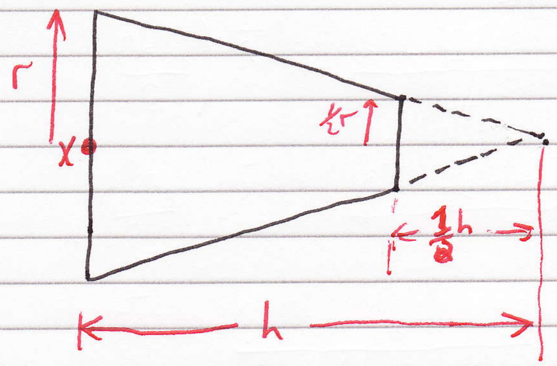


$$\left(\pi\times4^2\times6-\pi\times2^2\times3\right)\rho\,\bar{x}=\pi\times4^2\times6(3)\rho-\pi\times2^2\times3(1.5)\rho$$

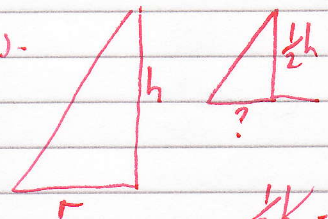
$$84\cancel{\rho}\cancel{\pi}\,\bar{x}=288\cancel{\rho}\cancel{\pi}-18\cancel{\rho}\cancel{\pi}$$

$$\bar{x}=\frac{270}{84}=3.21\text{ cm from E.}$$

(15)



Sim Δ's.



$$\frac{\frac{1}{2}\cancel{h}}{\cancel{h}}=\frac{?}{r}$$

$$?=\frac{1}{2}r$$

$$\cancel{\rho}\left(\frac{1}{3}\pi r^2h-\frac{1}{3}\pi\left(\frac{1}{2}r\right)^2\left(\frac{1}{2}h\right)\right)\bar{x}=\frac{1}{3}\pi r^2h\cancel{\rho}\left(\frac{1}{4}h\right)-\frac{1}{3}\pi\left(\frac{1}{2}r\right)^2\left(\frac{1}{2}h\right)\cancel{\rho}\left(\frac{1}{2}h+\frac{1}{4}\left(\frac{1}{2}h\right)\right.$$

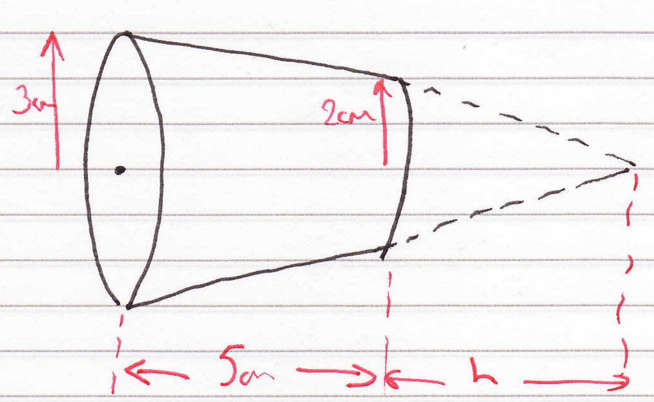
$$\left(\frac{1}{3}\cancel{\pi}\cancel{r^2}h-\frac{1}{24}\cancel{\pi}\cancel{r^2}h\right)\bar{x}=\frac{1}{12}\cancel{\pi}\cancel{r^2}h^2-\frac{\cancel{\pi}\cancel{r^2}h}{24}\left(\frac{5}{8}h\right)$$

$$\frac{7}{24}h\,\bar{x}=\frac{1}{12}h^2-\frac{5}{192}h^2$$

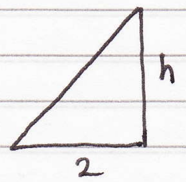
$$\frac{7}{24}\bar{x}=\frac{11}{192}h$$

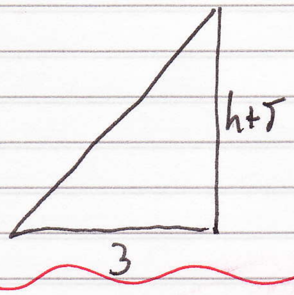
$$\bar{x}=\frac{11}{56}h\quad\text{As required.}$$

16



Sim△'s





$$\frac{h}{h+5} = \frac{2}{3}$$

$$3h = 2h+10$$

$$h = 10$$

Not Volume

$$\left(\frac{1}{3}\pi \times 3^2 \times 15 - \frac{1}{3}\pi \times 2^2 \times 10\right)\rho\bar{x} = \left(\frac{1}{3}\pi \times 3^2 \times 15\right)\rho\left(\frac{1}{3}\times 15\right) - \left(\frac{1}{3}\pi \times 2^2 \times 10\right)\rho\left(5+\frac{1}{3}\times 10\right)$$

$$\left(45\pi - \frac{40}{3}\pi\right)\bar{x} = 225\pi - \frac{1000\pi}{9}$$

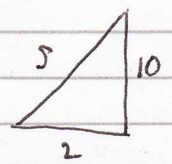
$$\frac{95}{3}\pi\bar{x} = \frac{1025\pi}{9}$$

$$\bar{x} = \frac{1025}{9} \times \frac{3}{95} = \frac{1025}{285} = 3.60\text{cm}$$

* 2.33cm in book

use Surface Area

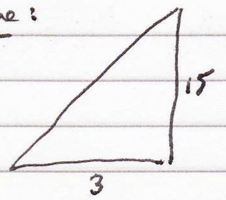
SA small cone:



$s = \sqrt{104}$

$SA = \pi \times 2 \times \sqrt{104} = 2\pi\sqrt{104}$

SA large cone:



$s = \sqrt{234}$

$SA = \pi \times 3 \times \sqrt{234} = 3\pi\sqrt{234}$

$$\left(3\cancel{\pi}\sqrt{234} - 2\cancel{\pi}\sqrt{104}\right)\cancel{\rho}\bar{x} = \left(3\cancel{\pi}\sqrt{234}\right)\cancel{\rho}\left(\frac{1}{3}\times 15\right) - \left(2\cancel{\pi}\sqrt{104}\right)\cancel{\rho}\left(5+\frac{1}{3}\times 10\right)$$

$$\left(3\sqrt{234} - 2\sqrt{104}\right)\bar{x} = 15\sqrt{234} - \frac{50}{3}\sqrt{104}$$

$$\bar{x} = 2.33.$$