Ex 1B

(1) $a = f$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = f$$

$$\frac{1}{2}v^2 = fx + c$$

when $x = 0$, $v = u$

$$\frac{1}{2}u^2 = c$$

$$\therefore \quad \frac{1}{2}v^2 = fx + \frac{1}{2}u^2$$

$$v^2 = 2fx + u^2$$

$$v = \pm\sqrt{2fx + u^2}$$

but a is +ve and x is +ve $\therefore$ v is +ve

$$\therefore \quad v = \sqrt{2fx + u^2}$$

(2) $a = -4x$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$$

$$\frac{1}{2}v^2 = -2x^2 + c$$

when $x = 5$, $v = 0$

$$0 = -50 + c \qquad c = 50$$

$$\therefore \quad \frac{1}{2}v^2 = -2x^2 + 50$$

$$v^2 = 100 - 4x^2$$

when $x = 3$ $\quad v^2 = 100 - 4(3)^2$

$$v^2 = 64$$

$$v = \pm 8 \qquad \text{Speed} = 8\,ms^{-1}$$

③ $a = -(20+2x)$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -20 - 2x$$

$$\frac{1}{2}v^2 = -20x - x^2 + c$$

when $x=0$, $v=40$

$$\frac{1}{2}(40)^2 = c \qquad c = 800$$

$$\frac{1}{2}v^2 = 800 - 20x - x^2$$

$$v^2 = 1600 - 40x - 2x^2$$

when $v=0$ $\quad 2x^2 + 40x - 1600 = 0$

$$x = 20 \text{ or } x = -40$$

retarding in a +ve direction $\therefore$ comes to rest when $x = 20$.

④ $a = -\lambda x^2$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\lambda x^2$$

$$\frac{1}{2}v^2 = -\frac{\lambda}{3}x^3 + c$$

when $x = a$, $v=0$

$$0 = -\frac{\lambda a^3}{3} + c \qquad c = \frac{\lambda a^3}{3}$$

$$\therefore \frac{1}{2}v^2 = -\frac{\lambda}{3}x^3 + \frac{\lambda}{3}a^3$$

when $x=0$ $\quad \frac{1}{2}v^2 = \frac{\lambda}{3}a^3$

$$v = \pm\sqrt{\frac{2\lambda}{3}a^3} \qquad \text{Speed} = \sqrt{\frac{2\lambda}{3}a^3}$$

(5) $a = \frac{k}{x}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{k}{x}$$

$$\frac{1}{2}v^2 = k\ln x + c$$

when $x=1$, $v=0$ $\therefore c=0$

$$\frac{1}{2}v^2 = k\ln x$$

$$\ln x = \frac{v^2}{2k}$$

$$x = e^{(v^2/2k)}$$

(6) $a = 4x^3$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4x^3$$

$$\frac{1}{2}v^2 = x^4 + c$$

when $x=0$, $v=2$ $\therefore c=2$

$$\frac{1}{2}v^2 = x^4 + 2$$

when $v=6$

$$18 = x^4 + 2$$

$$x^4 = 16$$

$$x = 2m$$

7 (a) $a = x+3$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x+3$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} + 3x + c$$

when $x=0, v=3$ $\therefore c = \frac{9}{2}$

$$\frac{1}{2}v^2 = \frac{x^2}{2} + 3x + \frac{9}{2}$$

$$v^2 = x^2 + 6x + 9$$

$$v^2 = (x+3)^2$$

$$v = \pm(x+3)$$

$$\text{Speed} = (x+3)$$

(b) Now $v = \frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = x+3$$

$$\int \frac{1}{x+3}\,dx = \int dt$$

$$\ln|x+3| = t + c$$

when $x=0, t=0$ $\therefore c = \ln 3$

$$\ln|x+3| = t + \ln 3$$

$$\ln|x+3| - \ln 3 = t$$

$$\ln\left|\frac{x+3}{3}\right| = t$$

$$\frac{x+3}{3} = e^t$$

$$x = 3e^t - 3$$

(8) (a) $a = -\frac{4}{x^3}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x^{-3}$

$\frac{1}{2}v^2 = +2x^{-2} + c$

when $t=0$, $v=2$, $x=1$

$\frac{1}{2}(2^2) = \frac{2}{1^2} + c$

$2 = 2 + c \quad c = 0$

$\frac{1}{2}v^2 = \frac{2}{x^2}$

$v^2 = \frac{4}{x^2}$

$v = \pm\frac{2}{x}$ but x is +ve $\therefore$ $v = \frac{2}{x}$

(b) $v = \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{2}{x}$

$x\,dx = 2\,dt$

$\frac{x^2}{2} = 2t + c$

when $t=0$, $x=1$ $\quad c = \frac{1}{2}$

$\frac{x^2}{2} = 2t + \frac{1}{2}$

$x^2 = 4t + 1$

$x = \pm\sqrt{4t+1}$ $\quad$ x is +ve $\therefore x = \sqrt{4t+1}$

(9) (a) $a = e^{2x}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = e^{2x}$$

$$\frac{1}{2}v^2 = \frac{1}{2}e^{2x} + c$$

when $v = -u$, $x = \ln u$

$$\frac{(-u)^2}{2} = \frac{1}{2}e^{2\ln u} + c$$

$$\frac{u^2}{2} = \frac{1}{2}e^{\ln u^2} + c$$

$$\frac{u^2}{2} = \frac{u^2}{2} + c \quad \therefore c = 0$$

$$\frac{1}{2}v^2 = \frac{1}{2}e^{2x}$$

$$v^2 = e^{2x}$$

$$v = \pm(e^{2x})^{\frac{1}{2}}$$

$v = \pm e^{x}$ but v is towards 0

$\therefore v = -e^{x}$

(b) $v = \frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = -e^{x}$$

$$\int e^{-x}\,dx = -\int dt$$

$$-e^{-x} = -t + c$$

when $t = 0$, $x = \ln u$

$c = -e^{-\ln u}$ $\quad c = -e^{\ln u^{-1}}$ $\quad c = -\frac{1}{u}$

9 (b) contd $-e^{-x} = -t - \frac{1}{u}$

$$e^{-x} = t + \frac{1}{u}$$

$$e^{-x} = \frac{ut+1}{u}$$

$$\frac{1}{e^{x}} = \frac{ut+1}{u}$$

$$e^{x} = \frac{u}{ut+1}$$

$$x = \ln\left|\frac{u}{ut+1}\right|$$

u is const, $t>0$ $\therefore$ $x = \ln\left(\frac{u}{ut+1}\right)$

10 $a = -\frac{1}{x^3}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -x^{-3}$$

$$\frac{1}{2}v^2 = +\frac{1}{2}x^{-2} + c$$

when $t=0$, $v=0$, $x=1$

$$0 = \frac{1}{2(1)^2} + c \qquad c = -\frac{1}{2}$$

$$\therefore \frac{1}{2}v^2 = \frac{1}{2}x^{-2} - \frac{1}{2}$$

$$v^2 = x^{-2} - 1$$

$$v^2 = \frac{1}{x^2} - 1$$

$$v^2 = \frac{1-x^2}{x^2}$$

(10) <u>Contd</u> $V = \pm\sqrt{\dfrac{1-x^2}{x^2}}$ but to go from $x=1$ to $x=\frac{1}{4}$ V must be -ve

$$V = -\frac{\sqrt{1-x^2}}{x}$$

Now $V = \dfrac{dx}{dt}$

$$\frac{dx}{dt} = -\frac{\sqrt{1-x^2}}{x}$$

$$\int \frac{-x}{\sqrt{1-x^2}}\,dx = \int dt$$

let $u = 1-x^2$

$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$\int \frac{-x}{u^{\frac{1}{2}}}\,\frac{du}{-2x} = \int dt$$

$$+\frac{1}{2}\int u^{-\frac{1}{2}}\,du = \int dt$$

$$+\frac{1}{2}\cdot 2u^{\frac{1}{2}} = t + c$$

$$+(1-x^2)^{\frac{1}{2}} = t + c$$

~~$-\frac{1}{2}\ln u = t + c$~~

~~$\frac{1}{2}\ln|1-x^2| = t + c$~~

~~when $t=0$, $x=1$~~

when $t=0$ $x=1$

$$+(1-1^2)^{\frac{1}{2}} = c \qquad c = 0$$

$$\therefore\ t = +(1-x^2)^{\frac{1}{2}}$$

when $x = \frac{1}{4}$ $t = +\left(1-\left(\frac{1}{4}\right)^2\right)^{\frac{1}{2}}$

$$t = +\left(1-\frac{1}{16}\right)^{\frac{1}{2}}$$

$$t = +\left(\frac{15}{16}\right)^{\frac{1}{2}} \qquad t = \frac{1}{4}\sqrt{15}$$

(11) $a = 4\sin x$

$\cancel{\frac{d}{dx}(4\sin x)}$ $\frac{d}{dx}\left(\frac{1}{2}V^2\right) = 4\sin x$

$\frac{1}{2}V^2 = -4\cos x + c$

when $x=0$, $V=6$

$18 = -4 + c$

$c = 22$

$\therefore \frac{1}{2}V^2 = -4\cos x + 22$

$V^2 = -8\cos x + 44$

Speed is max when $a=0$

$\therefore 4\sin x = 0$

$\sin x = 0$

$x = 0, \pi$ but when $x=0$, $V=6$

when $x=\pi$ $V^2 = -8\cos\pi + 44$

$V^2 = -8(-1) + 44$

$V^2 = 52$

$V = \sqrt{52} = 2\sqrt{13}$ m/s

(12) $a = e^{2x}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = e^{2x}$$

$$\frac{1}{2}v^2 = \frac{1}{2}e^{2x} + c$$

when $t=0$, $v=1$, $x=0$

$$\frac{1}{2} = \frac{1}{2} + c \qquad c = 0$$

$$\frac{1}{2}v^2 = \frac{1}{2}e^{2x}$$

$$v^2 = e^{2x}$$

$$v = \pm e^{x}$$ but v is in direction Ox $\therefore$ +ve

$$v = e^{x}$$

Now $v = \frac{dx}{dt} = e^{x}$

$$\int e^{-x}\,dx = \int dt$$

$$-e^{-x} = t + c$$

when $t=0$, $x=0$ $\quad c = -1$

$$-e^{-x} = t - 1$$

$$e^{-x} = 1 - t$$

$$e^{x} = (1-t)^{-1}$$

$$x = \ln|1-t|^{-1}$$

$$x = -\ln|1-t|$$

as $t \to 1$, $|1-t| \to 0$, $\ln|1-t| \to \infty$

(13) $a = 3\sqrt{x}$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 3x^{\frac{1}{2}}$$

$$\frac{1}{2}v^2 = \frac{2}{3} \cdot 3x^{3/2} + c$$

$$\frac{1}{2}v^2 = 2x^{3/2} + c$$

when $x=0, v=0 \quad \therefore \quad c=0$

$$\frac{1}{2}v^2 = 2x^{3/2}$$

$$v^2 = 4x^{3/2}$$

$$v = 2x^{3/4} \qquad (\text{accel inc +ve}, \therefore v \text{ +ve})$$

Now $v = \dfrac{dx}{dt} = 2x^{3/4}$

$$\int x^{-3/4}\,dx = \int 2\,dt$$

$$4x^{\frac{1}{4}} = 2t + c$$

when $x=0, t=0 \quad \therefore \quad c=0$

$$4x^{\frac{1}{4}} = 2t$$

$$x^{1/4} = \frac{t}{2}$$

$\therefore \quad x = \dfrac{t^4}{16}$ as required

(14)

$a = 2\sin^2 x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2\sin^2 x$

$\frac{1}{2}v^2 = 2\int 1 - \cos^2 x \, dx$

$\frac{1}{2}v^2 = 2\int 1 - \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$\frac{1}{2}v^2 = 2\int \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$

$\frac{1}{2}v^2 = \int 1 - \cos 2x \, dx$

$\frac{1}{2}v^2 = x - \frac{1}{2}\sin 2x + c$

when $x = \frac{\pi}{4}$, $v = 1$

$\frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} + c$

$\frac{1}{2} = \frac{\pi}{4} - \frac{1}{2} + c$

$c = 1 - \frac{\pi}{4}$

$\therefore \frac{1}{2}v^2 = x - \frac{1}{2}\sin 2x + 1 - \frac{\pi}{4}$

when $x = \frac{\pi}{3}$

$\frac{1}{2}v^2 = \frac{\pi}{3} - \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) + 1 - \frac{\pi}{4}$

$v^2 = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} + 2 - \frac{\pi}{2}$

$v = 1.29 \, ms^{-1}$

15 (a) $a = x + 2$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x + 2$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} + 2x + c$$

when $t = 0$, $v = 2$, $x = 0$

$$\frac{1}{2}(2)^2 = c \qquad c = 2$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} + 2x + 2$$

$$v^2 = x^2 + 4x + 4$$

$$v^2 = (x+2)^2$$

$v = \pm(x+2)$ but a is +ve + increasing $\therefore v > 0$

$v = x + 2$ as required

(b) Now $v = \frac{dx}{dt} = x + 2$

$$\int \frac{1}{x+2}\,dx = \int dt$$

$$\ln|x+2| = t + c$$

when $t = 0$, $x = 0$ $\therefore c = \ln 2$

$$\ln|x+2| = t + \ln 2$$

$$\ln\left|\frac{x+2}{2}\right| = t$$

$$\frac{x+2}{2} = e^t$$

$$x = 2e^t - 2$$