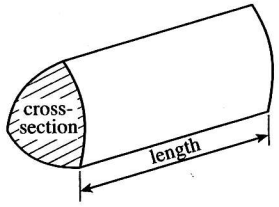
JUNE 08 PAPER 2 - SOLUTIONS

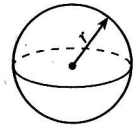
Formula List

Volume of prism = area of cross-section × length



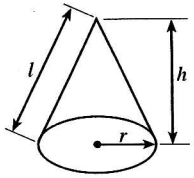
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

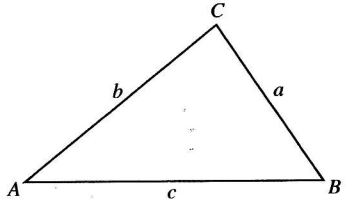


In any triangle ABC

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\frac{1}{2}ab\sin C$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

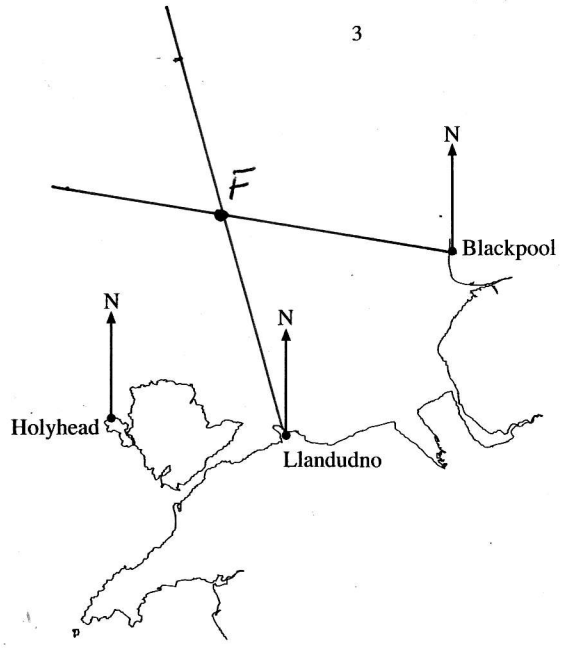
Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n$, having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad \textbf{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

1.



A fishing boat F is anchored in the bay.
The bearing of F from Llandudno is 345°.
The bearing of F from Blackpool is 280°.
By drawing suitable lines mark the position of F on the above diagram.

[3]

2. While on holiday in America, Katherine bought a camera for $470.
Colin, while on holiday in Spain, bought the same model camera for 324 euros.

The rates of exchange at the times the cameras were purchased were £1 = $1.88 and £1 = 1.44 euros.

Showing all your working, find out who purchased the camera for the lower price and write down the difference in the prices.

KATH pays 470 ÷ 1.88 = £250

Colin pays 324 ÷ 1.44 = £237.50

Price diff = £12.50

..... paid the lower price

The difference in the prices was .....

[5]

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Using the graph paper on the following page, draw the graph of the straight line $y = 2x - 3$ for values of x from -2 to $+3$.

[3]

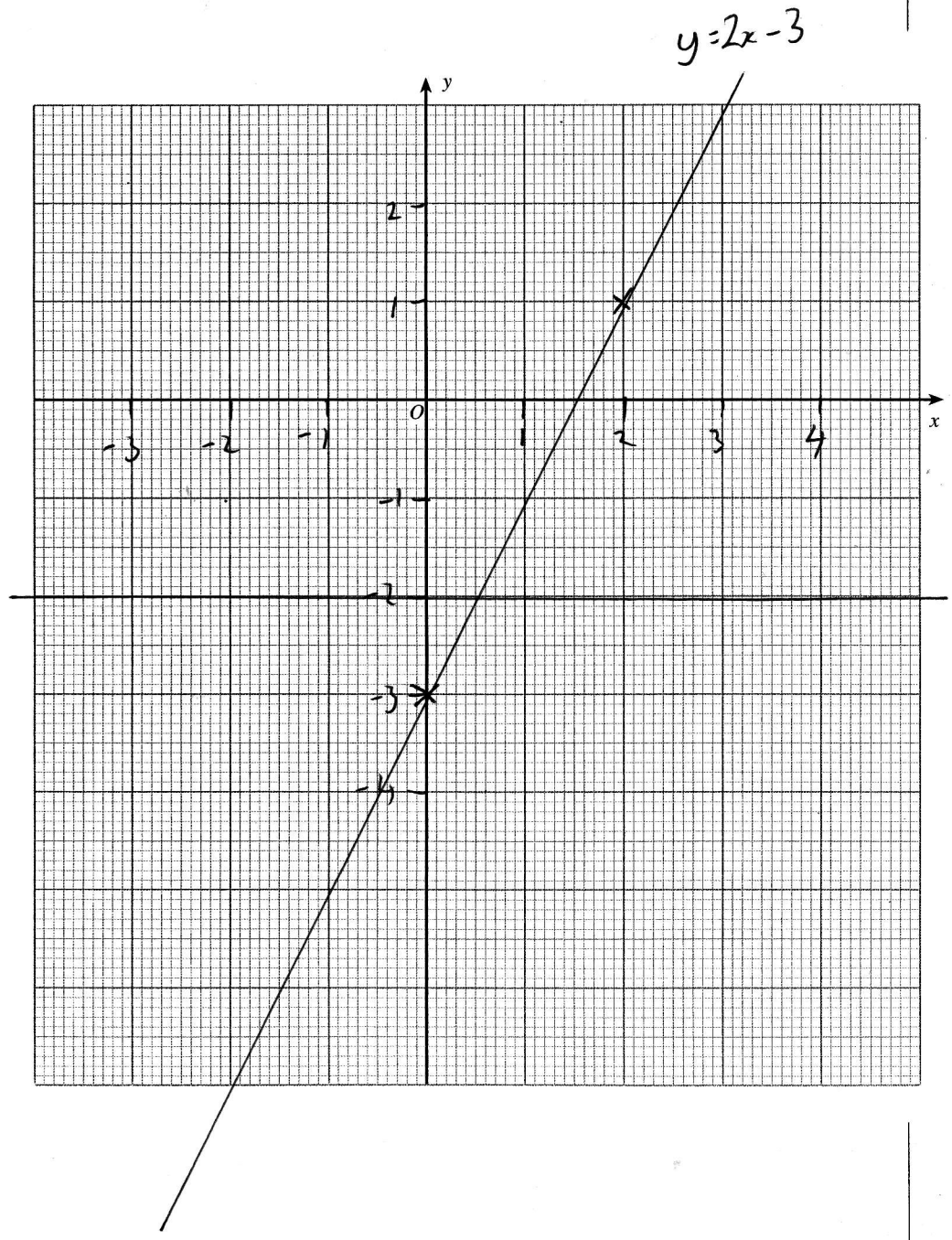
On the same graph paper draw the line $y = -2$.
Write down the coordinates of the point at which the straight line $y = 2x - 3$ cuts the line $y = -2$.

Coordinates are (0.5, −2)

[2]

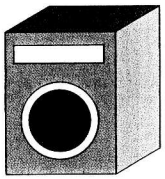
when $x=0$ $y=2\times0-3$
$y=-3$ $(0,-3)$

when $x=2$ $y=2\times2-3$
$=4-3$
$=1$ $(2,1)$



4.

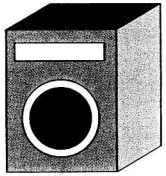
Sam's Electrical Shop



Turbo washing machine

£330 + VAT at 17·5%

Discount Electrics



Turbo washing machine

£408 including VAT

Ann decides to buy a new Turbo washing machine.
She notes the prices shown above, at **Sam's Electrical Shop** and at **Discount Electrics.**
Ann buys the machine in the shop offering the lower price.
In which shop does Ann buy the washing machine and how much cheaper is it in this shop than in the other shop?

@ Sam's 1.175 x 330 = £387.75

Sams is cheaper by £20.25

[4]

5. The diagram shows three points *A*, *B* and *C*, which are on level ground.
The point *B* is 55m due East of *A*.
The point *C* is due North of *A* and 95m from *B*.

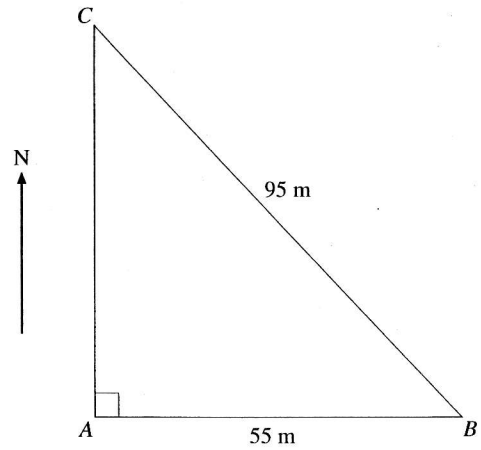


Diagram not drawn to scale.

Calculate the distance *AC*, giving your answer to an appropriate degree of accuracy.

$AC^2 = 95^2 - 55^2$

$AC^2 = 6000$

$AC = \sqrt{6000} = 77.5\text{m}$

[4]

Examiner only

6. The heights of 110 Christmas trees were measured to the nearest centimetre. The table below shows a grouped frequency distribution of the heights.

Height (h centimetres)	Number of Christmas trees
$191 \leqslant h \leqslant 197$	24
$198 \leqslant h \leqslant 204$	35
$205 \leqslant h \leqslant 211$	28
$212 \leqslant h \leqslant 218$	23

Find an estimate for the mean height of the Christmas trees.

$194 \times 24 = 4656$

$201 \times 35 = 7035$

$208 \times 28 = 5824$

$215 \times 23 = 4945$

$22460 \div 110 = 204.2$ cm

[4]

Examiner only

7. A solution of the equation $x^3 + 2x - 5 = 0$ lies between $x = 1$ and $x = 2$. Find this solution giving your answer correct to one decimal place.

$x = 1.5$ 1.375 too big

$x = 1.3$ -0.203 too small

$x = 1.4$ 0.544 too big

So x lies between 1.3 & 1.4

Test using $x = 1.35$ 0.160... — too big

$\therefore x = 1.3$ to 1dp

[4]

Examiner only

8. *(a)* Find the value of $\frac{9{\cdot}2 \times 23{\cdot}4}{8{\cdot}5 - 2{\cdot}6}$ correct to 2 decimal places.

36.49

[2]

(b) What percentage of 350 is 84?

$\frac{84}{350} \times 100 = 24\%$

[2]

(c) Linda's dog eats $\frac{2}{3}$ of a tin of food each day. What is the least number of tins needed to feed the dog for 7 days?

$\frac{2}{3} \times 7 = 4\frac{2}{3}$ So need 5 tins

[3]

Examiner only

9. Manon wants to carry out a survey in order to find out how often people visit a dentist.

(a) She wrote the following question.

How often do you visit a dentist?

☐ Not often ☐ Often ☐ Very often

What do you see wrong with this question and how would you improve it?

too vague

How many times in the last year have you visited a dentist

☐ never ☐ once ☐ twice ☐ more

[2]

(b) Another of the questions in her questionnaire was

Which age group are you in?

☐ 30 – 40 ☐ 40 – 50 ☐ 50 and above

Write down **two** criticisms of this question.

(i) overlap eg which box would you tick if you were 40?

(ii) Nowhere for under 30's to tick. Lots would tick 50 & above.

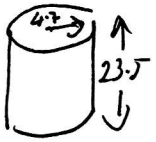
[2]

10. Theo invests £800 for 3 years at 5% per annum compound interest. How much money is in the account after 3 years?

$(1.05)^3 \times 800 = £926.40$

[3]

11. *(a)* A cylinder has a uniform circular cross section of radius of 4·7cm and a height of 23·5cm. Calculate the volume of the cylinder, stating the units of your answer.



Volume $= \pi \times 4.7^2 \times 23.5$

$= 1630.8 \text{ cm}^3$

[3]

(b)

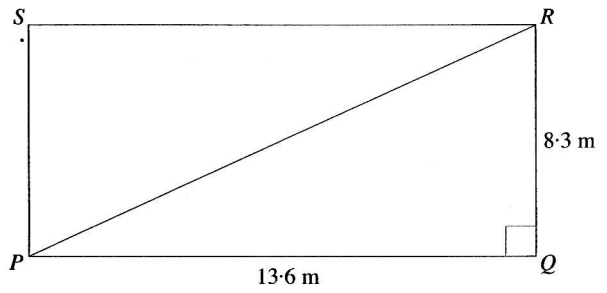


Diagram not drawn to scale.

PQRS is a rectangle in which *PQ* = 13·6 metres and *QR* = 8·3 metres. Calculate the length of the diagonal *PR*.

$PR^2 = 13.6^2 + 8.3^2$

$PR^2 = 253.85$

$PR = \sqrt{253.85} = 15.9 \text{ m}$

[3]

Examiner only

12. *(a)* Write **each** of the following numbers in standard form.

(i) 53 000 000 000

$53\,000\,000\,000 = 5{\cdot}3 \times 10^{10}$

[1]

(ii) 0·00000002

$0{\cdot}00000002 = 2 \times 10^{-8}$

[1]

(b) Find, in standard form, the value of:

$$(6{\cdot}8 \times 10^{-5}) \times (7{\cdot}3 \times 10^{-4})$$

4.964×10^{-8}

[2]

Examiner only

13. *(a)* Expand the following expression, simplifying your answer as far as possible.

$$(x + 2)(x - 6)$$

$x^2 - 6x + 2x - 12$

$x^2 - 4x - 12$

[2]

(b) Make m the subject of the formula

$$3(2m - t) = 2t + 7 .$$

$6m - 3t = 2t + 7$

$6m = 2t + 7 + 3t$

$6m = 5t + 7$

$m = \frac{5t + 7}{6}$

[3]

(c) Factorise $4x^2 - 4$.

$(2x - 2)(2x + 2)$

[2]

14. Solve the following equation.

$$\frac{3x - 7}{4} - \frac{4x + 5}{2} = \frac{3}{4}$$

$\times 4 \quad \frac{4(3x-7)}{4} - \frac{4(4x+5)}{2} = \frac{4 \times 3}{4}$

$3x - 7 - 2(4x + 5) = 3$

$3x - 7 - 8x - 10 = 3$

$-5x - 17 = 3$

$-5x = 20$

$x = \frac{20}{-5} = -4$

[4]

Turn over.

Examiner only

15. *(a)* A vertical post *AB* is 15 m from a point *C* on horizontal ground. The angle of elevation of the top of the post from the point *C* is 67°. Calculate the height of the post.

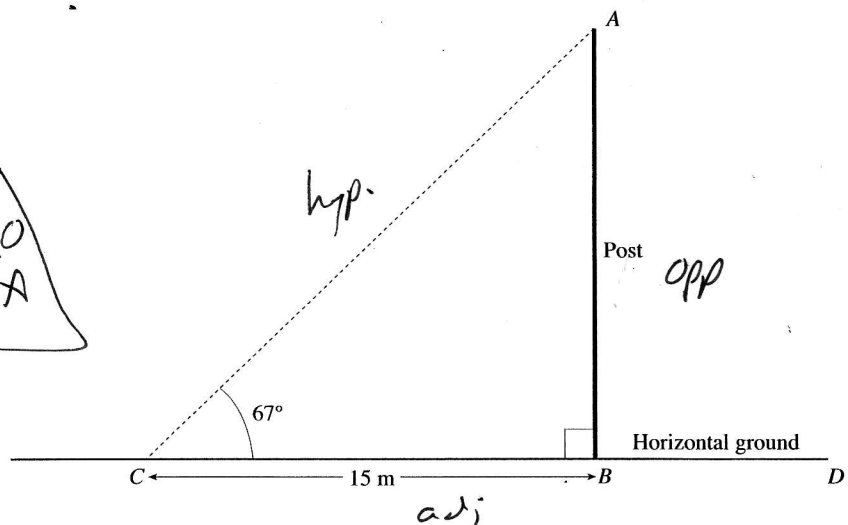


Diagram not drawn to scale.

$AB = \tan 67 \times 15$

$= 35.3\text{ m}$

[3]

(b) A ladder, 21 m long, is placed against a vertical wall. The foot of the ladder is 13 m from the wall on horizontal ground. Calculate the angle which the ladder makes with the horizontal.

$\cos\theta = \frac{13}{21}$

$\theta = \cos^{-1}\left(\frac{13}{21}\right) = 51.8°$

[3]

Examiner only

16. A European supermarket employs people from a number of countries.
The number of people employed by the company in each country is given in the following table.

Country	Number of employees
Germany	12355
France	8340
Spain	6860
Italy	4100
United Kingdom	3045

The company is organising a conference and decides to invite a total of 45 employees to represent the views of the entire workforce.

Use a stratified sampling method to calculate how many people from each country should be invited to the conference.

Total Employees = 34700

Germany = 12355/34700 × 45 = 16.0 → 16
France = 8340/34700 × 45 = 10.8 → 11
Spain = 6860/34700 × 45 = 8.9 → 9
Italy = 4100/34700 × 45 = 5.3 → 5
UK = 3045/34700 × 45 = 3.9 → 4

45 ✓

[4]

Turn over.

Examiner only

17. Given that y is inversely proportional to x^2, and that $y = 2$ when $x = 15$,

(a) find an expression for y in terms of x,

$y \propto \frac{1}{x^2}$ $\quad k = 2 \times 15^2 = 450$

$y = \frac{k}{x^2}$ $\quad \therefore y = \frac{450}{x^2}$

$2 = \frac{k}{15^2}$

[3]

(b) calculate y when $x = 10$.

$y = \frac{450}{10^2} = \frac{450}{100} = 4.5$

[1]

Examiner only

18. On the graph paper provided, draw the region which satisfies all of the following inequalities.

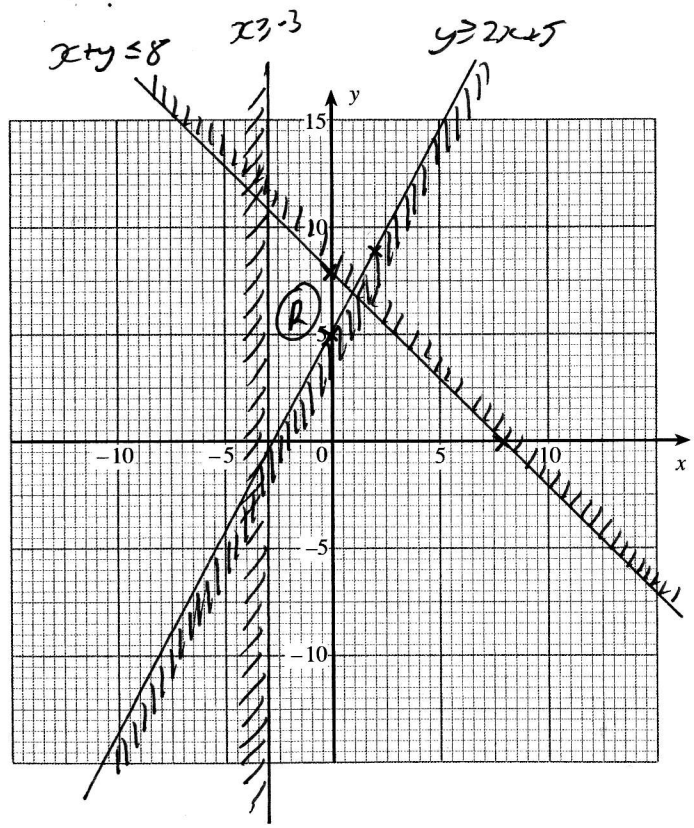
$$x + y \leqslant 8$$
$$y \geqslant 2x + 5$$
$$x \geqslant -3$$

Make sure that you clearly indicate the region that represents your answer.

$x+y=8$ when $x=0$ $y=8$ $(0,8)$
when $y=0$ $x=8$ $(8,0)$

$y=2x+5$ when $x=0$ $y=5$ $(0,5)$
when $x=2$ $y=2\times2+5=9$ $(2,9)$

[3]



Turn over.

Examiner only

19. *(a)* Factorise the expression $8x^2 - 26x - 7$ and hence solve the equation $8x^2 - 26x - 7 = 0$.

$56 \quad -28, +2$

$8x^2 + 2x - 28x - 7$

$2x(4x+1) - 7(4x+1)$

$(2x-7)(4x+1) = 0$

either $2x - 7 = 0$ or $4x + 1 = 0$

$x = \frac{7}{2}$ $\quad x = \frac{-1}{4}$

[3]

(b) Use the formula method to solve the equation $3x^2 + 6x - 11 = 0$, giving solutions correct to two decimal places.

$a = 3, \; b = 6, \; c = -11$

$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times -11}}{2 \times 3} = \frac{-6 \pm \sqrt{36 + 132}}{6}$

$x = \frac{-6 \pm \sqrt{168}}{6}$

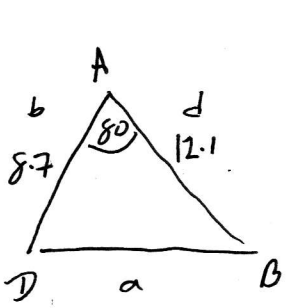
$\therefore$ either $x = \frac{-6 + \sqrt{168}}{6} = 1.16$

or $x = \frac{-6 - \sqrt{168}}{6} = -3.16$

[3]

Examiner only

20. In the diagram below $AD = 8{\cdot}7$ cm, $AB = 12{\cdot}1$ cm, $CD = 6{\cdot}3$ cm, $C\hat{D}B = 25°$ and $D\hat{A}B = 80°$.



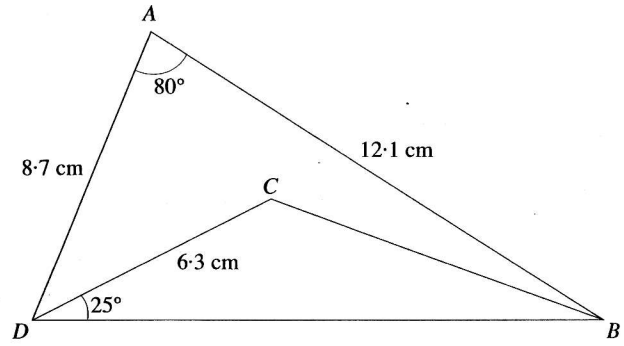


Diagram not drawn to scale.

(a) Calculate the length of *BD*.

Cosine rule $a^2 = b^2 + d^2 - 2bd\cos A$

$a^2 = 8{\cdot}7^2 + 12{\cdot}1^2 - (2 \times 8{\cdot}7 \times 12{\cdot}1 \times \cos 80)$

$a^2 = 222{\cdot}1 - 36{\cdot}56$

$a^2 = 185{\cdot}54$

$a = \sqrt{185{\cdot}54} = 13{\cdot}6$

[3]

(b) Calculate the area of the quadrilateral *ABCD*.

Area of quad $ABCD$ = Area of ΔABD − Area of ΔBCD

$= \left(\frac{1}{2} \times 8{\cdot}7 \times 12{\cdot}1 \times \sin 80\right) - \left(\frac{1}{2} \times 6{\cdot}3 \times 13{\cdot}6 \times \sin 25\right)$

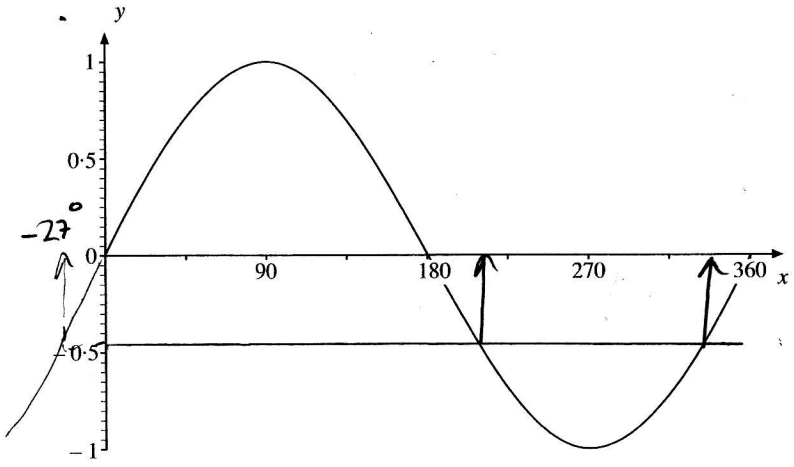
$= 51{\cdot}83 - 18{\cdot}10$

$= 33{\cdot}7 \text{ cm}^2$

[3]

Examiner only

21. The diagram below shows the sketch of $y = \sin x$ for values of x from 0° to 360°.



Find all solutions of the following equation in the range 0° to 360°.

$$\sin x = -0{\cdot}454$$

From calc $x = \sin^{-1}(-0.454) = -27^\circ$

From symmetry on graph

$x = 180 + 27 = 207^\circ$

$x = 360 - 27 = 333^\circ$

[2]

Examiner only

22. Solve the equation $\frac{n}{n+3} + \frac{7}{n+4} = 1$.

$\times (n+3)(n+4)$

$\frac{(n+3)(n+4)}{(n+3)} \times n + \frac{(n+3)(n+4)}{(n+4)} \times 7 = (n+3)(n+4) \times 1$

$n(n+4) + 7(n+3) = (n+3)(n+4)$

$n^2 + 4n + 7n + 21 = n^2 + 4n + 3n + 12$

$n^2 + 11n + 21 - n^2 - 7n - 12 = 0$

$4n + 9 = 0$

$n = -\frac{9}{4}$

[5]