Components of forces

We have seen that two forces can be combined into a single force which is called their *resultant*.

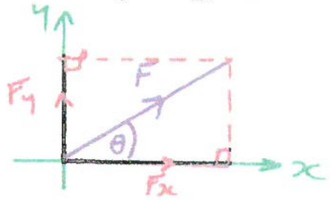
There is a reverse process which consists of expressing a single force in terms of its two *components*. These components are sometimes referred to as the *resolved parts* of the force.

It is particularly useful to find two mutually perpendicular components of a force.

The directions may, for example, be horizontal and vertical, or parallel and perpendicular to the surface of an inclined plane.

The component of the force F in any given direction is a measure of the effect of the force F in that direction.

Consider a force F acting at and angle θ to the x-axis as shown below. The components F_x and F_y being the horizontal and vertical components of F respectively.



$\frac{F_y}{F} = \frac{opp}{hyp} = \sin\theta$

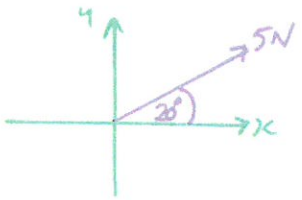
$F_y = F\sin\theta$

$\frac{F_x}{F} = \frac{adj}{hyp} = \cos\theta$

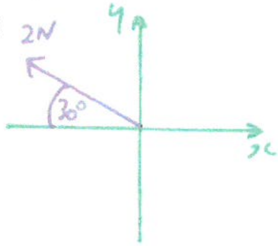
$F_x = F\cos\theta$

Eg13 Find the components F_x and F_y of the given forces:

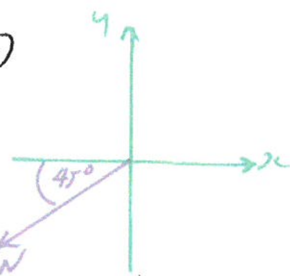
(a)



(b)



(c)

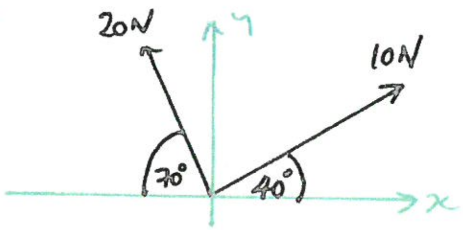


Exercise Q's 1 & 2

Eg14 A body of mass 4kg rests on an incline of 35°. Find the component of the weight of the body parallel and perpendicular to the plane.

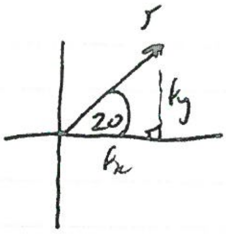
Exercise Q3

Eg15 Find the sum of the components of the given forces in the direction of (i) x-axis (ii) y-axis.



Exercise Q's 4 & 5

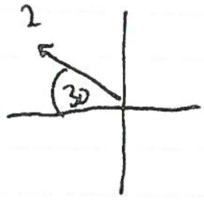
Eg13 (a)



$F_x = 5\cos 20 = 4.70\,N$

$F_y = 5\sin 20 = 1.71\,N$

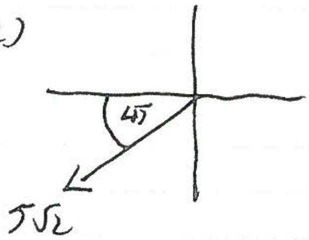
(b)



$F_x = -2\cos 30 = -2 \times \frac{\sqrt{3}}{2} = -\sqrt{3}\,N$

$F_y = +2\sin 30 = 2 \times \frac{1}{2} = 1\,N$

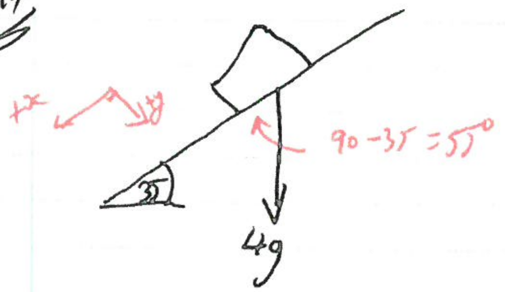
(c)



$F_x = -5\sqrt{2}\cos 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

$F_y = -5\sqrt{2}\sin 45 = -5\sqrt{2} \times \frac{1}{\sqrt{2}} = -5\,N$

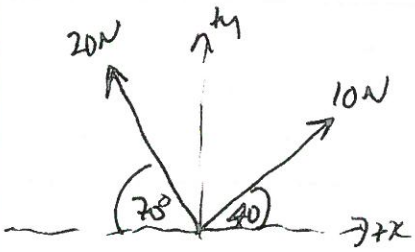
Eg14



$F_x = 4g\cos 55 = 22.5\,N$

$F_y = 4g\sin 55 = 32.1\,N$

Eg15



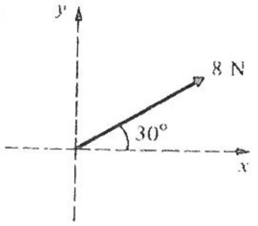
$\Sigma F_x = 10\cos 40 - 20\cos 70 = 0.82\,N$

$\Sigma F_y = 10\sin 40 + 20\sin 70 = 25.2\,N$

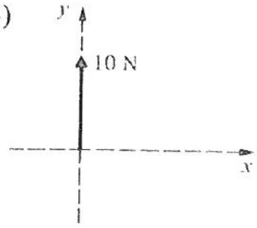
Components of Forces Exercise

1. For each of the forces shown below, find the components in the direction of
(i) the x-axis and (ii) the y-axis.

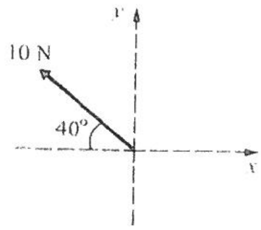
(a)



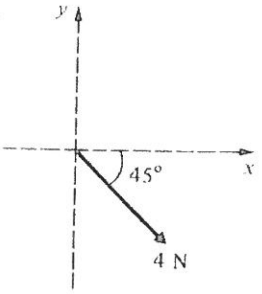
(b)



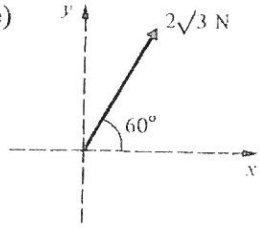
(c)



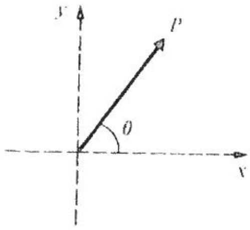
(d)



(e)

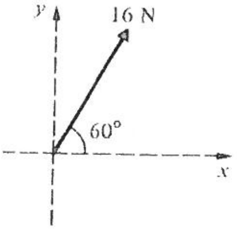


(f)

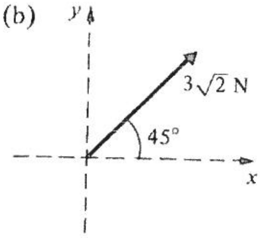


2. Express each of the following forces in the form $a\mathbf{i} + b\mathbf{j}$.

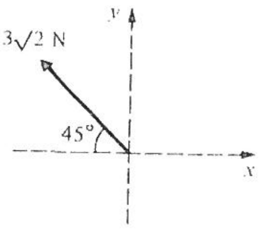
(a)



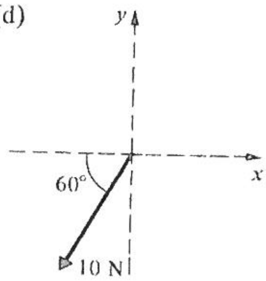
(b)



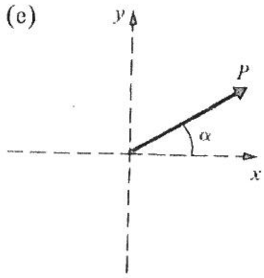
(c)



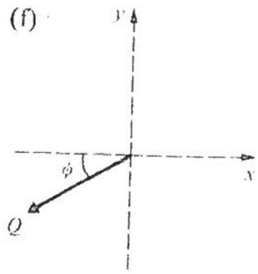
(d)



(e)

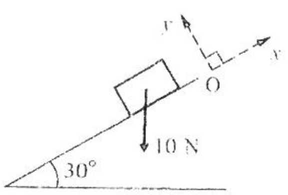


(f)

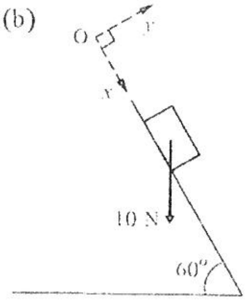


3. Each of the following diagrams shows a body of weight 10 N on an incline. In each case find the component of the weight of the body (i) in the Ox direction and (ii) in the Oy direction.

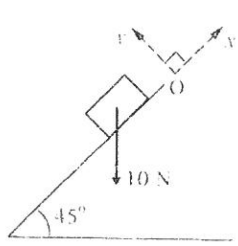
(a)



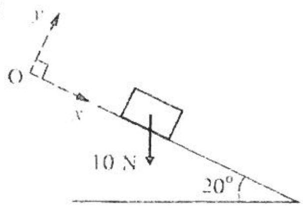
(b)



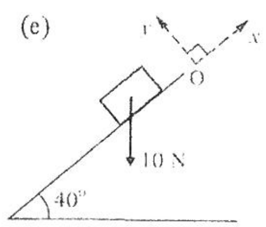
(c)



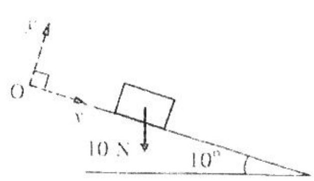
(d)



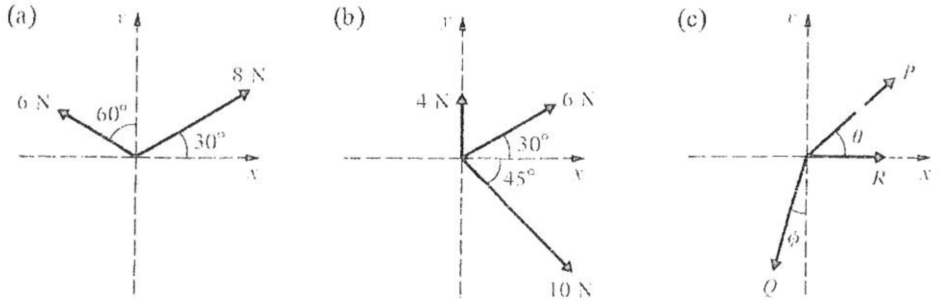
(e)



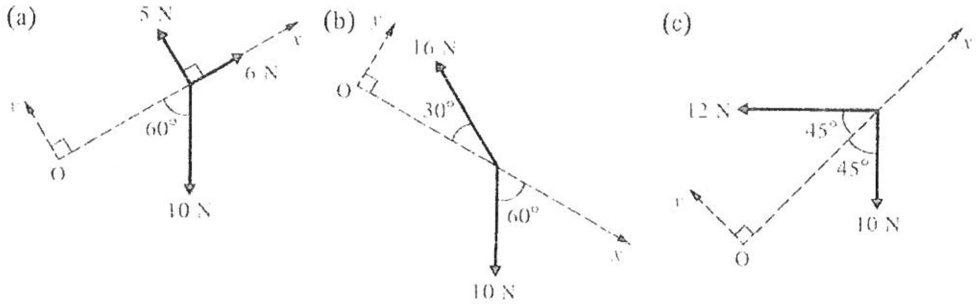
(f)

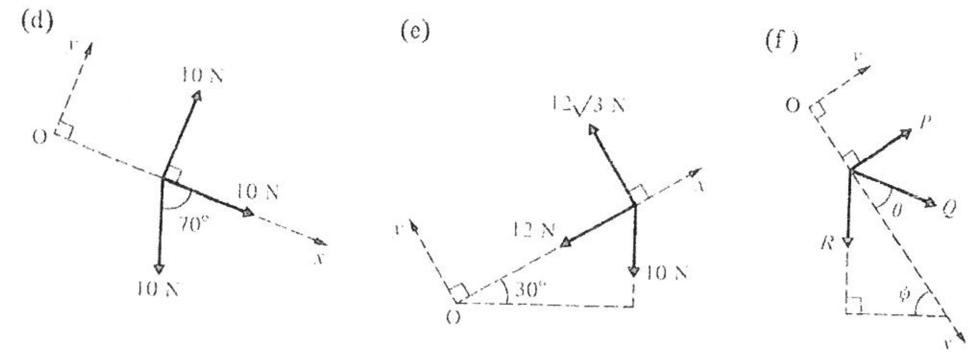


4. For each of the following systems of forces, find the sum of the components in the direction of (i) the x-axis and (ii) the y-axis.



5. For each of the following systems of forces, find the sum of the components (i) in the Ox direction and (ii) in the Oy direction.





ANSWERS

1. (a) (i) $4\sqrt{3}$ N (ii) 4 N (b) (i) 0 (ii) 10 N
   (c) (i) $-7{\cdot}66$ N (ii) $6{\cdot}43$ N (d) (i) $2\sqrt{2}$ N (ii) $-2\sqrt{2}$ N
   (e) (i) $\sqrt{3}$ N (ii) 3 N (f) (i) $P\cos\theta$ (ii) $P\sin\theta$
2. (a) $(8\mathbf{i} + 8\sqrt{3}\mathbf{j})$ N (b) $(3\mathbf{i} + 3\mathbf{j})$ N (c) $(-3\mathbf{i} + 3\mathbf{j})$ N (d) $(-5\mathbf{i} - 5\sqrt{3}\mathbf{j})$ N
   (e) $P\cos\alpha\mathbf{i} + P\sin\alpha\mathbf{j}$ (f) $-Q\cos\phi\mathbf{i} - Q\sin\phi\mathbf{j}$
3. (a) (i) -5 N (ii) $-5\sqrt{3}$ N (b) (i) $5\sqrt{3}$ N (ii) -5 N
   (c) (i) $-5\sqrt{2}$ N (ii) $-5\sqrt{2}$ N (d) (i) $3{\cdot}42$ N (ii) $-9{\cdot}40$ N
   (e) (i) $-6{\cdot}43$ N (ii) $-7{\cdot}66$ N (f) (i) $1{\cdot}74$ N (ii) $-9{\cdot}85$ N
4. (a) (i) $\sqrt{3}$ N (ii) 7 N (b) (i) $12{\cdot}3$ N (ii) $-0{\cdot}071$ N
   (c) (i) $P\cos\theta + R - Q\sin\phi$ (ii) $P\sin\theta - Q\cos\phi$
5. (a) (i) 1 N (ii) $-3{\cdot}66$ N (b) (i) $-8{\cdot}86$ N (ii) $-0{\cdot}66$ N
   (c) (i) $-11\sqrt{2}$ N (ii) $\sqrt{2}$ N (d) (i) $13{\cdot}4$ N (ii) $0{\cdot}60$ N
   (e) (i) -17 N (ii) $7\sqrt{3}$ N (f) (i) $R\sin\phi + Q\cos\theta$ (ii) $P + Q\sin\theta - R\cos\phi$

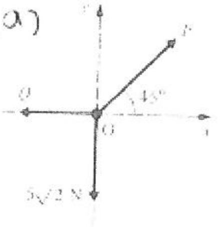
Statics

We have already seen how a system of forces acting on a particle in be resolved into the sums of vertical and horizontal (or parallel and perpendicular) forces. If the particle is to move, then these forces are resolved into one equivalent (resultant) force.

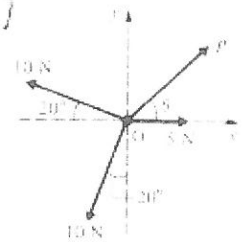
We will now consider what happens if the sum of these forces is zero. This will mean that the resultant force is zero and the particle remains at rest. In such a state, the particle is said to be in ***equilibrium***.

Eg1 Each of the diagrams below show a particle in equilibrium under the forces shown. In each case, by resolving in the directions *Ox* and *Oy*, find the unknown forces and angles:

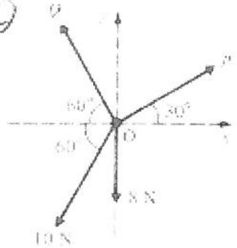
(a)



(b)

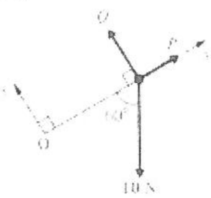


(c)

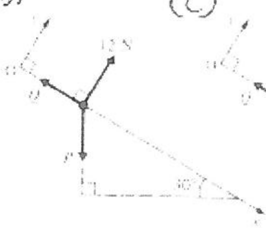


Eg2 Each of the diagrams below show a particle in equilibrium under the forces shown. In each case, by resolving in the directions *Ox* and *Oy*, find the unknown forces and angles:

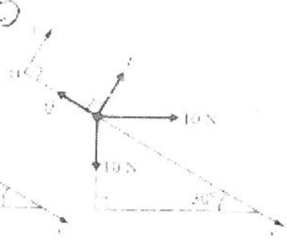
(a)



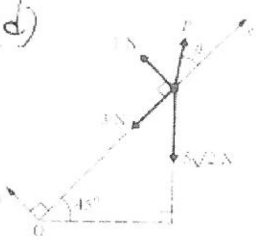
(b)



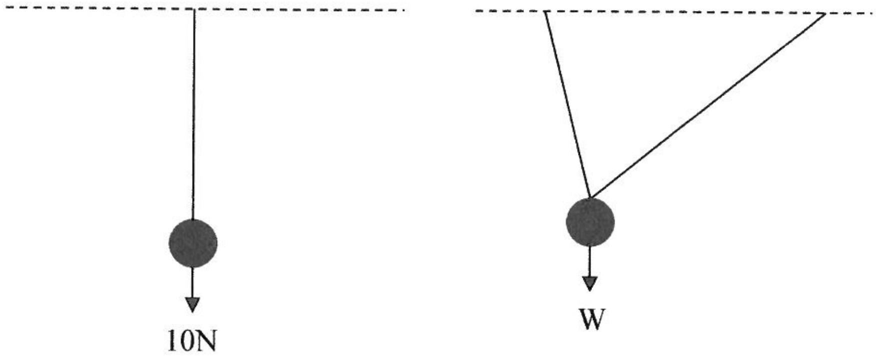
(c)



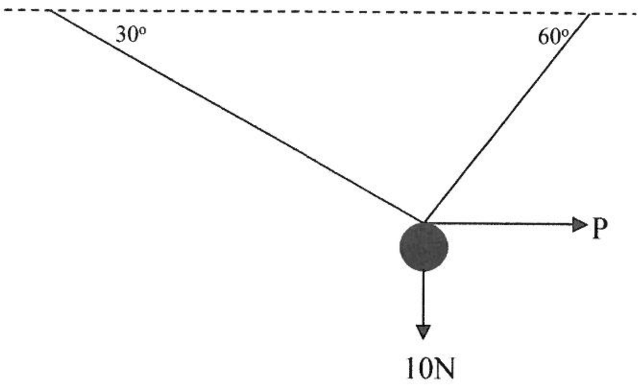
(d)



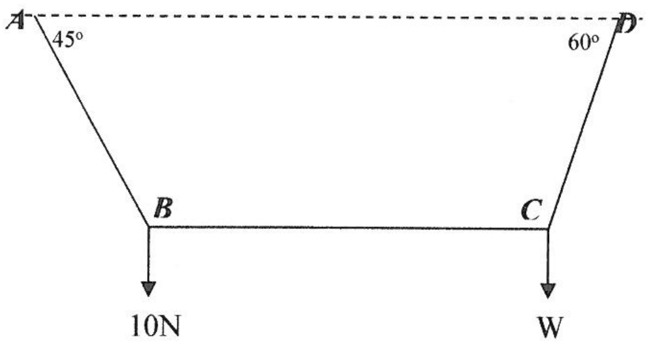
Each part of a system in equilibrium, is in equilibrium, ie



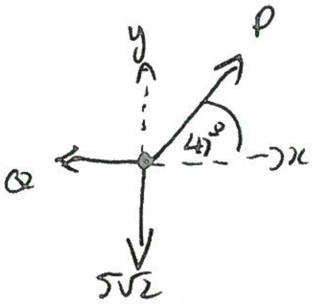
Eg3 A string is tied to two points on the same level and a ***smooth ring*** of weight 10N which can slide freely along the string is pulled by a horizontal force, P. For the position of equilibrium shown in the diagram, find P and the tension in the string.



Eg4 $ABCD$ is a string knotted at B and C. Find W and the tensions in AB, BC and CD.



Eg1 (a)



ΣF_x: $P\cos 45 - Q = 0$ — ①

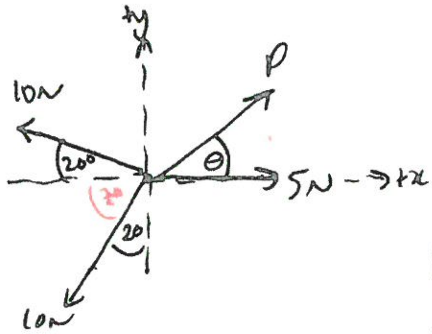
ΣF_y: $P\sin 45 - 5\sqrt{2} = 0$ — ②

$$\frac{P}{\sqrt{2}} = 5\sqrt{2}$$

$$P = 10\ N$$

in ① $Q = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}\ N$

(b)



ΣF_x: $P\cos\theta + 5 - 10\cos 20 - 10\cos 70 = 0$ —①

ΣF_y: $P\sin\theta + 10\sin 20 - 10\sin 70 = 0$ —②

From ① $P\cos\theta = 10\cos 20 + 10\cos 70 - 5$ —③

From ② $P\sin\theta = 10\sin 70 - 10\sin 20$ —④

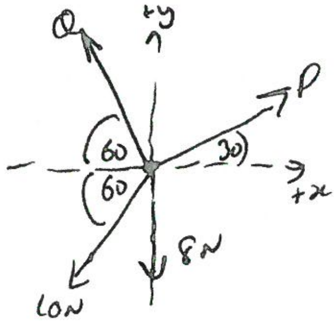
④ ÷ ③ $\frac{P\sin\theta}{P\cos\theta} = \tan\theta = \frac{10\sin 70 - 10\sin 20}{10\cos 20 + 10\cos 70 - 5}$

$$\tan\theta = 0.764\ldots$$

$$\theta = 37.4^\circ$$

in ④ $P = \frac{10\sin 70 - 10\sin 20}{\sin 37.4} = 9.84N$

(c)



ΣF_x: $P\cos 30 - Q\cos 60 - 10\cos 60 = 0$ — ①

ΣF_y: $P\sin 30 + Q\sin 60 - 10\sin 60 - 8 = 0$ — ②

From ① $\frac{P\sqrt{3}}{2} - \frac{Q}{2} - \frac{10}{2} = 0$

$$Q = P\sqrt{3} - 10 \quad \text{—③}$$

in ② $\frac{P}{2} + (P\sqrt{3} - 10)\cdot\frac{\sqrt{3}}{2} - \frac{10\cdot\sqrt{3}}{2} - 8 = 0$

$\times 2$ $P + 3P - 10\sqrt{3} - 10\sqrt{3} = 16$

$4P = 16 + 20\sqrt{3}$

$P = 4 + 5\sqrt{3} = 12.66N$

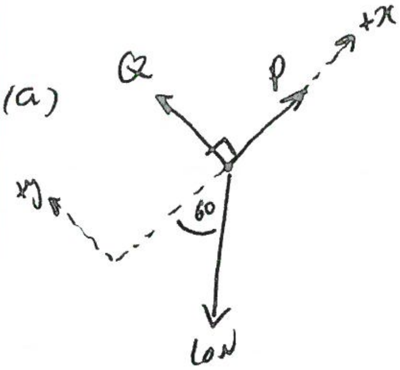
in ③ $Q = (4 + 5\sqrt{3})\sqrt{3} - 10$

$Q = 4\sqrt{3} + 15 - 10$

$Q = 4\sqrt{3} + 5\ N$

$= 11.9N$

Q2 (a)



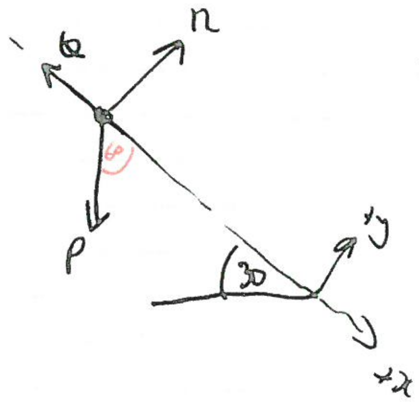
$\Sigma F_x: \; P - 10\cos 60 = 0$

$P = 5N$

$\Sigma F_y: \; Q - 10\sin 60 = 0$

$Q = 5\sqrt{3}\, N$

(b)



$\Sigma F_x: \; P\cos 60 - Q = 0$

$Q = \frac{P}{2} \quad — (1)$

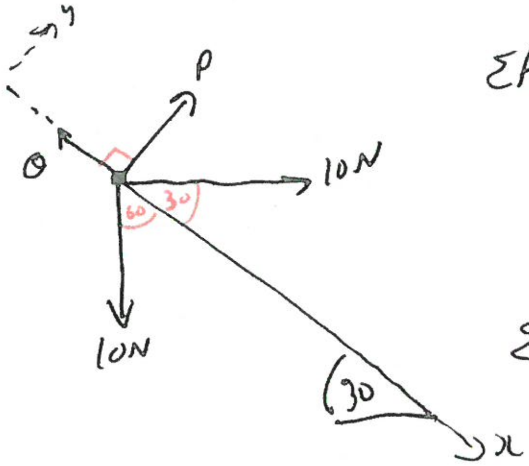
$\Sigma F_y: \; 12 - P\sin 60 = 0$

$12 = \frac{P\sqrt{3}}{2}$

$P = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3}\, N$

in (1) $\quad Q = 4\sqrt{3}\, N$

(c)



$\Sigma F_x: \; 10\cos 30 + 10\cos 60 - Q = 0$

$Q = \frac{10\sqrt{3}}{2} + \frac{10}{2} = 5\sqrt{3} + 5\, N$

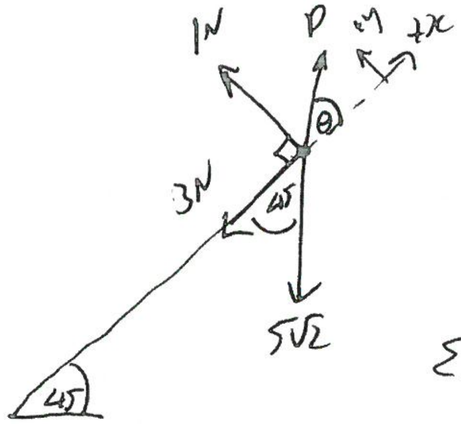
$= 13.66\, N$

$\Sigma F_y: \; P + 10\sin 30 - 10\sin 60 = 0$

$P = \frac{10\sqrt{3}}{2} - \frac{10}{2} = 5\sqrt{3} - 5\, N$

$= 3.66\, N$

Q2 (d)



$\Sigma F_x: \; P\cos\theta - 3 - 5\sqrt{2}\cos 45 = 0$

$P\cos\theta = 3 + 5\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 8$ — (1)

$\Sigma F_y: \; P\sin\theta + 1 - 5\sqrt{2}\sin 45 = 0$

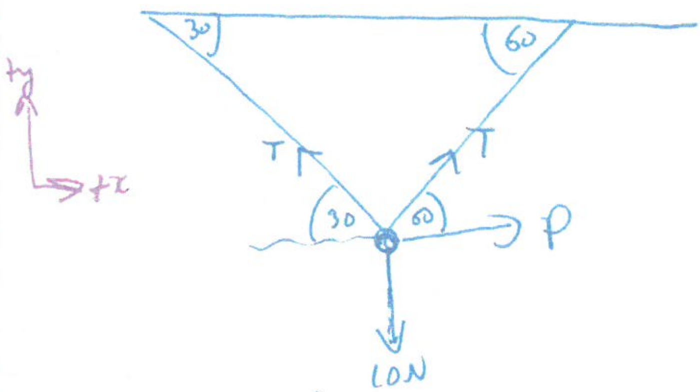
$P\sin\theta = 4$ — (2)

(2) ÷ (1) $\tan\theta = \frac{1}{2}$

$\theta = 26.6°$

in (2) $P = \frac{4}{\sin 26.6} = 8.94N$

Eg3/



Smooth ring
∴ tensions equal
throughout string.

System in equilibrium ∴ $\sum F_x = 0$
$\sum F_y = 0$

$\sum F_x$: $P + T\cos 60 - T\cos 30 = 0$ — (1)

$\sum F_y$: $T\sin 60 + T\sin 30 - 10 = 0$ — (2)

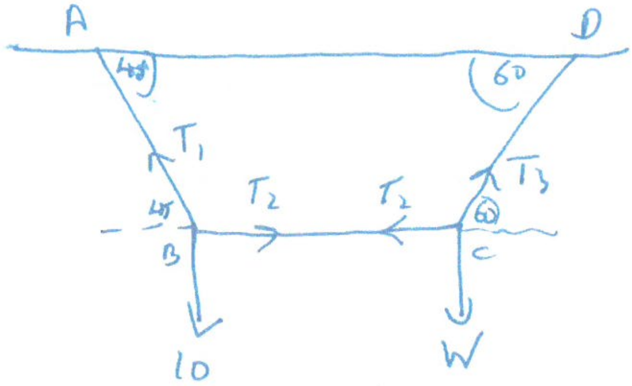
From (2) $\frac{T\sqrt{3}}{2} + \frac{T}{2} = 10$

$T(\sqrt{3}+1) = 20$

$T = \frac{20}{\sqrt{3}+1}$ N $\quad (7.32\text{N})$

In (1) $P = \frac{T\sqrt{3}}{2} - \frac{T}{2} = T\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right] = 2.68\text{N}$.

Eg4



Forces @ B : ΣF_x: $T_2 - T_1 \cos 45 = 0$ — ①

ΣF_y: $T_1 \sin 45 - 10 = 0$ — ②

$\frac{T_1}{\sqrt{2}} = 10$

$T_1 = 10\sqrt{2}$ N

in ① $T_2 = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10$ N

Forces @ C : ΣF_x : $T_3 \cos 60 - T_2 = 0$

$T_3 . \frac{1}{2} = 10$

$T_3 = 20$ N

ΣF_y: $T_3 \sin 60 - W = 0$

$20 . \frac{\sqrt{3}}{2} = W$

$W = 10\sqrt{3}$ N