Ex 1A

(1) $a = 4t$

$V = \int 4t \, dt$

$V = 2t^2 + c$

when $t=0$ $V=3$ $\therefore c=3$ $V = 2t^2+3$

@ $t=3$ $V = 2(3)^2 + 3 = 21 \, ms^{-1}$

(3) $a = 2t$

$V = \int 2t \, dt$

$V = t^2 + c$

when $t=0$, $V=0$ $\therefore c=0$

$\therefore V = t^2$

Now $x = \int t^2 \, dt$

$x = \frac{t^3}{3} + c$

when $t=0$, $x=0$ $\therefore c=0$ $x = \frac{t^3}{3}$

Now @ $t=3$ $V = 9 \, ms^{-1}$ $x = 9m$

(2) $a = 6t$

$V = \int 6t \, dt$

$V = 3t^2 + c$

when $t=0$ $V=0$ $\therefore c=0$

$V = 3t^2$

@ $t=4$ $V = 3(4)^2 = 48 \, ms^{-1}$

(4) $a = \frac{3t^2}{2}$

$$V = \int \frac{3t^2}{2}\, dt$$

$$V = \frac{t^3}{2} + c$$

when $t = 2$, $V = 1$

$$1 = \frac{8}{2} + c$$

$$c = -3$$

$$\therefore \quad V = \frac{t^3}{2} - 3$$

$$x = \int \frac{t^3}{2} - 3 \; dt$$

$$x = \frac{t^4}{8} - 3t + c$$

when $t = 2$, $x = 0$

$$0 = \frac{2^4}{8} - 3(2) + c$$

$$c = 4$$

$$\therefore \quad x = \frac{t^4}{8} - 3t + 4$$

when $t = 4$, $\quad V = \frac{4^3}{2} - 3 = 29\,ms^{-1}$

$$x = \frac{4^4}{8} - 3(4) + 4 = 24 \text{ metres.}$$

(5) between $0 \leq t < 3$ $\qquad a = 2t$

$$V = \int 2t \, dt$$

$$V = t^2 + c$$

when $t=0$ $V=0$ $\therefore c=0$ $\qquad V = t^2$

$$x = \int t^2 \, dt$$

$$x = \frac{t^3}{3} + c$$

when $t=0$, $x=0$ $\therefore c=0$ $\qquad x = \frac{t^3}{3}$

@ $t=3$, $V = 9ms^{-1}$, $x = \frac{3^3}{3} = 9$ metres.

Now when $t>3$, using const accel formulae $V=0$, $a=-2$, $u=9$, $t=?$

$V = u + at$
$0 = 9 - 2t$
$t = 4.5$ secs

$\therefore$ particle comes to rest after $3 + 4.5 = 7.5$ seconds

using $x = ut + \frac{1}{2}at^2$

$$x = 9(4.5) + \frac{1}{2} \times -2 \times (4.5)^2$$

$x = 20.25$ metres

$\therefore$ particle comes to rest a distance of $9 + 20.25 = 29.25$ metres from O.

(6) $a = 3t - 1$

$V = \int 3t - 1 \, dt$

$V = \frac{3t^2}{2} - t + c$

when $t = 2, V = 3$

$3 = \frac{3(2)^2}{2} - 2 + c$

$c = 3 + 2 - 6 = -1$

$\therefore \quad V = \frac{3t^2}{2} - t - 1$

when $t = 4 \quad V = \frac{3}{2}(4)^2 - 4 - 1 \quad = 19 ms^{-1}$

(7) $a = 3t - 6$

$V = \int 3t - 6 \, dt$

$V = \frac{3t^2}{2} - 6t + c$

when $t = 0, V = 4.5 \quad \therefore \quad c = 4.5$

$V = \frac{3t^2}{2} - 6t + 4.5$

(a) when $V = 0 \qquad \frac{3t^2}{2} - 6t + 4.5 = 0$

$t = 1, 3$

(b) $x = \int \frac{3t^2}{2} - 6t + 4.5 \, dt$

$x = \frac{t^3}{2} - 3t^2 + 4.5t + c$

when $t = 0, x = 2 \quad \therefore \quad c = 2$

$x = \frac{t^3}{2} - 3t^2 + 4.5t + 2$

when $t = 4 \qquad x = \frac{4^3}{2} - 3(4)^2 + 4.5(4) + 2 = 4$ metres

(8) $a = \sin 2t$

$V = \int \sin 2t \, dt$

$V = -\frac{1}{2}\cos 2t + c$

when $t=0, V=0$ $\quad 0 = -\frac{1}{2} + c \quad c = \frac{1}{2}$

$\therefore V = \frac{1}{2} - \frac{1}{2}\cos 2t$

Now $x = \int \frac{1}{2} - \frac{1}{2}\cos 2t \, dt$

$x = \frac{t}{2} - \frac{1}{4}\sin 2t + c$

when $t=0, x=0 \quad \therefore \quad c=0$

$x = \frac{t}{2} - \frac{1}{4}\sin 2t$

when $t = \frac{\pi}{2}$, $\quad V = \frac{1}{2} - \frac{1}{2}\cos\pi = \frac{1}{2} - \frac{1}{2}(-1) = 1 \text{ ms}^{-1}$

$x = \frac{\pi}{4} - \frac{1}{4}\sin\pi \quad = \frac{\pi}{4}$ metres

(9) $a = 6t$

$V = \int 6t \, dt$

$V = 3t^2 + A$, where A is a const.

$x = \int 3t^2 + A \, dt$

$x = t^3 + At + B$, where B is a const.

(9) contd when $t=1$, $x=5$ $\quad 5 = 1 + A + B$

$A + B = 4 \quad — (1)$

when $t = 2$, $x = 14$ $\quad 14 = 8 + 2A + B$

$2A + B = 6 \quad — (2)$

(2) − (1) $\quad A = 2$

in (1) $\quad B = 2$

$\therefore \quad x = t^3 + 2t + 2$

when $t = 3$ $\quad x = 27 + 6 + 2 = 35$ metres

(10) $\quad a = e^{-t}$

$V = \int e^{-t}\,dt$

$V = -e^{-t} + c$

when $t=0$, $V=4$ $\quad 4 = -e^{0} + c$

$c = 5$

$\therefore \quad V = 5 - e^{-t}$

$x = \int 5 - e^{-t}\,dt$

$x = 5t + e^{-t} + c$

when $t = 0$, $x = 5$ $\quad 5 = 0 + 1 + c$

$c = 4$

$\therefore \quad x = 5t + e^{-t} + 4$

11 $a = \frac{-1}{(4+t)^{1/2}}$

$V = \int -\frac{1}{(4+t)^{1/2}}\,dt$ let $u = 4+t$

$\frac{du}{dt} = 1$

$dt = du$

$V = \int -u^{-1/2}\,du$

$V = -2u^{1/2} + c$

$V = -2(4+t)^{1/2} + c$

when $t=0$, $V=2$

$2 = -2(4)^{1/2} + c$

$2 + 4 = c = 6$

$\therefore V = 6 - 2(4+t)^{1/2}$

when $V=0$ $6 - 2(4+t)^{1/2} = 0$

$2(4+t)^{1/2} = 6$

$(4+t)^{1/2} = 3$

$4+t = 9$

$t = 5$ sec

(12) $a = \frac{3}{(1+t)^2}$

$V = \int 3(1+t)^{-2}\, dt$

$V = -3(1+t)^{-1} + c$

when $t=0$, $V=0$

$0 = -3 + c \qquad c = 3$

$V = 3 - \frac{3}{1+t}$

$x = \int 3 - \frac{3}{1+t}\, dt$

$x = 3t - 3\ln|1+t| + c$

when $t=0$, $x=0$

$0 = 0 - 3\ln 1 + c \qquad c = 0$

$x = 3t - 3\ln|1+t|$

when $t=6$ $\quad x = 18 - 3\ln 7$ metres.

(13) $a \propto (t+t_0)^{-3}$

$a = k(t+t_0)^{-3}$

$V = \int k(t+t_0)^{-3}\,dt$

$V = -\frac{k}{2}(t+t_0)^{-2} + c$

when $t=0$, $V=u$

$u = -\frac{k}{2} \times \frac{1}{t_0^2} + c$

$u + \frac{k}{2t_0^2} = c$

$\therefore V = u + \frac{k}{2t_0^2} - \frac{k}{2}(t+t_0)^{-2}$

As $t \to \infty$, $-\frac{k}{2(t+t_0)^2} \to 0$ so $V \to u + \frac{k}{2t_0^2}$ = Constant.

Now if limiting value is $2u$, $2u = u + \frac{k}{2t_0^2}$

$u = \frac{k}{2t_0^2}$

$k = 2ut_0^2$

hence $V = u + \frac{\cancel{2}u\cancel{t_0^2}}{\cancel{2t_0^2}} - \frac{\cancel{2}ut_0^2}{\cancel{2}}(t+t_0)^{-2}$

$V = u + u - ut_0^2(t+t_0)^{-2}$

$V = 2u - ut_0^2(t+t_0)^{-2}$

Now $x = \int 2u - ut_0^2(t+t_0)^{-2}\,dt$

(13) contd

$$x = 2ut + ut_0^2(t+t_0)^{-1} + c$$

$$x = 2ut + \frac{ut_0^2}{t+t_0} + c$$

$$\cancel{x = \left(\frac{2ut(t+t_0) + ut_0^2}{t+t_0} \right) + c}$$

at $t=0$ $x=0$

$$0 = 0 + \frac{ut_0^{\cancel{2}}}{\cancel{t_0}} + c$$

$\therefore$ $c = -ut_0$

$$x = 2ut + \frac{ut_0^2}{t+t_0} - ut_0$$

$$x = \frac{2ut(t+t_0) + ut_0^2 - ut_0(t+t_0)}{t+t_0}$$

$$x = \frac{2ut^2 + 2utt_0 + \cancel{ut_0^2} - ut_0t - \cancel{ut_0^2}}{t+t_0}$$

$$x = \frac{2ut^2 + utt_0}{t+t_0}$$

$$x = \frac{ut(2t+t_0)}{t+t_0}$$ As required.