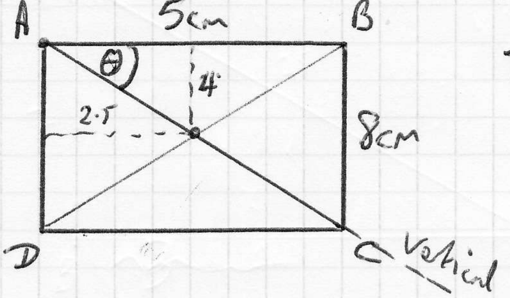
Ex ~~6E~~ 2C

(1)

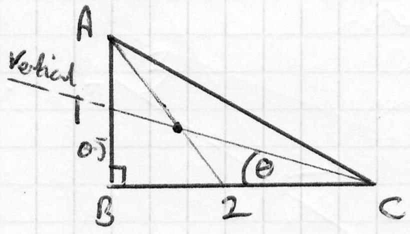


COM @ (2.5, 4)

$\tan\theta = \frac{4}{2.5}$

$\theta = 58^\circ$

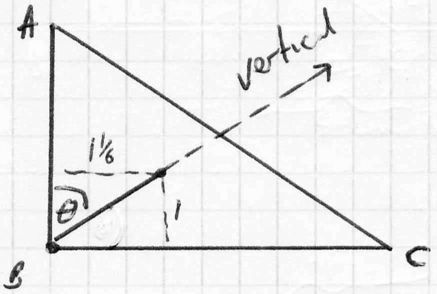
(2)



$\tan\theta = \frac{0.5}{2}$

$\theta = 14^\circ$

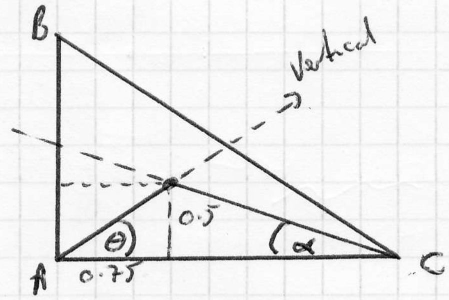
(3)



$\tan\theta = \frac{1\frac{1}{6}}{1}$

$\theta = 49.4^\circ$

(4)



(a) $\tan\theta = \frac{0.5}{0.75}$

$\theta = 33.7^\circ$

(b) $\tan\alpha = \frac{0.5}{1.25}$

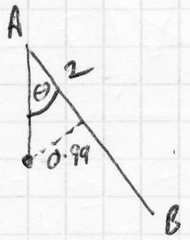
$\alpha = 21.8^\circ$

(5)

$$\left(\frac{\pi\times 2^2}{2} - \frac{\pi\times 1^2}{2}\right)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 2\pi\begin{pmatrix}0\\ 8/3\pi\end{pmatrix} - \frac{\pi}{2}\begin{pmatrix}0\\ 4/3\pi\end{pmatrix}$$

$$\frac{3\pi}{2}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}0\\ 14/3\end{pmatrix}$$

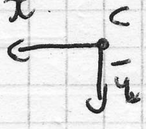
$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}0\\ 28/9\pi\end{pmatrix} = \begin{pmatrix}0\\ 0.99\end{pmatrix}$$



$\tan\theta = \frac{0.99}{2}$

$\theta = 26.3^\circ$

(6)

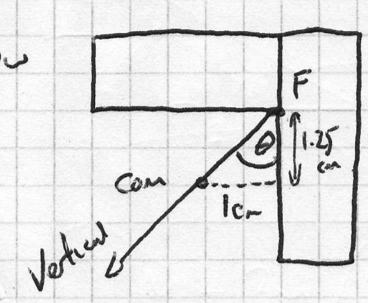


$$(M+3M)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = M\begin{pmatrix}14\\ 2.5\end{pmatrix} + 3M\begin{pmatrix}2\\ 7.5\end{pmatrix}$$

$$4\cancel{M}\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \cancel{M}\begin{pmatrix}20\\ 25\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}5\\ 6.25\end{pmatrix}$$

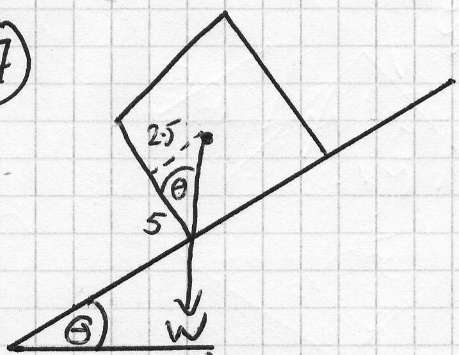
Now



$$\tan\theta = \frac{1}{1.25} \qquad \theta = 38.7^\circ$$

Frictionless Peg modelled as a point.

(7)

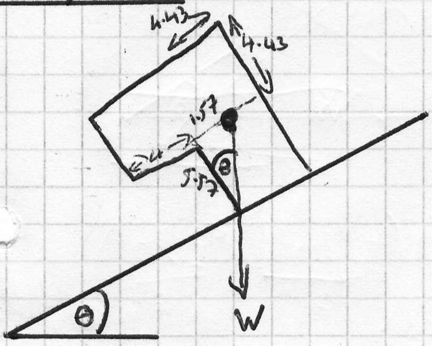


At point of toppling

$$\tan\theta = \frac{2.5}{5}$$

$\theta = 26.6^\circ$ ∴ when slope is at 25°, rectangle remains in equilib

(8)



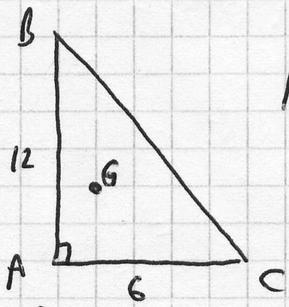
On point of toppling

$$\tan\theta = \frac{1.57}{5.57}$$

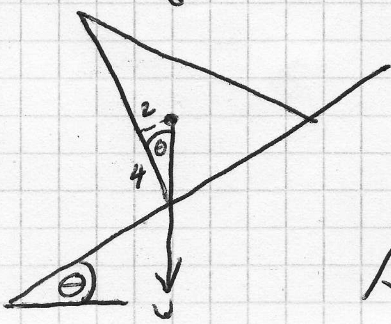
$\theta = 15.7^\circ$ ∴ (a) when $\theta = 10^\circ$ equilib ok

(b) when $\theta = 25^\circ$ no equilib.

(9)



Now $G\left(\frac{1}{3}\times 6, \frac{1}{3}\times 12\right) = (2, 4)$



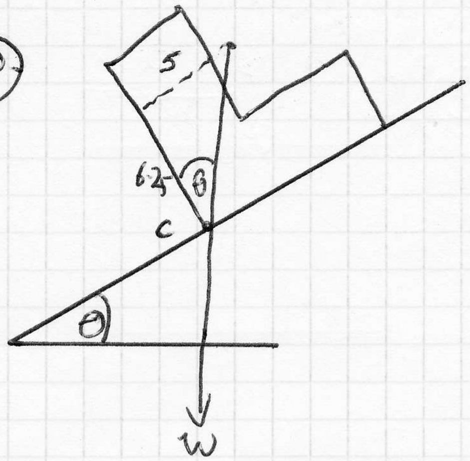
on point of toppling

$$\tan\theta = \frac{2}{4} \qquad \theta = 26.6^\circ$$

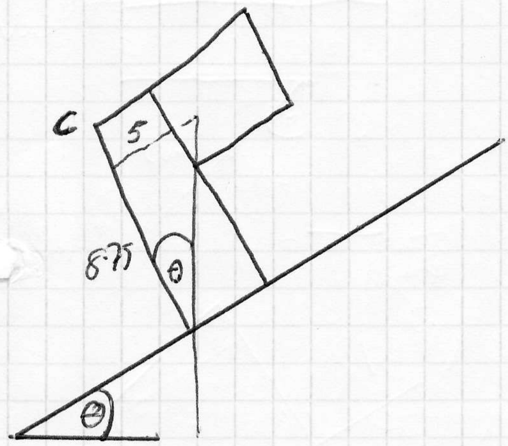
Sliding could have taken place.

Assume that μ is high enough to prevent sliding below this value of θ.

(18)



$\theta_{Max} = \tan^{-1}\left(\frac{5}{6.25}\right) = 38.7^\circ$

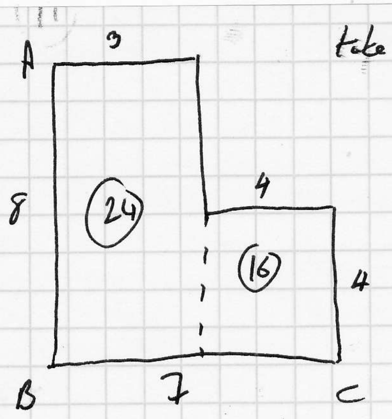


$\theta_{MAX} = \tan^{-1}\left(\frac{5}{8.75}\right) = 29.7^\circ$

∴ (a) when $\theta = 10^\circ$ eq ok.

(b) when $\theta = 30^\circ$ eq not ok.

(11)



take A as origin:

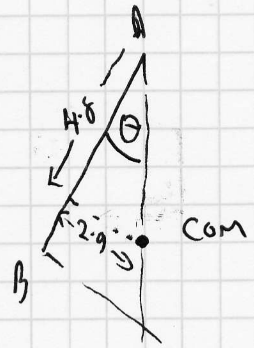
$$(24\mu + 16\mu)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 24\mu\begin{pmatrix}1.5\\ -4\end{pmatrix} + 16\mu\begin{pmatrix}5\\ -6\end{pmatrix}$$

$$40\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}116\\ -192\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}2.9\\ 4.8\end{pmatrix}$$

So (a) 2.9 cm from AB and (b) 3.2 from BC

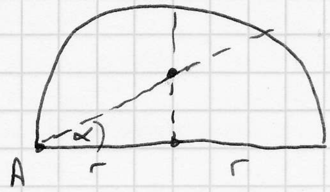
(c)



$$\tan\theta = \frac{2.9}{4.8}$$

$$\theta = 31.1^\circ$$

(12)



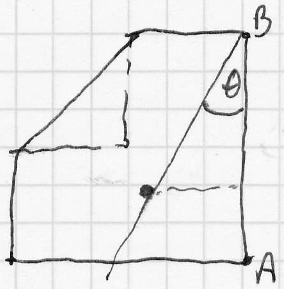
COM $\dfrac{2r\sin\frac{\pi}{2}}{\frac{3\pi}{2}} = \dfrac{4r}{3\pi}$

If suspended from A: $\tan\alpha = \dfrac{4r/3\pi}{r}$

$\tan\alpha = \dfrac{4}{3\pi}$

$\alpha = 0.401^c$

(13)



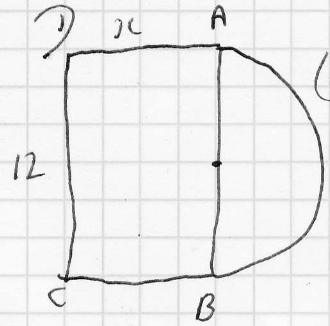
Com from A $\left(3\frac{5}{6}, 3\frac{5}{6}\right)$

Com from B $\left(3\frac{5}{6}, 8-3\frac{5}{6}=6\frac{1}{6}\right)$

$\tan\theta = \dfrac{3\frac{5}{6}}{6\frac{1}{6}}$

$\theta = 42.6^\circ$

(14)



(a) $\left(12x\mu + \frac{\pi 6^2}{2}\mu\right)\bar{x} = 12x\mu\left(\frac{x}{2}\right) + 18\pi\mu\left(x + \dfrac{2.6.\sin\frac{\pi}{2}}{\frac{3\pi}{2}}\right)$

$(12x+18\pi)x = 6x^2 + 18\pi\left(x+\frac{8}{\pi}\right)$

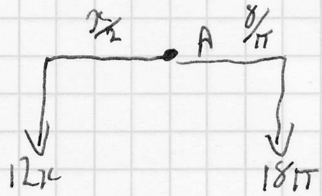
$12x^2 + 18\pi x = 6x^2 + 18\pi x + 144$

$6x^2 - 144 = 0$

$x^2 = 24$

$x = \sqrt{24} = 2\sqrt{6} = 4.9$

Alternatively: centre of mass lies on AB so balances

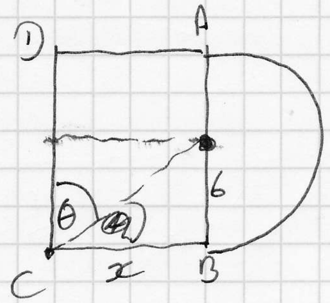


Equilib about A: $\overset{6}{\cancel{12}}x \cdot \frac{x}{\cancel{2}} = \frac{8}{\cancel{\pi}} \cdot 18\cancel{\pi}$

$6x^2 = 144$

$x = 4.9.$

(14) (b)



$\cancel{\tan\theta = \frac{6}{\sqrt{24}}}$

$\tan\theta = \frac{x}{6} = \frac{\sqrt{24}}{6} = 39.2^\circ$