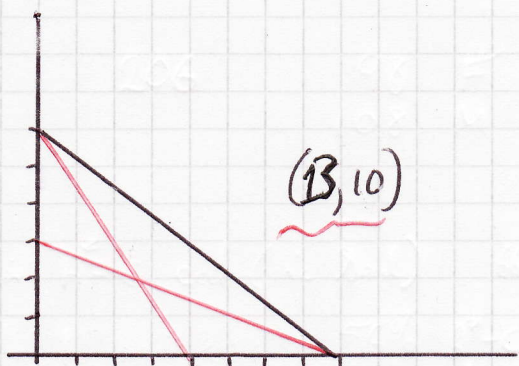


Ex 2B \* Some differences on arcs of o's \*

① (a)  $(2.5, 1.5)$  (b)  $(4.5, 0.5)$  (c)  $(6, 6)$  (d)  $(4\frac{1}{3}, 6, 0) = (3, 0)$

(e)



(f)  $(8, 4 + \frac{2}{3} \times 8) = (8, 9\frac{1}{3})$

② (a)  $[4+10] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 10 \begin{pmatrix} 3 \\ 2.5 \end{pmatrix}$

$$14 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 34 \\ 29 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 2\frac{3}{7} \\ 2\frac{1}{4} \end{pmatrix}$$

(b)  $[24-6] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 24 \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 6 \begin{pmatrix} 3 \\ 3.5 \end{pmatrix}$

$$18 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 30 \\ 51 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1\frac{2}{3} \\ 2\frac{5}{6} \end{pmatrix}$$

(c)  $[9+18] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 9 \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 18 \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}$

$$27 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 99 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 3\frac{2}{3} \\ 0 \end{pmatrix}$$

(d)  $[48+4+16] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 48 \begin{pmatrix} 6 \\ 6 \end{pmatrix} + 4 \begin{pmatrix} 9 \\ 5 \end{pmatrix} + 16 \begin{pmatrix} 10 \\ 10 \end{pmatrix}$

$$68 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 484 \\ 468 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7.12 \\ 6.88 \end{pmatrix}$$



$$(e) [16 - 1] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 16 \begin{pmatrix} 2 \\ 2 \end{pmatrix} - 1 \begin{pmatrix} 2.5 \\ 1.5 \end{pmatrix}$$

$$15 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 29.5 \\ 30.5 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1.97 \\ 2.03 \end{pmatrix}$$

$$(f) [168 - 8 - 8] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 168 \begin{pmatrix} 7 \\ 6 \end{pmatrix} - 8 \begin{pmatrix} 3 \\ 8 \end{pmatrix} - 8 \begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

$$152 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1072 \\ 920 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7.05 \\ 6.05 \end{pmatrix}$$

$$(g) [18 - 3] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 18 \begin{pmatrix} 3 \\ 2 \end{pmatrix} - 3 \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

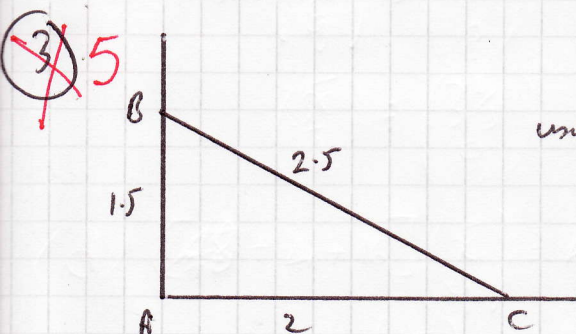
$$15 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 45 \\ 33 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 3 \\ 2.2 \end{pmatrix}$$

$$(h) [18 - 2] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 18 \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2 \begin{pmatrix} 3 \\ 4.5 \end{pmatrix}$$

$$16 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 48 \\ 63 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 3 \\ 3.94 \end{pmatrix}$$



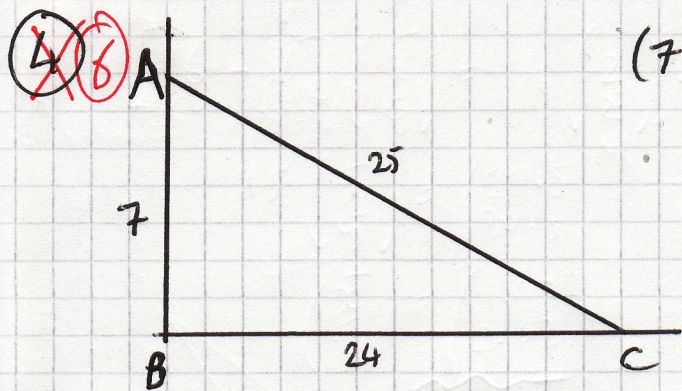
using pythg  $AC = \sqrt{2.5^2 - 1.5^2} = 2$

$$\text{Now } [1.5 + 2.5 + 2] \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 1.5 \begin{pmatrix} 0 \\ 0.75 \end{pmatrix} + 2.5 \begin{pmatrix} 1 \\ 0.75 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$6 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 4.5 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.5 \end{pmatrix}$$

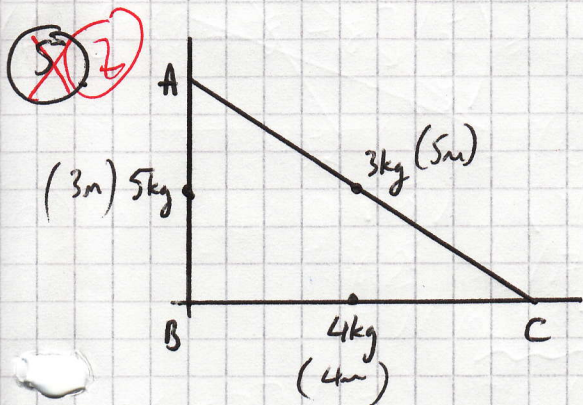




$$(7+25+24)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 7\begin{pmatrix} 0 \\ 3.5 \end{pmatrix} + 25\begin{pmatrix} 12 \\ 3.5 \end{pmatrix} + 24\begin{pmatrix} 12 \\ 0 \end{pmatrix}$$

$$56\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 588 \\ 112 \end{pmatrix}$$

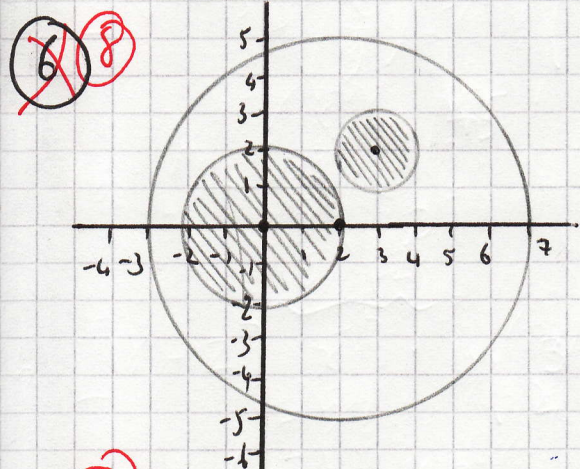
$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 10.5 \\ 2 \end{pmatrix}$$



$$(5+3+4)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 5\begin{pmatrix} 0 \\ 1.5 \end{pmatrix} + 3\begin{pmatrix} 2 \\ 1.5 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$12\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 14 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1\frac{1}{6} \\ 1 \end{pmatrix}$$



$$[\pi \times 5^2 - \pi \times 2^2 - \pi \times 1^2]\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 25\pi\begin{pmatrix} 2 \\ 0 \end{pmatrix} - 4\pi\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \pi\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

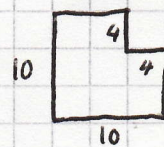
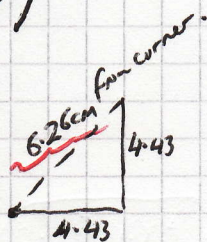
$$20\pi\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 47\pi \\ -2\pi \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 2.35 \\ -0.1 \end{pmatrix}$$

$$[100 - 16]\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 100\begin{pmatrix} 5 \\ 5 \end{pmatrix} - 16\begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$84\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 372 \\ 372 \end{pmatrix}$$

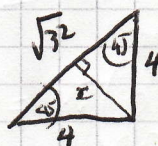
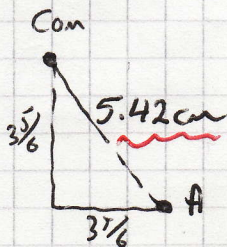
$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 4.43 \\ 4.43 \end{pmatrix}$$



$$64\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 16\begin{pmatrix} 6 \\ 2 \end{pmatrix} + 32\begin{pmatrix} 2 \\ 4 \end{pmatrix} + 16\begin{pmatrix} 5\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} \Rightarrow$$

$$64\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 245\frac{1}{3} \\ 245\frac{1}{3} \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 3\frac{5}{6} \\ 3\frac{5}{6} \end{pmatrix}$$

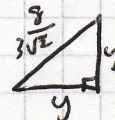
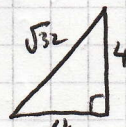


$$\text{Com @ } \frac{2}{3} \times x$$

$$x = 4.5 = 4.5$$

$$\therefore \text{Com @ } \frac{2}{3} \times \frac{4}{\sqrt{2}} = \frac{8}{3\sqrt{2}}$$

Now sim Δ



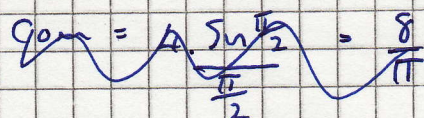
$$\frac{4}{y} = \frac{8/3\sqrt{2}}{\sqrt{32}}$$

$$\therefore y = \frac{4}{3}$$





Q3 COM of arc =  $\frac{r \sin \alpha}{\alpha}$  \*for centre  $2\alpha$ \* i centre  $\alpha$ , Com =  $\frac{r \sin(\frac{\alpha}{2})}{(\frac{\alpha}{2})}$

(a) 

(a)  $Com = \frac{4 \cdot \sin \frac{\pi}{4}}{\pi/4} = 3.60 \text{ cm}$

(b)  $Com = \frac{6 \cdot \sin \frac{1}{2}}{\frac{1}{2}} = 5.75 \text{ cm}$

(c)  $Com = \frac{12 \cdot \sin \frac{\pi}{2}}{\pi/2} = 7.64 \text{ cm}$

(d)  $60^\circ = \frac{\pi}{3}$   $\therefore Com = \frac{5 \cdot \sin \frac{\pi}{6}}{\pi/6} = 4.77 \text{ cm}$

(e)  $120^\circ = \frac{120}{180} \pi = \frac{2\pi}{3}$   $\therefore Com = \frac{8 \cdot \sin \frac{2\pi}{6}}{\frac{2\pi}{6}} = 6.62 \text{ cm}$

4 COM of sector =  $\frac{2r \sin \alpha}{3\alpha}$  \*for centre  $2\alpha$ \* i centre  $\alpha$ , Com =  $\frac{2r \sin(\frac{\alpha}{2})}{3\frac{\alpha}{2}}$

(a)  $Com = \frac{2 \cdot 9 \cdot \sin \frac{\pi}{2}}{3 \cdot \frac{\pi}{2}} = 3.82 \text{ cm}$

(b)  $Com = \frac{2 \cdot 6 \cdot \sin \frac{\pi}{6}}{3 \cdot \frac{\pi}{6}} = 3.82 \text{ cm}$

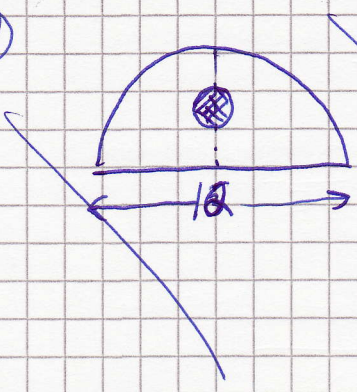
(c)  $Com = \frac{2 \cdot 8 \cdot \sin \frac{\pi}{2}}{3 \cdot \frac{\pi}{2}} = 5.28 \text{ cm}$

(d)  $90^\circ = \frac{\pi}{2}$   $Com = \frac{2 \cdot 12 \cdot \sin \frac{\pi}{4}}{3 \cdot \frac{\pi}{4}} = 7.20 \text{ cm}$

(e)  $45^\circ = \frac{\pi}{4}$   $Com = \frac{2 \cdot 10 \cdot \sin \frac{\pi}{8}}{3 \cdot \frac{\pi}{8}} = 6.50 \text{ cm}$



10



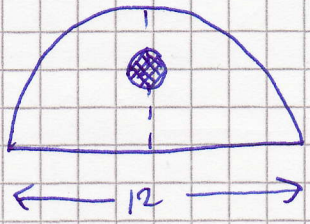
$$(\frac{\pi \cdot 3^2}{2} - \pi \cdot 1.5^2) \bar{y} = \frac{\pi \cdot 3^2}{2} \left( \frac{2.35 \sin \frac{\pi}{2}}{3 \frac{\pi}{2}} \right) - \pi \cdot 1.5^2 (2)$$

$$(4.5 - 2.25) \bar{y} = \frac{9}{2} \left( \frac{4}{\pi} \right) - 4.5$$

$$2.25 \bar{y} = 4.5 \left( \frac{4}{\pi} - 1 \right)$$

$$\bar{y} = 0.55 \text{ cm}$$

11

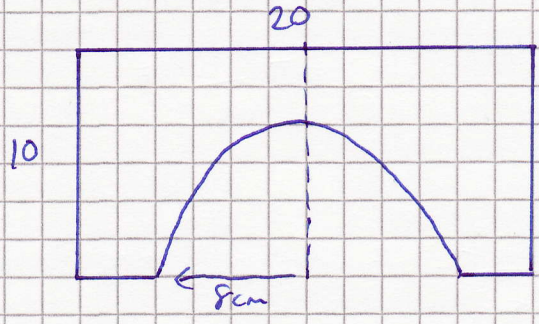


$$(\frac{\pi \cdot 6^2}{2} - \pi \cdot 1.5^2) \bar{y} = \frac{\pi \cdot 6^2}{2} \left( \frac{2.6 \cdot 5 \sin \frac{\pi}{2}}{3 \frac{\pi}{2}} \right) - \pi \cdot 1.5^2 (2)$$

$$(18 - 2.25) \bar{y} = 18 \left( \frac{8}{\pi} \right) - 4.5$$

$$\bar{y} = 2.62 \text{ cm}$$

12



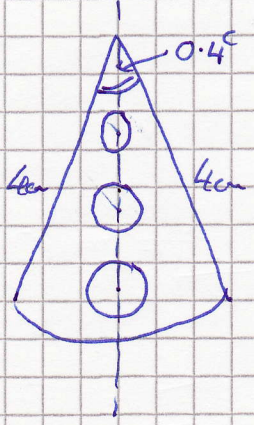
$$(200\pi - \frac{\pi \cdot 8^2}{2}) \bar{y} = 200(5) - \frac{\pi \cdot 8^2}{2} \left( \frac{2 \cdot 8 \cdot 5 \sin \frac{\pi}{2}}{3 \frac{\pi}{2}} \right)$$

$$(200 - 32\pi) \bar{y} = 1000 - 32\pi \left( \frac{32}{3\pi} \right)$$

$$(200 - 32\pi) \bar{y} = 658 \frac{2}{3}$$

$$\bar{y} = 6.62 \text{ cm}$$

13



$$\left[ \frac{1}{2} \cdot 4^2 \cdot 0.4 \text{ M} - \pi \cdot 0.1^2 \text{ M} - \pi \cdot 0.2^2 \text{ M} - \pi \cdot 0.3^2 \text{ M} \right] \bar{y}$$

$$= \frac{1}{2} \cdot 4^2 \cdot 0.4 \text{ M} \left( \frac{2.4 \cdot 5 \sin \frac{\pi}{2}}{3 \left( \frac{0.4}{2} \right)} \right) - \pi \cdot 0.1^2 \text{ M} (1) - \pi \cdot 0.2^2 \text{ M} (2) - \pi \cdot 0.3^2 \text{ M} (3)$$

$$(3.2 - 0.01\pi - 0.04\pi - 0.09\pi) \bar{y} = 3.2 \left( \frac{2.5}{1.2} \right) - 0.01\pi - 0.08\pi - 0.27\pi$$

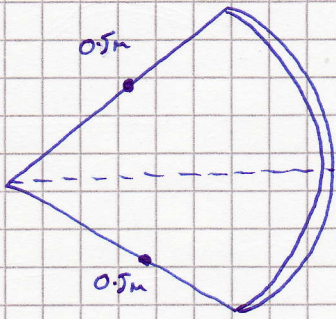
$$\bar{y} = 7.35$$

$$\bar{y} = 2.66 \text{ cm} *$$

TFK agrees with me!



14

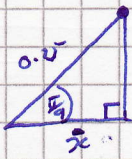


$$40^\circ = \frac{40\pi}{180} = \frac{2\pi}{9} \text{ rad}$$

$$20^\circ = \frac{\pi}{9} \text{ rad}$$

$$\text{Arc length} = 0.5 \times \frac{2\pi}{9} = \frac{\pi}{9}$$

$$\text{Arc doubled} \therefore \text{doubled for mass} = \frac{2\pi}{9} M$$



$$x = 0.25 \cos \frac{\pi}{9}$$

$$\text{Com of Sector} = \frac{0.5 \sin\left(\frac{2\pi/9}{2}\right)}{\frac{2\pi/9}{2}} = \frac{0.5 \sin\left(\frac{\pi}{9}\right)}{\frac{\pi}{9}} = \frac{9}{2\pi} \sin\left(\frac{\pi}{9}\right)$$

$$\text{Now } \left[ 0.5M + 0.5M + \frac{2\pi}{9}M \right] \bar{x} = 0.5M \left( 0.25 \cos \frac{\pi}{9} \right) + 0.5M \left( 0.25 \cos \frac{\pi}{9} \right) + \frac{2\pi}{9}M \left( \frac{9}{2\pi} \sin \left( \frac{\pi}{9} \right) \right)$$

$$\left[ 1 + \frac{2\pi}{9} \right] \bar{x} = 0.25 \cos \frac{\pi}{9} + \sin \frac{\pi}{9}$$

$$1.698 \bar{x} = 0.577$$

$$\bar{x} = 0.340 \text{ metres.}$$



15

$$(M+m)\bar{y} = M\left(\frac{2r\sin\frac{\pi}{2}}{3\frac{\pi}{2}}\right) + m\bar{r}$$

but  $\bar{y} = \frac{r}{2}$

$$(M+m)\frac{r}{2} = M \cdot \frac{4r}{3\pi} + mr$$

$$\frac{(M+m)}{2} = \frac{4M}{3\pi} + m$$

$$\frac{M+m}{2} = \frac{4M + 3\pi m}{3\pi}$$

$$3\pi(M+m) = 2(4M + 3\pi m)$$

$$3\pi M + 3\pi m = 8M + 6\pi m$$

$$3\pi M - 8M = 6\pi m - 3\pi m$$

$$M(3\pi - 8) = 3\pi m$$

$$\frac{M}{m} = \frac{3\pi}{3\pi - 8}$$

∴ a ratio  $M:m$   $3\pi : 3\pi - 8$