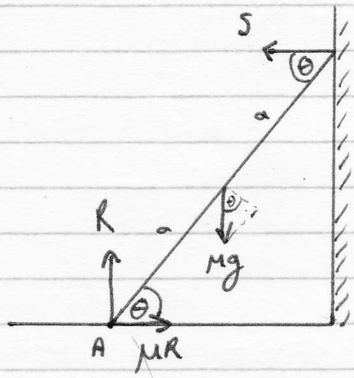
Ex5B

(1)



let ladder be of length $2l$ & Mass M kg.

Σf_x : $\mu R - S = 0$ — (1)

Σf_y : $R - Mg = 0$ — (2)

C_A : $Mg\cos\theta \times a - 2aS\sin\theta = 0$ — (3)

from (2) $R = Mg$

in (1) $S = \mu Mg$

in (3) $\cancel{a}\cancel{M}g\cos\theta - 2\cancel{a}\mu\cancel{M}g\sin\theta = 0$

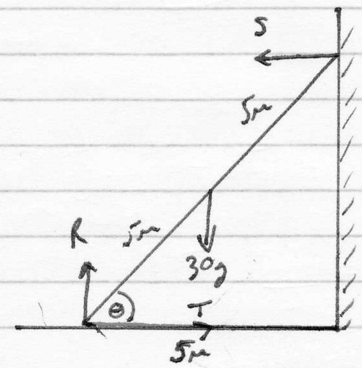
$\cos\theta - 2\mu\sin\theta = 0$

$\div\cos\theta$

$1 - 2\mu\tan\theta = 0$

$2\mu\tan\theta = 1$ as required.

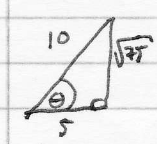
(2)



Σf_x : $T - S = 0$ — (1)

Σf_y : $R - 30g = 0$ — (2)

C_A : $30g\cos\theta \times 5 - 10S\sin\theta = 0$ — (3)



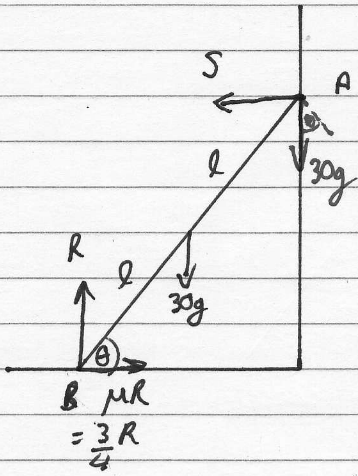
$\sin\theta = \frac{\sqrt{75}}{10}$ $\cos\theta = \frac{5}{10}$

from (3) $150g \times \frac{5}{10} - 10S \times \frac{\sqrt{75}}{10} = 0$

$75g - \sqrt{75}S = 0$

$S = \frac{75g}{\sqrt{75}}$ from (1) $= T = 84.9N.$

(3)



$\Sigma F_x: \quad \frac{3}{4}R = S$

$\Sigma F_y: R = 60g$

$\therefore \quad S = \frac{3}{4} \times 60g = 45g$

$\circlearrowright_B: \quad 30g\cos\theta . l - 2l . S\sin\theta + 30g\cos\theta . 2l = 0$

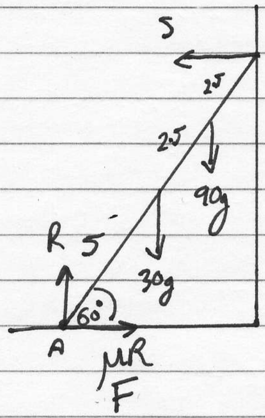
$30g\cos\theta - 2.45g\sin\theta + 60g\cos\theta = 0$

$90g\cos\theta - 90g\sin\theta = 0$

$\tan\theta = 1$

$\theta = 45°$

(4)



$\Sigma F_x: \quad S = \mu R = F$

$\Sigma F_y: R = 120g$

$\therefore \quad S = 120\mu g$

$\circlearrowright_A: \quad 30g\cos60 \times 5 + 90g\cos60 \times 7.5 - 10S\sin60 = 0 \quad - (3)$

$150g \times \frac{1}{2} + 675g \times \frac{1}{2} - 10 \times 120\mu g \times \frac{\sqrt{3}}{2} = 0$

$75g + 337.5g - 600\mu \; 600\sqrt{3}g\mu = 0$

$600\sqrt{3}\mu = 412.5$

$\mu = \frac{412.5}{600\sqrt{3}} =$

(a) $R = 120g = 1176N$

From (3) $\quad 150g \times \frac{1}{2} + 675g \times \frac{1}{2} - 10F \times \frac{\sqrt{3}}{2} = 0$

$75g + 337.5g - 5\sqrt{3}F = 0$

$5\sqrt{3}F = 412.5g$

$F = \frac{412.5g}{5\sqrt{3}} = 467N.$

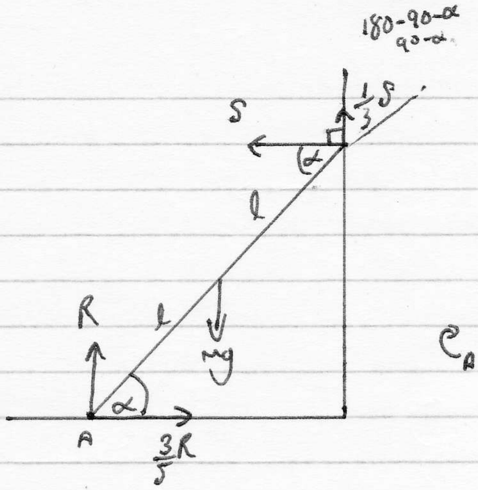
(b) Man @ top of ladder: $\circlearrowright_A: \quad 30g\cos60 \times 5 + 90g\cos60 \times 10 - 10F\sin60 = 0$

$75g + 450g = 5\sqrt{3}F$

$F = \frac{5145}{5\sqrt{3}}$

Now $\mu = \frac{F}{R} = \frac{5145/5\sqrt{3}}{120g} = 0.505$

(5)



$\Sigma F_x \quad S = \frac{3}{5}R$ — (1)

$\Sigma F_y \quad R + \frac{1}{3}S = Mg$ — (2)

$C_A: \quad Mg\cos\alpha\, l - S\sin\alpha . 2l - \frac{1}{3}S\sin(90-\alpha) . 2l = 0$ — (3)

From (2) $R = Mg - \frac{1}{3}S$

in (1) $S = \frac{3}{5}\left(Mg - \frac{1}{3}S\right)$

$S = \frac{3}{5}Mg - \frac{1}{5}S$

$\cancel{\frac{6}{5}S =}$ $\quad 5S = 3Mg - S$

$6S = 3Mg$

$S = \frac{3Mg}{6} = \frac{Mg}{2}$

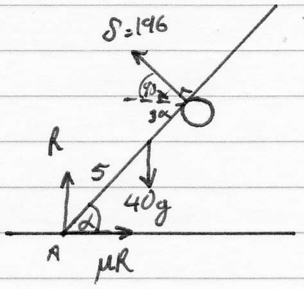
in (3) $\cancel{Mg}\cos\alpha \cancel{l} - \frac{\cancel{Mg}}{\cancel{2}}\sin\alpha . \cancel{2l} - \frac{1}{3} . \frac{\cancel{Mg}}{\cancel{2}}\cos\alpha . \cancel{2l} = 0$

$\cos\alpha - \sin\alpha - \frac{1}{3}\cos\alpha = 0$

$\frac{2}{3}\cos\alpha = \sin\alpha$

$\tan\alpha = \frac{2}{3}$

(6)



$$\Sigma F_x: \quad \mu R - 196\cos(90-\alpha) = 0 \quad \text{— (1)}$$

$$\Sigma F_y: \quad R + 196\sin(90-\alpha) - 40g = 0 \quad \text{— (2)}$$

$$\circlearrowright_A: \quad 40g\cos\alpha \times 5 - 196 \times 8 = 0 \quad \text{— (3)}$$

$$1960\cos\alpha = 1568$$

$$\cos\alpha = 0.8$$

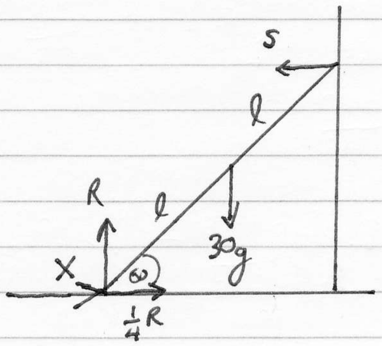
$$\alpha = 36.9^\circ$$

in (2) $R = 40g - 196\sin(90-36.9) = 235.2$

in (1) $\mu\, 235.2 = 196\cos(90-36.9)$

$$\mu = 0.5$$

(7)



$$\Sigma F_x: \quad X + \tfrac{1}{4}R - S = 0 \quad \text{— (1)}$$

$$\Sigma F_y \quad R = 30g \quad \text{— (2)}$$

$$\circlearrowright_A: \quad 30g\cos 60 \times l - S\sin 60 \times 2l = 0 \quad \text{— (3)}$$

$$30g \times \frac{1}{2} - S \times \frac{\sqrt{3}}{2} \times 2 = 0$$

$$15g = \sqrt{3}\,S$$

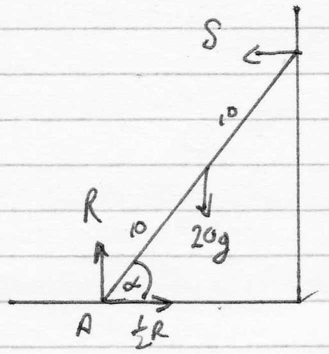
$$S = \frac{15g}{\sqrt{3}}$$

in (1) $X + \frac{1}{4}.30g - \frac{15g}{\sqrt{3}} = 0$

$$X = \frac{15g}{\sqrt{3}} - 7.5g$$

$$X = 11.4\,N.$$

(8)



(a) $\Sigma F_x: \quad S = \frac{1}{2}R$

$\Sigma F_y: \quad R = 20g$

$\therefore \quad S = 10g$

$C_A: \quad 20g\cos\alpha \cdot 10 - S\sin\alpha \times 20 = 0$

$200g\cos\alpha - 200g\sin\alpha = 0$

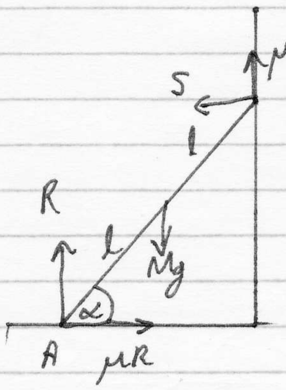
$\tan\alpha = 1$

$\alpha = 45^\circ$

(b) Uniform ladder against fairly smooth wall

(c) Take into account roughness of wall. Model ladder by non-uniform rod.

(9)



$\Sigma F_x: \quad \mu R - S = 0$ — (1)

$\Sigma F_y: \quad \mu' S + R - Mg = 0$ — (2)

$C_A \quad Mg\cos\alpha \cdot l - 2l \times S\sin\alpha - 2l \times \mu' S \sin(90-\alpha) = 0$

$Mg\cos\alpha\, l - 2lS\sin\alpha - 2l\mu' S\cos\alpha = 0$ — (3)

from (2) $R = Mg - \mu' S$

in (1) $\mu(Mg - \mu' S) - S = 0$

$\mu Mg - \mu\mu' S - S = 0$

$S(1 + \mu\mu') = \mu Mg$

$S = \dfrac{\mu Mg}{1 + \mu\mu'}$

in (3) $\cancel{M}\cancel{g}\cos\alpha\,\cancel{l} - 2\cancel{l}\sin\alpha\left(\dfrac{\mu \cancel{M}\cancel{g}}{1+\mu\mu'}\right) - 2\cancel{l}\mu'\cos\alpha\left(\dfrac{\mu \cancel{M}\cancel{g}}{1+\mu\mu'}\right) = 0$

$\cos\alpha - 2\sin\alpha\left(\dfrac{\mu}{1+\mu\mu'}\right) - 2\cos\alpha\left(\dfrac{\mu\mu'}{1+\mu\mu'}\right) = 0$

$\times(1+\mu\mu') \quad \cos\alpha(1+\mu\mu') - 2\mu\sin\alpha - 2\mu\mu'\cos\alpha = 0$

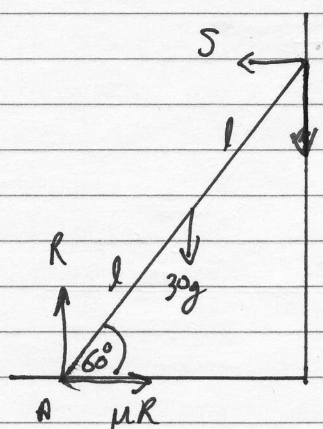
$\cos\alpha + \cos\alpha\,\mu\mu' - 2\mu\mu'\cos\alpha = 2\mu\sin\alpha$

$\cos\alpha(1 + \mu\mu' - 2\mu\mu') = 2\mu\sin\alpha$

$\cos\alpha(1 - \mu\mu') = 2\mu\sin\alpha$

$\tan\alpha = \cancel{\dfrac{2\mu}{1+\mu\mu'}} \; \dfrac{1-\mu\mu'}{2\mu}$ As required.

10



(a) $\Sigma F_x: \; S = \mu R$

$\Sigma F_y: \; R = 30g$

$\therefore \; S = 30\mu g$

$\circlearrowleft_A: \quad 30g\cos 60 \times \cancel{l} - S\sin 60 \times 2\cancel{l} = 0$

$15g - \sqrt{3}S = 0$

$S = \dfrac{15g}{\sqrt{3}} = 84.9N$

Now $\quad 84.9 = 30\mu g$

$\cancel{\mu = \dfrac{84.9}{30g}} = \dfrac{15\cancel{g}}{\sqrt{3}} = 30\mu g$

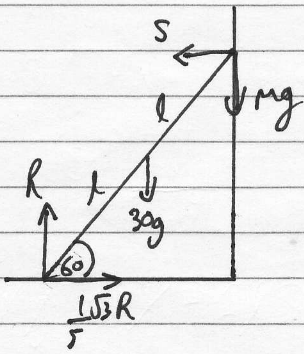
$\mu = \dfrac{15}{30\sqrt{3}}$

$\mu = \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$\mu = \dfrac{1}{6}\sqrt{3}$

μ is this value at limiting equilibrium $\therefore$ for ladder not to slip, $\mu \geq \frac{1}{6}\sqrt{3}$

(b)



$\Sigma F_x: \; S = \frac{1}{5}\sqrt{3}R$ — (1)

$\Sigma F_y: \; R = 30g + Mg$ — (2)

$\circlearrowleft_A: \; 30g\cos 60 \times l - S\sin 60 \times 2l + Mg\cos 60 \times 2l = 0$ — (3)

From (2) in (1) $\quad S = \frac{1}{5}\sqrt{3}\cancel{R}(30g + Mg) = 6\sqrt{3}g + \frac{1}{5}\sqrt{3}Mg$

in (3) $\quad 30\cancel{g}\cdot\frac{1}{2}\cdot\cancel{l} - \left[6\sqrt{3}\cancel{g} + \frac{1}{5}\sqrt{3}M\cancel{g}\right]\cdot\frac{\sqrt{3}}{2}\cdot 2\cancel{l} + M\cancel{g}\cdot\frac{1}{2}2\cancel{l} = 0$

$15 - \left[6\sqrt{3} + \frac{1}{5}\sqrt{3}M\right]\cdot\sqrt{3} + M = 0$

$15 - 18 - \frac{3}{5}M + M = 0$

$\frac{2}{5}M = 3$

$M = 7.5\,kg$