Ex5A

① $y = \frac{4x}{3}$ $y=0$, $x=3$.

Mass of lamina $M = \int_0^3 \frac{4}{3}x\rho\, dx = \rho\left[\frac{4x^2}{6}\right]_0^3 = 6\rho$

$M\bar{x} = \int_0^3 \rho x f(x)\, dx = \rho\int_0^3 \frac{4}{3}x^2\, dx = \rho\left[\frac{4}{9}x^3\right]_0^3 = 12\rho$

$6\rho\bar{x} = 12\rho$
$\bar{x} = 2$

$M\bar{y} = \frac{1}{2}\int_0^3 \rho\,(f(x))^2\, dx = \frac{1}{2}\rho\int_0^3 \frac{16x^2}{9}\, dx = \frac{8}{27}\rho\left[x^3\right]_0^3 = 8\rho$

$6\rho\bar{y} = 8\rho$

$\bar{y} = \frac{4}{3}$

$\therefore$ com @ $\left(2, \frac{4}{3}\right)$

② $y = x^2$ $x=0$ $x=1$

$M = \int_0^1 \rho y\, dx = \rho\int_0^1 x^2\, dx = \rho\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}\rho$

$M\bar{x} = \int_0^1 \rho x f(x)\, dx = \rho\int_0^1 x^3\, dx = \rho\left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}\rho$

$\frac{1}{3}\rho\bar{x} = \frac{1}{4}\rho$ $\bar{x} = \frac{3}{4}$

$M\bar{y} = \frac{1}{2}\int_0^1 \rho\,(f(x))^2\, dx = \frac{1}{2}\rho\int_0^1 x^4\, dx = \frac{1}{2}\rho\left[\frac{x^5}{5}\right]_0^1 = \frac{1}{10}\rho$

$\frac{1}{3}\bar{y}\rho = \frac{1}{10}\rho$
$\bar{y} = \frac{3}{10}$

$\therefore$ com @ $\left(\frac{3}{4}, \frac{3}{10}\right)$

(3) $y = x^3 \quad x = 1,\ x = 2$

$$M = \int_1^2 \rho x^3 \, dx = \rho \left[\frac{x^4}{4}\right]_1^2 = \rho\left(\frac{16}{4} - \frac{1}{4}\right) = \frac{15}{4}\rho$$

$$M\bar{x} = \int_1^2 \rho x f(x)\, dx = \rho \int_1^2 x^4 \, dx = \rho\left[\frac{x^5}{5}\right]_1^2 = \rho\left(\frac{32}{5} - \frac{1}{5}\right) = \frac{31}{5}\rho$$

$$\frac{15}{4}\rho\bar{x} = \frac{31}{5}\rho$$

$$\bar{x} = \frac{124}{75}$$

$$M\bar{y} = \frac{1}{2}\int_1^2 \rho (f(x))^2 \, dx = \frac{1}{2}\rho \int_1^2 x^6 \, dx = \frac{1}{2}\rho\left[\frac{x^7}{7}\right]_1^2 = \frac{1}{2}\rho\left[\frac{128}{7} - \frac{1}{7}\right] = \frac{127}{14}\rho$$

$$\frac{15}{4}\cancel{\frac{31}{5}}\rho\bar{y} = \frac{127}{14}\rho$$

$$\bar{y} = \cancel{\frac{635}{217}}\ \frac{254}{105}$$

$\therefore$ com@ $\left(\frac{124}{75}, \frac{254}{105}\right)$

(4) $y^2 = 4ax \quad x = 0 \quad x = b$

$$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}$$

$$M = \int_0^b \rho 2a^{\frac{1}{2}}x^{\frac{1}{2}} \, dx = \rho 2a^{\frac{1}{2}}\left[\frac{2}{3}x^{3/2}\right]_0^b = \frac{4}{3}a^{\frac{1}{2}}b^{3/2}\rho$$

$$M\bar{x} = \int_0^b \rho x \, . \, 2a^{\frac{1}{2}}x^{\frac{1}{2}} \, dx = 2\rho a^{\frac{1}{2}} \int_0^b x^{3/2} \, dx = 2\rho a^{\frac{1}{2}}\left[\frac{2}{5}x^{5/2}\right]_0^b$$

$$\frac{\cancel{4}}{3}\cancel{a^{1/2}}b^{3/2}\cancel{\rho}\bar{x} = \frac{\cancel{4}}{5}\cancel{\rho}\cancel{a^{1/2}}b^{5/2}$$

$$\bar{x} = \frac{3}{5}\frac{b^{5/2}}{b^{3/2}} = \frac{3}{5}b$$

④ contd $M\bar{y} = \frac{1}{2}\rho\int_0^b \left(2a^{1/2}x^{1/2}\right)^2 dx = \frac{1}{2}\rho 4a\int_0^b x\,dx$

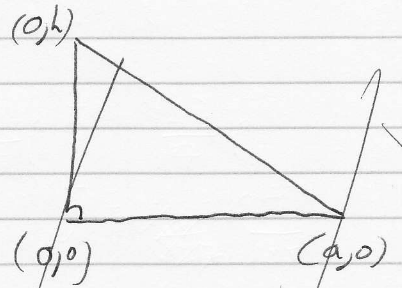
$$= 2\rho a\left[\frac{x^2}{2}\right]_0^b$$

$$\frac{4}{3}a^{1/2}b^{3/2}\cancel{\rho}\bar{y} = \cancel{2}\cancel{\rho}a\frac{b^2}{\cancel{2}}$$

$$\bar{y} = \frac{3ab^2}{4a^{1/2}b^{3/2}} = \frac{3}{4}a^{1/2}b^{1/2}$$

$\therefore$ com @ $\left(\frac{3}{5}b, \frac{3}{4}a^{1/2}b^{1/2}\right)$ * $\left(\frac{3}{5}b, 0\right)$ in book?

⑤



$$\frac{y}{x} = -\frac{h}{a}$$

$$y = -\frac{h}{a}x$$

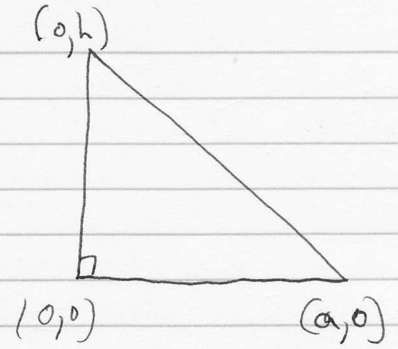
$$M = \frac{1}{2}ah\rho$$

$$M\bar{x} = \rho\int_0^a -x\frac{h}{a}x\,dx = -\frac{\rho h}{a}\left[\frac{x^3}{3}\right]_0^a = -\frac{\rho h}{a}\frac{a^3}{3} = -\frac{\rho h a^2}{3}$$

$$\cancel{\frac{1}{2}ah\rho\bar{x} = -\frac{\rho h a}{2}}$$

$$\frac{1}{2}ah\rho\bar{x}$$

5



$$\frac{y-h}{0-h} = \frac{x-0}{a-0}$$

$$\cancel{hh}\ \frac{y-h}{-h} = \frac{x}{a}$$

$$y = -\frac{hx}{a} + h$$

$$y = h - \frac{hx}{a}$$

$$M = \frac{1}{2}ah\rho$$

$$M\bar{x} = \rho\int_0^a x\left(h - \frac{hx}{a}\right)dx = \rho\int_0^a hx - \frac{hx^2}{a}\,dx$$

$$M\bar{x} = \rho\left[\frac{hx^2}{2} - \frac{hx^3}{3a}\right]_0^a$$

$$\frac{1}{2}ah\rho\bar{x} = \rho\left(\frac{ha^2}{2} - \frac{ha^2}{3}\right)$$

$$\frac{1}{2}a\cancel{h}\cancel{\rho}\,\bar{x} = \cancel{\rho}\frac{\cancel{h}a^2}{6}$$

$$\bar{x} = \frac{a}{3}$$

$$M\bar{y} = \frac{1}{2}\rho\int_0^a\left(h - \frac{hx}{a}\right)^2 dx = \frac{1}{2}\rho\int_0^a h^2 - 2\frac{h^2x}{a} + \frac{h^2x^2}{a^2}\,dx$$

$$M\bar{y} = \frac{1}{2}\rho\left[h^2x - 2\frac{h^2x^2}{2a} + \frac{h^2x^3}{3a^2}\right]_0^a$$

$$M\bar{y} = \frac{1}{2}\rho\left(h^2a - h^2a + \frac{h^2a}{3}\right)$$

$$\cancel{\frac{1}{2}}\cancel{a}\cancel{h}\cancel{\rho}\,\bar{y} = \cancel{\frac{1}{2}}\cancel{\rho}\frac{h^{\cancel{2}}\cancel{a}}{3}$$

$$\bar{y} = \frac{h}{3}$$

$$\therefore \text{Com} @ \left(\frac{a}{3}, \frac{h}{3}\right)$$

(6) $y = x^2 + 1 \quad x = 0 \quad x = 1$

$$M = \int_0^1 \rho (x^2+1)\,dx = \rho\left[\frac{x^3}{3} + x\right]_0^1 = \frac{4\rho}{3}$$

$$M\bar{x} = \rho\int_0^1 x(x^2+1)\,dx = \rho\left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 = \frac{3}{4}\rho$$

$$\frac{4}{3}\rho\bar{x} = \frac{3}{4}\rho$$

$$\bar{x} = \frac{9}{16}$$

$$M\bar{y} = \frac{1}{2}\rho\int_0^1 (x^2+1)^2\,dx = \frac{1}{2}\rho\int_0^1 x^4 + 2x^2 + 1\,dx$$

$$M\bar{y} = \frac{1}{2}\rho\left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_0^1 = \frac{14\rho}{15}$$

$$\frac{4}{3}\rho\,\bar{y} = \frac{14}{15}\rho$$

$$\bar{y} = \frac{7}{10}$$

$\therefore$ Com @ $\left(\frac{9}{16}, \frac{7}{10}\right)$

(7) eq^n of circle centre $(0,0)$ radius r $\quad x^2 + y^2 = r^2$

$$y = (r^2 - x^2)^{1/2}$$

Area $= \frac{\pi r^2}{4}$

Mass $M = \frac{\pi r^2 \rho}{4}$

$$M\bar{x} = \rho\int_0^r x(r^2-x^2)^{1/2}\,dx$$

let $u = r^2 - x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$x=0 \quad u = r^2$

$x = r \quad u = 0$

$$M\bar{x} = \rho\int_{r^2}^0 x\,u^{1/2} \cdot \frac{du}{-2x}$$

(7) contd $M\bar{x} = -\frac{\rho}{2}\int_{r^2}^{0} u^{\frac{1}{2}}\,du = -\frac{\rho}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2} = -\frac{\rho}{3}\left[u^{3/2}\right]_{r^2}^{0}$

$$\frac{\pi r^2}{4}\rho\,\bar{x} = -\frac{\rho}{3}\left(0-(r^2)^{3/2}\right)$$

$$\frac{\pi \cancel{r^2}}{4}\cancel{\rho}\,\bar{x} = \cancel{\rho}\frac{r^{\cancel{3}}}{3}$$

$$\bar{x} = \frac{4r}{3\pi}$$

$$M\bar{y} = \frac{1}{2}\rho\int_{0}^{r}\left((r^2-x^2)^{\frac{1}{2}}\right)^2 dx = \frac{1}{2}\rho\int_{0}^{r} r^2-x^2\,dx$$

$$M\bar{y} = \frac{1}{2}\rho\left[r^2x-\frac{x^3}{3}\right]_0^r$$

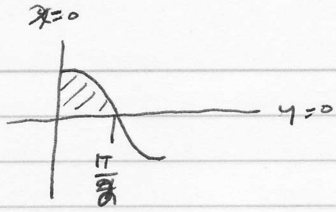
$$\frac{\pi r^2}{4}\rho\,\bar{y} = \frac{1}{2}\rho\left(r^3-\frac{r^3}{3}\right)$$

$$\frac{\pi \cancel{r^2}}{4}\cancel{\rho}\,\bar{y} = \frac{1}{\cancel{2}}\cancel{\rho}\cdot\frac{\cancel{2}}{3}r^{\cancel{3}}$$

$$\bar{y} = \frac{4r}{3\pi}$$

$\therefore$ COM @ $\left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$

⑧ $y = \cos x$



$$\text{Mass} = \rho \int_0^{\pi/2} \cos x \, dx$$

$$= \rho \left[\sin x \right]_0^{\pi/2} = \rho$$

$$M\bar{x} = \rho \int_0^{\pi/2} x \cos x \, dx$$ let $V = x$ $\quad \frac{du}{dx} = \cos x$

$\frac{dV}{dx} = 1 \quad u = \sin x$

$$M\bar{x} = \rho \left[x \sin x - \int_0^{\pi/2} \sin x \, dx \right]$$

$$M\bar{x} = \rho \left[x \sin x + \cos x \right]_0^{\pi/2}$$

$$M\bar{x} = \rho \left[\left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} \right) - \left(0 + \cos 0 \right) \right]$$

$$\cancel{\rho}\bar{x} = \cancel{\rho} \left[\frac{\pi}{2} - 1 \right]$$

$$M\bar{y} = \frac{1}{2}\rho \int_0^{\pi/2} \cos^2 x \, dx$$

$$M\bar{y} = \frac{1}{2}\rho \int_0^{\pi/2} \frac{1}{2} + \frac{1}{2} \cos 2x \, dx$$

$$M\bar{y} = \frac{1}{4}\rho \int_0^{\pi/2} 1 + \cos 2x \, dx$$

$$\cancel{\rho}\bar{y} = \frac{1}{4}\cancel{\rho} \left[x + \frac{1}{2} \sin 2x \right]_0^{\pi/2}$$

$$\bar{y} = \frac{1}{4} \left[\left(\frac{\pi}{2} + \frac{1}{2} \sin \pi \right) - \left(0 + \frac{1}{2} \sin 0 \right) \right]$$

$$\bar{y} = \frac{1}{4} \left(\frac{\pi}{2} \right) = \frac{\pi}{8}$$ $\therefore$ COM @ $\left(\left(\frac{\pi}{2} - 1 \right), \frac{\pi}{8} \right)$

(9) $y = e^x \quad x=0 \quad x=1$

$$M = \rho \int_0^1 e^x \, dx = \rho \left[e^x \right]_0^1 = \rho(e-1)$$

$$M\bar{x} = \rho \int_0^1 x e^x \, dx \qquad v = x \qquad \frac{du}{dx} = e^x$$

$$\frac{dv}{dx} = 1 \qquad u = e^x$$

$$M\bar{x} = \rho \left[x e^x - \int e^x \, dx \right]_0^1$$

$$M\bar{x} = \rho \left[x e^x - e^x \right]_0^1$$

$$\cancel{\rho}(e-1)\bar{x} = \cancel{\rho} \left[(e^1 - e^1) - (0-1) \right]$$

$$(e-1)\bar{x} = 1$$

$$\bar{x} = \frac{1}{e-1}$$

$$M\bar{y} = \frac{1}{2}\rho \int_0^1 e^{2x} \, dx$$

$$M\bar{y} = \frac{1}{2}\rho \left[\frac{1}{2} e^{2x} \right]_0^1$$

$$\cancel{\rho}(e-1)\bar{y} = \frac{1}{2}\cancel{\rho} \left[\frac{1}{2}e^2 - \frac{1}{2} \right]$$

$$\bar{y} = \frac{e^2-1}{4(e-1)} = \frac{(e+1)\cancel{(e-1)}}{4\cancel{(e-1)}}$$

$$\bar{y} = \frac{1}{4}(e+1)$$

$$\therefore \text{COM @} \left(\frac{1}{e-1}, \frac{e+1}{4} \right)$$

(10) $\left(\frac{\pi.6^2}{2} - \frac{\pi.3^2}{2}\right)\rho\bar{y} = \frac{\pi.6^2}{2}.\rho.\frac{4.6}{3\pi} - \frac{\pi.3^2}{2}\rho.\frac{4.3}{3\pi}$

$$\frac{27}{2}\pi\,\bar{y} = 144 - 18$$

$$\bar{y} = \cancel{\frac{126}{27\pi}}\ \frac{252}{27\pi} = \frac{28}{3\pi}$$

(11) Line & curve intersect when $x = x^2$ $\quad x = 1$

$$\text{Mass} = \rho\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right]$$

$$= \rho\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{\rho}{6}$$

$$M\bar{x} = \rho\left[\int_0^1 x.x\,dx - \int_0^1 x.x^2\,dx\right]$$

$$M\bar{x} = \rho\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$\frac{\cancel{\rho}}{6}\bar{x} = \cancel{\rho}.\frac{1}{12}$$

$$\bar{x} = \frac{1}{2}$$

$$M\bar{y} = \frac{1}{2}\rho\left[\int_0^1 x^2\,dx - \int_0^1 x^4\,dx\right]_0^1$$

$$\frac{\cancel{\rho}}{6}\bar{y} = \frac{\cancel{\rho}}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$\bar{y} = 3\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\bar{y} = \frac{2}{5}$$

$\therefore$ com @ $\left(\frac{1}{2}, \frac{2}{5}\right)$.