Mark Scheme January 2009

GCE

GCE Mathematics (8371/8373,9371/9373)

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January 2009

Publications Code UA020941

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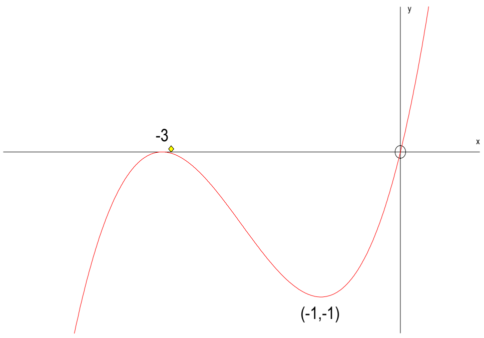
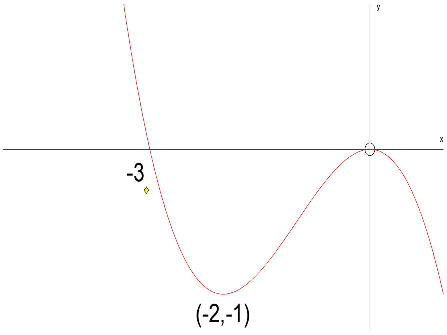
January 2009
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
1 (a)	**5** $\quad (\pm 5$ is B0)	B1 (1)
(b)	$\dfrac{1}{(\text{their } 5)^2}$ or $\left(\dfrac{1}{\text{their } 5}\right)^2$	M1
	$= \dfrac{1}{25}$ **or 0.04** $\quad (\pm\dfrac{1}{25}$ is A0)	A1 (2) **[3]**
(b)	M1 follow through their value of 5. Must have reciprocal and square. 5^{-2} is <u>not</u> sufficient to score this mark, unless $\dfrac{1}{5^2}$ follows this. A negative introduced at any stage can score the M1 but not the A1, e.g. $125^{-2/3} = \left(-\dfrac{1}{5}\right)^2 = \dfrac{1}{25}$ scores M1 A0 $125^{-2/3} = -\left(\dfrac{1}{5}\right)^2 = -\dfrac{1}{25}$ scores M1 A0. Correct answer with no working scores both marks. <u>Alternative</u>: $\dfrac{1}{\sqrt[3]{125^2}}$ or $\dfrac{1}{(125^2)^{1/3}}$ M1 (reciprocal and the correct number squared) $\left(= \dfrac{1}{\sqrt[3]{15625}}\right)$ $= \dfrac{1}{25}$ A1	

Question Number	Scheme	Marks
2	$\left(I=\right)\frac{12}{6}x^6-\frac{8}{4}x^4+3x+c$ $=2x^6-2x^4+3x+c$	M1 A1A1A1 **[4]**
	M1 for an attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the + c (seen at some stage), even if no other terms are correct. 1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x + c$ (or $3x+k$, etc., any appropriate letter can be used as the constant) Allow $3x^1+c$, but <u>not</u> $\frac{3x^1}{1}+c$. Note that the A marks can be awarded at separate stages, e.g. $\frac{12}{6}x^6-2x^4+3x$ scores 2nd A1 $\frac{12}{6}x^6-2x^4+3x+c$ scores 3rd A1 $2x^6-2x^4+3x$ scores 1st A1 (even though the c has now been lost). Remember that all the A marks are dependent on the M mark. If applicable, isw (ignore subsequent working) after a correct answer is seen. Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6-2x^4+3x+c\,\mathrm{d}x$.	

Question Number	Scheme	Marks
3	$\sqrt{7}^2 + 2\sqrt{7} - 2\sqrt{7} - 2^2$, or $7 - 4$ or an exact equivalent such as $\sqrt{49} - 2^2$ $= \mathbf{3}$	M1 A1 **[2]**
	M1 for an expanded expression. At worst, there can be <u>one wrong term</u> and <u>one wrong sign</u>, or <u>two wrong signs</u>. e .g. $7 + 2\sqrt{7} - 2\sqrt{7} - 2$ is M1 (one wrong term -2) $7 + 2\sqrt{7} + 2\sqrt{7} + 4$ is M1 (two wrong signs $+2\sqrt{7}$ and $+4$) $7 + 2\sqrt{7} + 2\sqrt{7} + 2$ is M1 (one wrong term $+2$, one wrong sign $+2\sqrt{7}$) $\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} + 4$ is M1 (one wrong term $\sqrt{7}$, one wrong sign $+4$) $\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} - 2$ is M0 (two wrong terms $\sqrt{7}$ and -2) $7 + \sqrt{14} - \sqrt{14} - 4$ is M0 (two wrong terms $\sqrt{14}$ and $-\sqrt{14}$) If only 2 terms are given, they must be correct, i.e. (7 – 4) or an equivalent unsimplified version to score M1. The terms can be seen <u>separately</u> for the M1. Correct answer with <u>no working</u> scores both marks.	

Question Number	Scheme	Marks
4	$\left(\mathrm{f}(x)=\right)\frac{3x^3}{3}-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x\left(+c\right)$ $= x^3-2x^{\frac{3}{2}}-7x \quad (+c)$ $\mathrm{f}(4)=22 \quad \Rightarrow \quad 22=64-16-28+c$ $\boldsymbol{c=2}$	M1 A1A1 M1 A1cso (5) **[5]**
	1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The x term is insufficient for this mark and similarly the + c is insufficient. 1st A1 for $\frac{3}{3}x^3$ or $-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form) 2nd A1 for all three x terms correct and simplified… (the simplification may be seen later). The + c is not required for this mark. Allow $-7x^1$, but <ins>not</ins> $-\frac{7x^1}{1}$. 2nd M1 for an attempt to use $x = 4$ <ins>and</ins> $y = 22$ in a changed function (even if differentiated) to form an equation in c. 3rd A1 for $c = 2$ with no earlier incorrect work (a final expression for f(x) is not required).	

Question Number	Scheme	Marks
5 (a)		
	Shape  , touching the x-axis at its maximum.	M1
	Through (0,0) & -3 marked on x-axis, or $(-3,0)$ seen. Allow $(0,-3)$ if marked on the x-axis. Marked in the correct place, but 3, is A0.	A1
	Min at $(-1,-1)$	A1 (3)
(b)		
	Correct shape  (top left - bottom right)	B1
	Through -3 and max at (0, 0). Marked in the correct place, but 3, is B0.	B1
	Min at $(-2,-1)$	B1 (3) **[6]**
(a)	M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 1st A1 for curve passing through -3 and the origin. Max at $(-3,0)$ 2nd A1 for minimum at $(-1,-1)$. Can simply be indicated on sketch.	
(b)	1st B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for curve passing through $(-3,0)$ having a max at (0, 0) and no other max. 3rd B1 for minimum at $(-2,-1)$ and no other minimum. If in correct quadrant but labelled, e.g. $(-2,1)$, this is B0. In each part the (0, 0) does not need to be written to score the second mark… having the curve pass through the origin is sufficient. The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, $(-2,-1)$ marked in the wrong quadrant). The mark for the minimum is not given for the coordinates just marked on the axes unless these are clearly linked to the minimum by vertical and horizontal lines.	

Question Number	Scheme	Marks
6 (a) (b)	$2x^{3/2}$ or $p=\frac{3}{2}$ (<u>Not</u> $2x\sqrt{x}$) $-x$ or $-x^1$ or $q=1$ $\left(\frac{dy}{dx}=\right)$ $20x^3+2\times\frac{3}{2}x^{1/2}-1$ $=\underline{20x^3+3x^{\frac{1}{2}}-1}$	B1 B1 (2) M1 A1A1ftA1ft (4) **[6]**
(a) (b)	1st B1 for $p = 1.5$ or exact equivalent 2nd B1 for $q = 1$ M1 for an attempt to differentiate $x^n \to x^{n-1}$ (for any of the 4 terms) 1st A1 for $20x^3$ (the -3 must 'disappear') 2nd A1ft for $3x^{\frac{1}{2}}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a <u>fraction</u>, and the coefficient must be simplified if necessary. 3rd A1ft for -1 (<u>not</u> the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1). If ft is applied, the coefficient must be simplified if necessary. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +). If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Multiplying</u> by $\sqrt{x}$: (assuming this is a restart) e.g. $y=5x^4\sqrt{x}-3\sqrt{x}+2x^2-x^{3/2}$ $\left(\frac{dy}{dx}=\right)\frac{45}{2}x^{7/2}-\frac{3}{2}x^{-1/2}+4x-\frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft. <u>Extra term</u> included: This invalidates the final mark. e.g. $y=5x^4-3+2x^2-x^{3/2}-x^{1/2}$ $\left(\frac{dy}{dx}=\right)20x^3+4x-\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0. <u>Numerator and denominator differentiated separately</u>: For this, neither of the last two (ft) marks should be awarded. <u>Quotient/product rule</u>: Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)	

Question Number	Scheme	Marks
7 (a)	$b^2-4ac>0 \Rightarrow \quad 16-4k(5-k)>0$ or equiv., e.g. $16>4k(5-k)$	M1A1
	So $k^2-5k+4>0$ (Allow any order of terms, e.g. $4-5k+k^2>0$) (*)	A1cso (3)
(b)	<u>Critical Values</u> $(k-4)(k-1)=0 \quad k=\ldots.$	M1
	$k = 1$ or 4	A1
	Choosing "outside" region	M1
	<u>$\boldsymbol{k<1}$ or $\boldsymbol{k>4}$</u>	A1 (4) **[7]**

For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.

(a)

M1 for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).

If the formula b^2-4ac is seen, at least 2 of a, b and c must be correct.

If the formula b^2-4ac is <u>not</u> seen, all 3 (a, b and c) must be correct.

This mark can still be scored if substitution in b^2-4ac is within the quadratic formula.

This mark can also be scored by comparing b^2 and $4ac$ (with substitution).

However, use of b^2+4ac is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2-4ac>0$ or 'discriminant positive'.
Condone a bracketing slip, e.g. $16-4\times k\times 5-k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen.
Condone a bracketing slip if otherwise correct and convincing.

<u>Using</u> $\sqrt{b^2-4ac}>0$:
Only available mark is the first M1 (unless recovery is seen).

(b)

1st M1 for attempt to solve an appropriate 3TQ

1st A1 for both $k = 1$ and 4 (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$). **

2nd M1 for choosing the "outside" region. A diagram or table alone is not sufficient.
Follow through their values of k.
The set of values must be 'narrowed down' to score this M mark… listing everything $k<1$, $1<k<4$, $k>4$ is M0.

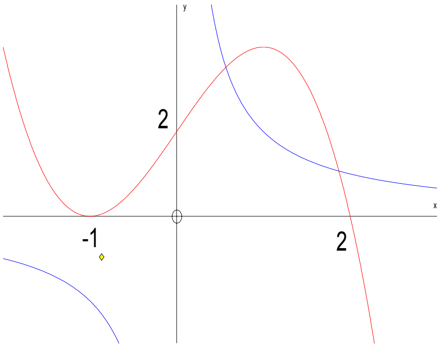
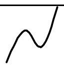
2nd A1 for correct answer only, condone "$k<1$, $k>4$" and even "$k<1$ and $k>4$",
but "$1>k>4$" is A0.

** Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks.

<u>Seeing 1 and 4 used as critical values</u> gives the first M1 A1 by implication.

In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of ≤ (or ≥) in the final answer loses the final mark.

Question Number	Scheme	Marks
8 (a)	$(a=)\ (1+1)^2(2-1)=\underline{\mathbf{4}}$ (1, 4) or $y=4$ is also acceptable	B1 (1)
(b)	 (i) Shape  or  anywhere	B1
	Min at $(-1,0)$ … can be -1 on x-axis. Allow $(0,-1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0.	B1
	(2, 0) and (0, 2) can be 2 on axes	B1
	(ii) Top branch in 1st quadrant with 2 intersections	B1
	Bottom branch in 3rd quadrant (ignore any intersections)	B1 (5)
(c)	(2 intersections therefore) $\underline{\mathbf{2}}$ (roots)	B1ft (1) **[7]**
(b)	1st B1 for shape  or  Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for minimum at $(-1,0)$ (even if there is an additional minimum point shown) 3rd B1 for the sketch meeting axes at (2, 0) and (0, 2). They can simply mark 2 on the axes. The marks for minimum and intersections are dependent upon having a sketch. Answers on the diagram for min. and intersections take precedence over answers seen elsewhere. 4th B1 for the branch fully within 1st quadrant having 2 intersections with (not just ‘touching’) the other curve. The curve can ‘touch’ the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join. Allow, for example, shapes like these: 5th B1 for a branch fully in the 3rd quadrant (ignore any intersections with the other curve for this branch). The curve can ‘touch’ the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.	
(c)	B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0. The answer 2 <u>incompatible with the sketch</u> is B0 (ignore any algebra seen). If the sketch shows the 2 correct intersections <u>and</u>, for example, one other intersection, the answer here should be 3, not 2, to score the mark.	

Question Number	Scheme	Marks
9 (a)	$a+17d=25$ or equiv. (for 1st B1), $a+20d=32.5$ or equiv. (for 2nd B1),	B1, B1 (2)
(b)	Solving (Subtract) $3d=7.5$ so $\boldsymbol{d}$ **= 2.5** $a=32.5-20\times 2.5$ so $\boldsymbol{a}$ **= -17.5** (*)	M1 A1cso (2)
(c)	$2750=\frac{n}{2}\left[-35+\frac{5}{2}(n-1)\right]$	M1A1ft
	$\{\ 4\times 2750=n(5n-75)\ \}$	
	$4\times 550=n(n-15)$	M1
	$n^2-15n=55\times 40$ (*)	A1cso (4)
(d)	$n^2-15n-55\times 40=0$ or $n^2-15n-2200=0$	M1
	$(n-55)(n+40)=0$ $n=\ldots$	M1
	$\boldsymbol{n}$ **= 55** (ignore - 40)	A1 (3) **[11]**
	Mark parts (a) and (b) as 'one part', ignoring labelling.	
(a)	Alternative: 1st B1: $d=2.5$ or equiv. or $d=\frac{32.5-25}{3}$. No method required, but $a=-17.5$ must not be assumed. 2nd B1: Either $a+17d=25$ or $a+20d=32.5$ seen, or used with a value of *d*… or for 'listing terms' or similar methods, 'counting back' 17 (or 20) terms.	
(b)	M1: In main scheme: for a full method (allow numerical or sign slips) leading to solution for *d* or *a* without assuming $a=-17.5$ In alternative scheme: for using a *d* value to find a value for *a*. A1: Finding correct values for both *a* and *d* (allowing equiv. fractions such as $d=\frac{15}{6}$), with no incorrect working seen.	
(c)	In the main scheme, if the given *a* is used to find *d* from one of the equations, then allow M1A1 if both values are checked in the 2nd equation. 1st M1 for attempt to form equation with correct S_n formula and 2750, with values of *a* and *d*. 1st A1ft for a correct equation following through their *d*. 2nd M1 for expanding and simplifying to a 3 term quadratic. 2nd A1 for correct working leading to printed result (no incorrect working seen).	
(d)	1st M1 forming the correct 3TQ = 0 . Can condone missing "= 0" but all terms must be on one side. First M1 can be implied (perhaps seen in (c), but there must be an attempt at (d) for it to be scored). 2nd M1 for attempt to solve 3TQ, by factorisation, formula or completing the square (see general marking principles at end of scheme). If this mark is earned for the 'completing the square' method or if the factors are written down directly, the 1st M1 is given by implication. A1 for *n* = 55 dependent on both Ms. Ignore – 40 if seen. No working or 'trial and improvement' methods in (d) score all 3 marks for the answer 55, otherwise no marks.	

Question Number	Scheme	Marks
10 (a)	$y-5=-\frac{1}{2}(x-2)$ or equivalent, e.g. $\frac{y-5}{x-2}=-\frac{1}{2}$, $y=-\frac{1}{2}x+6$	M1A1, A1cao (3)
(b)	$x=-2\Rightarrow\ y=-\frac{1}{2}(-2)+6\ =\ 7$ (therefore *B* lies on the line) (or equivalent verification methods)	B1 (1)
(c)	$\left(AB^2=\right)\ (2--2)^2+(7-5)^2$, $=16+4=20$, $AB=\sqrt{20}=2\sqrt{5}$	M1, A1, A1 (3)
(d)	*C* is $(p,\ -\frac{1}{2}p+6)$, so $AC^2=(p-2)^2+\left(-\frac{1}{2}p+6-5\right)^2$ Therefore $25=p^2-4p+4+\frac{1}{4}p^2-p+1$ $25=1.25p^2-5p+5$ or $100=5p^2-20p+20$ (or better, RHS simplified to 3 terms) Leading to: $0=p^2-4p-16$ (*)	M1 M1 A1 A1cso (4) **[11]**
(a)	M1 A1 The version in the scheme above can be written down directly (for 2 marks), and M1 A0 can be allowed if there is just one slip (sign or number). If the 5 and 2 are the wrong way round the M mark can still be given if a correct formula (e.g. $y-y_1=m(x-x_1)$) is seen, otherwise M0. If (2, 5) is substituted into $y=mx+c$ to find *c*, the M mark is for attempting this and the 1st A mark is for *c* = 6. Correct answer without working or from a sketch scores full marks.	
(b)	A conclusion/comment is not required, except when the method used is to establish that the line through $(-2,7)$ with gradient $-\frac{1}{2}$ has the same eqn. as found in part (a), or to establish that the line through $(-2,7)$ and $(2,5)$ has gradient $-\frac{1}{2}$. In these cases a comment 'same equation' or 'same gradient' or 'therefore on same line' is sufficient.	
(c)	M1 for attempting AB^2 or *AB*. Allow one slip (sign or number) <u>inside</u> a bracket, i.e. do <u>not</u> allow $(2--2)^2-(7-5)^2$. 1st A1 for 20 (condone bracketing slips such as $-2^2=4$) 2nd A1 for $2\sqrt{5}$ or *k* = 2 (Ignore ± here).	
(d)	1st M1 for $(p-2)^2+(\text{linear function of } p)^2$. The linear function may be unsimplified but must be equivalent to $ap+b,\ a\neq 0,\ b\neq 0$. 2nd M1 (dependent on 1st M) for forming an equation in *p* (using 25 or 5) and attempting (perhaps not very well) to multiply out both brackets. 1st A1 for collecting like *p* terms and having a correct expression. 2nd A1 for correct work leading to printed answer. <u>Alternative, using the result</u>: Solve the quadratic $\left(p=2\pm 2\sqrt{5}\right)$ and use one or both of the two solutions to find the length of AC^2 or $C_1C_2{}^2$: e.g. $AC^2=\left(2+2\sqrt{5}-2\right)^2+\left(5-\sqrt{5}-5\right)^2$ scores 1st M1, and 1st A1 if fully correct. Finding the length of *AC* or AC^2 for both values of *p*, or finding C_1C_2 with some evidence of halving (or intending to halve) scores the 2nd M1. Getting *AC* = 5 for both values of *p*, or showing $\frac{1}{2}C_1C_2=5$ scores the 2nd A1 (cso).	

Question Number	Scheme	Marks
11 (a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2\Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks could be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2)\to \ y=1-2x$ (*)	M1 A1cso
		(6)
(b)	Gradient of normal = $\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1
		(3)
(c)	$(A:)\ \frac{1}{2}$, $(B:)\ 8$	B1, B1
	Area of triangle is: $\frac{1}{2}(x_B \pm x_A)\times y_P$ with values for all of x_B, x_A and y_P	M1
	$\frac{1}{2}\left(8-\frac{1}{2}\right)\times 3 = \frac{45}{4}$ or 11.25	A1 (4) **[13]**
(a)	1st M1 for 4 or $8x^{-2}$ (ignore the signs). 1st A1 for both terms correct (including signs). 2nd M1 for substituting $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ (must be different from their y) B1 for $y_P=-3$, but not if clearly found from the given equation of the tangent. 3rd M1 for attempt to find the equation of tangent at *P*, follow through their *m* and y_P. Apply general principles for straight line equations (see end of scheme). NO DIFFERENTIATION ATTEMPTED: Just assuming $m=-2$ at this stage is M0 2nd A1cso for correct work leading to printed answer (allow equivalents with 2*x*, *y*, and 1 terms… such as $2x+y-1=0$).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their *m*, but if $m\neq-2$ there must be clear evidence that the *m* is thought to be the gradient of the tangent. M1 for an attempt to find normal at *P* using their changed gradient and their y_P. Apply general principles for straight line equations (see end of scheme). A1 for any correct form as specified above (correct answer only).	
(c)	1st B1 for $\frac{1}{2}$ and 2nd B1 for 8. M1 for a full method for the area of triangle *ABP*. Follow through their x_A, x_B and their y_P, but the mark is to be awarded 'generously', condoning sign errors.. The final answer must be positive for A1, with negatives in the working condoned. Determinant: Area = $\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=\frac{1}{2}\begin{vmatrix}2 & -3 & 1\\ 0.5 & 0 & 1\\ 8 & 0 & 1\end{vmatrix}=\ldots$ (Attempt to multiply out required for M1) Alternative: $AP=\sqrt{(2-0.5)^2+(-3)^2}$, $BP=\sqrt{(2-8)^2+(-3)^2}$, Area = $\frac{1}{2}AP\times BP=\ldots$ M1 Intersections with *y*-axis instead of *x*-axis: Only the M mark is available B0 B0 M1 A0.	

January 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1	$(3-2x)^5 = 243$, $\quad \ldots\ldots + 5\times(3)^4(-2x) = -810x \quad \ldots\ldots$ $+\frac{5\times 4}{2}(3)^3(-2x)^2 = \qquad +1080x^2$	B1, B1 M1 A1 (4) **[4]**
Notes	First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below). Term must be simplified to –810*x* for **B1** The *x* is required for this mark**.** The **method** mark **(M1) is generous** and is awarded for an attempt at Binomial to get the third term. There must be an x^2 (or no *x*- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip). So allow $\binom{5}{2}$ or $\binom{5}{3}$ or ${}^5C_2$ or ${}^5C_3$ or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle) May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\tfrac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the **M1** **A1**is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **M1 A1.)**	
Special cases	$243+810x+1080x^2$ is **B1B0M1A1** (condone no negative signs) Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full marks for correct answer Misreads *ascending* and gives $-32x^5+240x^4-720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)* Ignores 3 and expands $(1\pm 2x)^5$ is **0/4** 243, -810*x*, $1080x^2$ is full marks but 243, -810, 1080 is **B1,B0,M1,A0** NB Alternative method $3^5(1-\tfrac{2}{3}x)^5 = 3^5 - 5\times 3^5\times\left(\tfrac{2}{3}x\right)+\binom{5}{3}3^5\left(-\tfrac{2}{3}x\right)^2+..$ is **B0B0M1A0** – answers must be simplified to 243 –810*x* +1080x^2 for full marks (awarded as before) Special case $3(1-\tfrac{2}{3}x)^5 = 3-5\times 3\times\left(\tfrac{2}{3}x\right)+\binom{5}{3}3\left(-\tfrac{2}{3}x\right)^2+..$ is **B0, B0, M1, A0** Or $\quad 3(1-2x)^5$ is **B0B0M0A0**	

Question Number	Scheme	Marks
2	$y=(1+x)(4-x)=4+3x-x^2$ M: Expand, giving 3 (or 4) terms $\int\left(4+3x-x^2\right)\mathrm{d}x=4x+\frac{3x^2}{2}-\frac{x^3}{3}$ M: Attempt to integrate $=[..............]_{-1}^{4}=\left(16+24-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right)=\frac{125}{6}$ $\left(=20\frac{5}{6}\right)$	M1 M1 A1 M1 A1 (5) **[5]**
Notes	**M1** needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1\times4=5$, but there needs to be a 'constant' an 'x term' and an 'x^2 term'. The x terms do not need to be collected. (Need not be seen if next line correct) Attempt to integrate means that $x^n\to x^{n+1}$ for at least one of the terms, then **M1** is awarded (even 4 becoming 4*x* is sufficient) – one correct power sufficient. **A1** is for correct answer only, not follow through. But allow $2x^2-\frac{1}{2}x^2$ or any correct equivalent. Allow **+ *c***, and even allow an evaluated extra constant term. **M1**: Substitute limit 4 and limit –1 into a changed function (must be –1) and indicate subtraction (either way round). **A1** must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.	
Special cases	(i) Uses calculator method: **M1** for expansion (if seen) **M1** for limits if answer correct, so 0 , 1 or 2 marks out of 5 is possible (Most likely **M0 M0 A0 M1 A0**) (ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark **M1** gained. (iii) Using original method, but then change all signs after expansion is likely to lead to: **M1 M1 A0, M1 A0 i.e. 3/5**	

Question Number	Scheme	Marks
3 (a)	3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)	B1 B1 (2)
(b)	$\frac{1}{2}\times 0.4,\ \{(3+4.58)+2(3.47+3.84+4.14+4.39)\}$ = 7.852 (awrt 7.9)	B1, M1 A1ft A1 (4) **[6]**
Notes (a)	**B1** for one answer correct Second **B1** for all three correct Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and $\sqrt{\left(\frac{429}{25}\right)}$ or $\frac{\sqrt{429}}{5}$, score the marks.	
(b)	**B1** is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$. **M1** requires first bracket to contain first plus last values and second bracket to include no additional values from those in the table. If the only mistake is to omit one value from 2nd bracket this may be regarded as a slip ar can be allowed (An extra repeated term forfeits the **M** mark however) x values: **M0** if values used in brackets are x values instead of y values. Separate trapezia may be used : **B1** for 0.2, **M1** for $\frac{1}{2}h(a+b)$ used 4 or 5 times (and **A1**ft all e.g.. $0.2(3+3.47)+0.2(3.47+3.84)+0.2(3.84+4.14)+0.2(4.14+4.58)$ is **M1 A0** equivalent to missing one term in { } in main scheme **A1ft** follows their answers to part (a) and is for {correct expression} Final **A1** must be correct. (No follow through)	
Special cases	Bracketing mistake: i.e. $\frac{1}{2}\times 0.4(3+4.58)+2(3.47+3.84+4.14+4.39)$ scores **B1 M1 A0 A0** <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). **Need to see trapezium rule – answer only (with no working) is 0/4.**	

Question Number	Scheme	Marks
4	$2\log_5 x = \log_5\left(x^2\right)$, $\log_5(4-x) - \log_5(x^2) = \log_5 \frac{4-x}{x^2}$ $\log\left(\frac{4-x}{x^2}\right) = \log 5$ $5x^2 + x - 4 = 0$ **or** $5x^2 + x = 4$ **o.e.** $(5x-4)(x+1) = 0$ $x = \frac{4}{5}$ $(x = -1)$	B1, M1 M1 A1 dM1 A1 (6) **[6]**
Notes	**B1** is awarded for $2\log x = \log x^2$ anywhere. **M1** for correct use of $\log A - \log B = \log \frac{A}{B}$ **M1** for replacing 1 by $\log_k k$. **A1** for correct quadratic ($\log(4-x) - \log x^2 = \log 5 \Rightarrow 4 - x - x^2 = 5$ is **B1M0M1A0 M0A0**) **dM1** for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded) **A1** for 4/5 or 0.8 or equivalent (Ignore extra answer).	
Alternative 1	$\log_5(4-x) - 1 = 2\log_5 x$ so $\log_5(4-x) - \log_5 5 = 2\log_5 x$ $\log_5 \frac{4-x}{5} = 2\log_5 x$ then could complete solution with $2\log_5 x = \log_5\left(x^2\right)$ $\left(\frac{4-x}{5}\right) = x^2$ $5x^2 + x - 4 = 0$ Then as in first method $(5x-4)(x+1) = 0$ $x = \frac{4}{5}$ $(x = -1)$	M1 M1 B1 A1 dM1 A1 (6) **[6]**
Special cases	Complete trial and error yielding 0.8 is **M3** and **B1** for 0.8 **A1, A1** awarded for each of two tries evaluated. i.e. 6/6 Incomplete trial and error with wrong or no solution is 0/6 Just answer 0.8 with no working is **B1**	
	If log base 10 or base e used throughout - can score **B1M1M1A0M1A0**	

Question Number	Scheme	Marks
5 (a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(= \frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1 : \quad \frac{8}{12} \times \frac{6}{9-a} = -1 \qquad a = 13 \qquad (*)$	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$\left(9-(-3)\right)^2 + (10-2)^2$, (*i.e.*208) , $(9-a)^2 + (10-4)^2$, $\left(a-(-3)\right)^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, a = 13 (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)$ $(10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)$ $(13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, \quad r^2 = 65$	A1, A1, B1cao (5) **[8]**
Notes (a)	**M1**-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains a = 13 with no errors by solution or verification. Verification can score 3/3.	
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is **M1** They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9}=-\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip) They then complete to give (a)= 13 **A1** (ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2+7^2=(c+3)^2+1^2$ (equal radii) or $\frac{3-6}{c-3}=-\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord) Then using c (= 5) to find a is **M1** Finally a = 13 **A1** (iii) Vector Method: States **PQ. QR** = 0, with vectors stated 12i +8j and (9 – a)**i + 6**j is **M1** Evaluates scalar product so 108 – 12 a + 48 = 0 (**M1**) solves to give a = 13 (**A1**)	M1 M1 A1 M1 M1 A1 M1 M1 A1

Question Number	Scheme	Marks
6 (a)	$\mathrm{f}(2)=16+40+2a+b$ **or** $\mathrm{f}(-1)=1-5-a+b$	M1 A1
	Finds 2nd remainder and equates to 1st ⇒ $16+40+2a+b=1-5-a+b$	M1 A1
	$a=-20$	A1cso (5)
(b)	$\mathrm{f}(-3)=(-3)^4+5(-3)^3-3a+b=0$	M1 A1ft
	81 – 135 + 60 + b = 0 gives b = -6	A1 cso (3) **[8]**
Alternative for (a)	(a) Uses long division, to get remainders as b + 2a + 56 or b – a - 4 or correct equivalent	M1 A1
	Uses second long division as far as remainder term, to get b + 2a + 56 = b – a - 4 or correct equivalent	M1 A1
	$a=-20$	A1cso (5)
Alternative for (b)	(b) Uses long division of $x^4+5x^3-20x+b$ by (x + 3) to obtain $x^3+2x^2-6x+a+18$ (with their value for a)	M1 A1ft
	Giving remainder b + 6 = 0 and so b = -6	A1 cso (3) **[8]**
Notes (a)	**M1** : Attempts f(±2) or f(±1) **A1** is for the answer shown (or simplified with terms collected) for one remainder **M1:** Attempts other remainder and puts one equal to the other **A1:** for correct equation in a (and b) then **A1 for** $a=-20$ **cso**	
(b)	**M1** : Puts $\mathrm{f}(\pm 3)=0$ **A1** is for f(-3) = 0, (where f is original function), with no sign or substitution errors (follow through on 'a' and could still be in terms of a) **A1:** b = -6 is cso.	
Alternatives	(a) **M1**: Uses long division of x^4+5x^3+ax+b by (x ±2) **or** by (x ±1) as far as three term quotient **A1**: Obtains at least one correct remainder **M1:** Obtains second remainder and puts two remainders (no x terms) equal **A1:** correct equation **A1**: correct answer a = -20 following correct work. (b) **M1**: complete long division as far as constant (ignore remainder) **A1ft:** needs correct answer for their a **A1:** correct answer	
Beware: It is possible to get **correct answers with wrong working**. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0		

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 2.2 = 39.6 \quad \left(\text{cm}^2\right)$	M1 A1 (2)
(b)	$\left(\frac{2\pi - 2.2}{2} = \right)\ \pi - 1.1 = 2.04 \quad (\text{rad})$	M1 A1 (2)
	(c) $\Delta DAC = \frac{1}{2}\times 6\times 4\sin 2.04 \qquad (\approx 10.7)$	M1 A1ft
	Total area = sector + 2 triangles = 61 $\left(\text{cm}^2\right)$	M1 A1 (4) **[8]**
(a)	**M1:** Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1:** Does not need units. Answer should be 39.6 exactly. Answer with no working is **M1 A1**. This **M1A1** can only be awarded in part (a).	
(b)	**M1:** Needs full method to give angle in radians **A1:** Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2)	
(c)	**M1:** Use $\frac{1}{2}\times 6\times 4\sin A$ (if any other triangle formula e.g. $\frac{1}{2}b\times h$ is used the method must be complete for this mark) (No value needed for A, but should not be using 2.2) **A1:** ft the value obtained in part (b) – need not be evaluated- could be in degrees **M1:** Uses Total area = sector + 2 triangles or other complete method **A1:** Allow answers which round to 61. (Do not need units) Special case degrees: Could get M0A0, M0A0, M1A1M1A0 Special case: Use $\Delta\ BDC - \Delta\ BAC$ Both areas needed for first **M1** Total area = sector + area found is second **M1** **NB** Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4	

Question Number	Scheme	Marks
8 (a)	$4(1-\cos^2 x)+9\cos x-6=0 \qquad 4\cos^2 x-9\cos x+2=0$ (*)	M1 A1 (2)
(b)	$(4\cos x-1)(\cos x-2)=0 \qquad \cos x=...,\ \frac{1}{4}$	M1 A1
	$x=75.5 \qquad (\alpha)$	B1
	$360-\alpha, \qquad 360+\alpha$ or 720 - α	M1, M1
	284.5, 435.5, 644.5	A1 (6) **[8]**
(a)	**M1:** Uses $\sin^2 x=1-\cos^2 x$ (may omit bracket) **not** $\sin^2 x=\cos^2 x-1$ **A1:** Obtains the printed answer without error – **must have = 0**	
(b)	**M1:** Solves the quadratic with usual conventions **A1:** Obtains ¼ accurately- ignore extra answer 2 but penalise e.g. -2. **B1:** allow answers which round to 75.5 **M1:** $360-\alpha$ ft their value, **M1:** $360+\alpha$ ft their value or 720 - α ft **A1:** Three **and only three** correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic (4cosx-1)(cosx+2) Could yield, **M1A0B1M1M1A1** losing one mark for the error Works in radians: Complete work in radians :Obtains 1.3 **B0**. Then allow **M1 M1** for $2\pi-\alpha$, $2\pi+\alpha$ or $4\pi-\alpha$ Then gets 5.0, 7.6, 11.3 **A0 so 2/4** Mixed answer 1.3, 360 – 1.3, 360 + 1.3, 720 – 1.3 still gets **B0M1M1A0**	

Question Number	Scheme	Marks
9		
(a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$	
	Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$,	M1
	Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$	
	$k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$	M1, A1
	Proceed to $k^2 - 7k - 60 = 0$ (*)	A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}$ $\left(= \frac{3}{4} \text{ or } 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) **[10]**
(a)	**M1:** The 'initial step', scoring the first M mark, may be implied by next line of proof **M1**: Eliminates *a* and *r* to give valid equation in *k* only. Can be awarded for equation involving fractions. **A1 :** need some correct expansion and working and answer equivalent to required quadratic but with uncollected terms. Equations involving fractions do not get this mark. (No fractions, no brackets – could be a cubic equation) **A1:** as answer is printed this mark is for cso (Needs = 0) **All four marks must be scored in part (a)**	
(b)	**M1:** Attempt to solve quadratic **A1:** This is for correct factorisation or solution and $k = 12$. Ignore the extra solution ($k = -5$ or even $k = 5$), if seen. Substitute and verify is **M1 A0** **Marks must be scored in part (b)**	
(c)	**M1:** Complete method to find *r* Could have answer in terms of *k* **A1:** 0.75 or any correct equivalent **Both Marks must be scored in (c)**	
(d)	**M1:** Tries to use $\frac{a}{1-r}$, (even with $r>1$). Could have an answer still in terms of *k*. **A1:** This answer is 64 cao.	

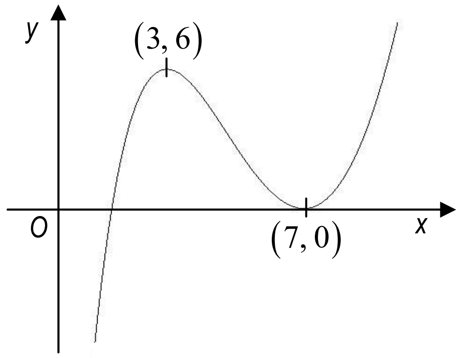
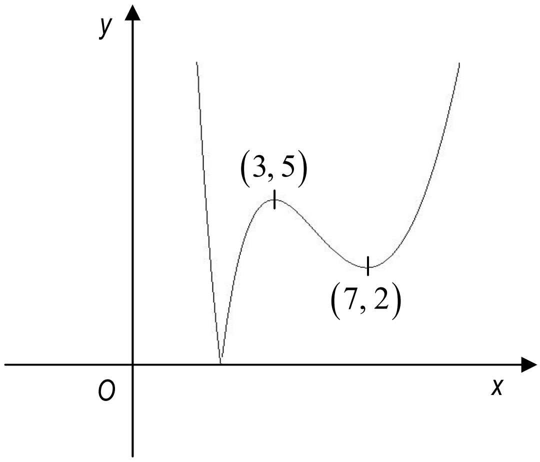
Question Number	Scheme	Marks
10 (a)	$2\pi rh + 2\pi r^2 = 800$	B1
	$h = \frac{400 - \pi r^2}{\pi r}$, $\quad V = \pi r^2 \left(\frac{400 - \pi r^2}{\pi r} \right) = 400r - \pi r^3 \quad$ (*)	M1, M1 A1 (4)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}r} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi \quad r^2 = 0 \qquad r^2 = \ldots, \qquad r = \sqrt{\frac{400}{3\pi}} \qquad$ (= 6.5 (2 s.f.))	M1 A1
	$V = 400r - \pi r^3 = 1737 = \frac{800}{3}\sqrt{\frac{400}{3\pi}} \ \left(\mathrm{cm}^3\right)$ (accept awrt 1737 or exact answer)	M1 A1 (6)
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}r^2} = -6\pi r$, Negative, $\therefore$ maximum (Parts (b) and (c) should be considered together when marking)	M1 A1 (2) **[12]**
Other methods for part (c):	Either:M: Find value of $\frac{\mathrm{d}V}{\mathrm{d}r}$ on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and consider sign. A: Indicate sign change of positive to negative for $\frac{\mathrm{d}V}{\mathrm{d}r}$, and conclude max. Or: M: Find value of V on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and compare with "1737". A: Indicate that both values are less than 1737 or 1737.25, and conclude max.	
Notes (a)	**B1:** For any correct form of this equation (may be unsimplified, may be implied by 1st M1) **M1 :** Making h the subject of their three or four term formula **M1:** Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only. **A1:** cso	
(b)	**M1:** At least one power of r decreased by 1 **A1:** cao **M1:** Setting $\frac{\mathrm{d}V}{\mathrm{d}r}$ =0 and finding a value for correct power of r for candidate **A1 :** This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5)or be exact answer **M1:** Substitute a positive value of r to give V **A1:** 1737 or 1737.25….. or exact answer	

(c)	**M1:** needs complete method **e.g.**attempts differentiation (power reduced) of their first derivative and considers its sign **A1(first method)** should be $-6\pi r$ (do not need to substitute *r* and can condone wrong *r* if found in (b)) Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as d*y*/d*x* for d*V*/d*r*
Alternative for (a)	$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is **M1** Equate to 400*r* **B1** Then $V = 400r - \pi r^3$ is **M1 A1**

January 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	$\text{M1}\ \frac{\text{A1+A1}}{\text{A1}}$ (4) **[10]**
	Alternative to (b)	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^2}-\frac{2\sin 2x}{x^3}\right)$	A1 (4)

Question Number	Scheme	Marks
2 (a)	$\frac{2x+2}{x^2-2x-3}-\frac{x+1}{x-3}=\frac{2x+2}{(x-3)(x+1)}-\frac{x+1}{x-3}$	
	$=\frac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\frac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\frac{1-x}{x-3}$ Accept $-\frac{x-1}{x-3},\ \frac{x-1}{3-x}$	A1 (4)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1-x}{x-3}\right)=\frac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1
	$=\frac{-x+3-1+x}{(x-3)^2}=\frac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
		[7]
	Alternative to (a)	
	$\frac{2x+2}{x^2-2x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3}$	M1 A1
	$\frac{2}{x-3}-\frac{x+1}{x-3}=\frac{2-(x+1)}{x-3}$	M1
	$=\frac{1-x}{x-3}$	A1 (4)
	Alternatives to (b)	
	① $\mathrm{f}(x)=\frac{1-x}{x-3}=-1-\frac{2}{x-3}=-1-2(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1
	$=\frac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$	M1
	$=-\frac{1}{x-3}-\frac{1-x}{(x-3)^2}=\frac{-(x-3)-(1-x)}{(x-3)^2}$	A1
	$=\frac{2}{(x-3)^2}$ $\ast$	A1 (3)

Question Number	Scheme		Marks	
3 (a)		Shape	B1	
		$(3, 6)$	B1	
		$(7, 0)$	B1	(3)
(b)		Shape	B1	
		$(3, 5)$	B1	
		$(7, 2)$	B1	(3)
				[6]

Question Number	Scheme		Marks
4	$x = \cos(2y + \pi)$		
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = -2\sin(2y + \pi)$		M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2\sin(2y + \pi)}$	Follow through their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ before or after substitution	A1ft
	At $y = \dfrac{\pi}{4}$, $\quad \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2\sin\dfrac{3\pi}{2}} = \dfrac{1}{2}$		B1
	$y - \dfrac{\pi}{4} = \dfrac{1}{2}x$		M1
	$y = \dfrac{1}{2}x + \dfrac{\pi}{4}$		A1 (6)
			[6]

Question Number	Scheme	Marks
5 (a)	$\mathrm{g}(x) \geq 1$	B1 (1)
(b)	$\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\,\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\,\mathrm{e}^{x^2}$ ∗	A1 (2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\,\mathrm{e}^{x^2}\right)$	
(c)	$\mathrm{fg}(x) \geq 3$	B1 (1)
(d)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\,\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 (6) **[10]**

Question Number	Scheme	Marks
6 (a)(i)	$\sin 3\theta = \sin\left(2\theta + \theta\right)$	
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	
	$= 2\sin\theta\cos\theta.\cos\theta + \left(1 - 2\sin^2\theta\right)\sin\theta$	M1 A1
	$= 2\sin\theta\left(1 - \sin^2\theta\right) + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ ∗ cso	A1 (4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$	
	$-2\sin 3\theta + 1 = 0$	M1 A1
	$\sin 3\theta = \frac{1}{2}$	M1
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$	A1 A1 (5)
(b)	$\sin 15° = \sin\left(60° - 45°\right) = \sin 60° \cos 45° - \cos 60° \sin 45°$	M1
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$ ∗ cso	A1 (4)
		[13]
	Alternatives to (b)	
	① $\sin 15° = \sin\left(45° - 30°\right) = \sin 45° \cos 30° - \cos 45° \sin 30°$	M1
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$ ∗ cso	A1 (4)
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$	
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$	
	$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$	M1 A1
	$\left(\frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)\right)^2 = \frac{1}{16}\left(6 + 2 - 2\sqrt{12}\right) = \frac{2 - \sqrt{3}}{4}$	M1
	Hence $\sin 15° = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$ ∗ cso	A1 (4)

Question Number	Scheme	Marks
7 (a)	$f'(x) = 3e^x + 3xe^x$	M1 A1
	$3e^x + 3xe^x = 3e^x(1+x) = 0$	
	$x = -1$	M1 A1
	$f(-1) = -3e^{-1} - 1$	B1 (5)
(b)	$x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 (3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$f(0.257\,55) = -0.000\,379\ ...$	
	$f(0.257\,65) = 0.000\,109\ \dots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ∗ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	(3) **[11]**
	Note: $x = 0.257\,627\,65\ \dots$ is accurate	

Question Number	Scheme	Marks
8 (a)	$R^2 = 3^2 + 4^2$	M1
	$R = 5$	A1
	$\tan\alpha = \frac{4}{3}$	M1
	$\alpha = 53\ ...\ ^\circ$ awrt 53°	A1 (4)
(b)	Maximum value is 5 ft their R	B1 ft
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$	M1
	$\theta = \alpha = 53\ ...\ ^\circ$ ft their α	A1 ft (3)
(c)	$\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$	
	Minimum occurs when $\cos(15t - \alpha)^\circ = -1$	M1
	The minimum temperature is $(10 - 5)^\circ = 5^\circ$	A1 ft (2)
(d)	$15t - \alpha = 180$	M1
	$t = 15.5$ awrt 15.5	M1 A1 (3) **[12]**

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Mark Scheme

Question Number	Scheme		Marks
1 (a)	$C\text{:}\ \ y^2 - 3y = x^3 + 8$		
	$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}x}} \cancel{\times}\right\}\quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x}$'. Can be implied.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow\ 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow\ \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2,3)\ \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(4)}{6-3}\ \Rightarrow\ \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$, then an answer of $\frac{\mathrm{d}y}{\mathrm{d}x}$ = their x^2, ***may*** indicate a correct follow through.		
			[7]

Question Number	Scheme	Marks
2 (a)	Area$(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$	
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$ ***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. <u>Correct integration.</u> Ignore limits.	M1 A1
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$	
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$ $\underline{3}$	<u>A1</u> (4)
	(Answer of 3 with no working scores M0A0M0A0.)	
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 \mathrm{d}x}$ Use of $V = \underline{\pi \int y^2}\,\mathrm{d}x$. Can be implied. Ignore limits and $\mathrm{d}x$.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,\mathrm{d}x$	
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$ $\pm k\ln\lvert 1+4x\rvert$ $\frac{9}{4}\ln\lvert 1+4x\rvert$	M1 A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$ Substitutes limits of 2 and 0 and subtracts the correct way round. Note that $\ln 1$ can be implied as equal to 0.	dM1
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$ $\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$ Note the answer must be a one term exact value. Note, also you can ignore subsequent working here. Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	A1 oe isw (5)
		[9]

Question Number	Scheme	Marks
3 (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$ Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16 = 25C \Rightarrow 75 = 25C \Rightarrow C=3$ Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
	Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A$ $\Rightarrow A=0$ $x=0,\quad 16 = 2A+B+4C$ $\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$ Compares coefficients or substitutes in a third *x*-value or uses simultaneous equations to show $A=0$.	B1
		(4)
(b)	$\mathrm{f}(x) = \frac{4}{(3x+2)^2}+\frac{3}{(1-x)}$	
	$= 4(3x+2)^{-2}+3(1-x)^{-1}$ Moving powers to top on any one of the two expressions	M1
	$= 4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$	
	$= 1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$	
	$= 1\left\{\underline{1+(-2)(\tfrac{3x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2+\ldots}\right\}$ Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+\,3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$ Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots}\}$ expansion.	A1
	Both $\{\underline{\ldots\ldots\ldots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$	
	$=4+0x\,;+\frac{39}{4}x^2$ $4+(0x)\,;\ \frac{39}{4}x^2$	A1; A1
		(6)

Question Number	Scheme		Marks
(c)	$\text{Actual} = \text{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	
	Or $\text{Actual} = \text{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	M1
	$\text{Estimate} = \text{f}(0.2) = 4 + \tfrac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{\ }$
	$\text{\%age error} = \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\% (2\text{sf})$	1.1%	A1 **cao** (4) **[14]**

Question Number	Scheme	Marks
4 (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	
	As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$ Apply dot product calculation between two direction vectors, ie. $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso (2)
(b)	Lines meet where:	
	$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$	
	First two of $\mathbf{i}: \; 11 - 2\lambda = -5 + q\mu \quad (1)$ $\mathbf{j}: \; 2 + \lambda = 11 + 2\mu \quad (2)$ $\mathbf{k}: \; 17 - 4\lambda = p + 2\mu \quad (3)$ Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$ Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$ Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$ Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$ $\underline{p = 1}$	A1 cso (6)
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$ Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$ $\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1 (2)

Question Number	Scheme	Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection	
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{\ }$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$	
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
		(3)
		[13]

Question Number	Scheme	Marks
5 (a)	Similar triangles $\Rightarrow \underline{\frac{r}{h}=\frac{16}{24}} \Rightarrow \underline{r=\frac{2h}{3}}$ Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ **AG** Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1 (2)
(b)	From the question, $\frac{\mathrm{d}V}{\mathrm{d}t}=8$ $\frac{\mathrm{d}V}{\mathrm{d}t}=8$	B1
	$\underline{\frac{\mathrm{d}V}{\mathrm{d}h}}=\underline{\frac{12\pi h^2}{27}}=\underline{\frac{4\pi h^2}{9}}$ $\underline{\frac{\mathrm{d}V}{\mathrm{d}h}}=\underline{\frac{12\pi h^2}{27}}$ or $\underline{\frac{4\pi h^2}{9}}$	B1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{dh} = \underline{8\times\frac{9}{4\pi h^2}} = \underline{\frac{18}{\pi h^2}}$ Candidate's $\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}$; $\underline{8\div\left(\frac{12\pi h^2}{27}\right)}$ or $\underline{8\times\frac{9}{4\pi h^2}}$ or $\underline{\frac{18}{\pi h^2}}$ oe	M1; A1
	When $h=12$, $\frac{\mathrm{d}h}{\mathrm{d}t}=\underline{\frac{18}{144\pi}}=\underline{\frac{1}{8\pi}}$ $\underline{\frac{18}{144\pi}}$ or $\underline{\frac{1}{8\pi}}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\underline{\frac{18}{144\pi}}$.	(5) **[7]**

Question Number	Scheme		Marks
6 (a)	$\int \tan^2 x \, \mathrm{d}x$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, \mathrm{d}x$		
	$= \underline{\tan x \; - x} \; (+ c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3} \ln x \, \mathrm{d}x$		
	$\left\{ \begin{array}{ll} u = \ln x & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} & \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} . \frac{1}{x} \, \mathrm{d}x$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, \mathrm{d}x$	An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ , $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \; (+ c)$	… "integrate"(process the result);	M1
		correct solution with/without + c	A1 oe
			(4)

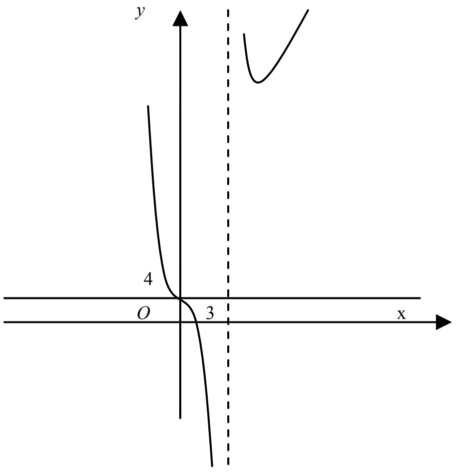
Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{u=1+e^x \Rightarrow \underline{\frac{du}{dx}=e^x}\ ,\ \underline{\frac{dx}{du}=\frac{1}{e^x}}\ ,\ \underline{\frac{dx}{du}=\frac{1}{u-1}}\right\}$	Differentiating to find any one of the three underlined	B1
	$=\int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $=\int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x}=f(u)$, their $\frac{dx}{du}=\frac{1}{e^x}$ and $u=1+e^x$ or $e^{3x}=f(u)$, their $\frac{dx}{du}=\frac{1}{u-1}$ and $u=1+e^x$.	M1*
	$=\int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$=\int \frac{u^2-2u+1}{u}\,du$ $=\int u-2+\frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$=\frac{u^2}{2}-2u+\ln u\ (+c)$	Correct integration with/without +c	A1
	$=\frac{(1+e^x)^2}{2}-2(1+e^x)+\ln(1+e^x)+c$	Substitutes $u=1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$=\frac{1}{2}+e^x+\frac{1}{2}e^{2x}-2-2e^x+\ln(1+e^x)+c$		
	$=\frac{1}{2}+e^x+\frac{1}{2}e^{2x}-2-2e^x+\ln(1+e^x)+c$		
	$=\frac{1}{2}e^{2x}-e^x+\ln(1+e^x)-\frac{3}{2}+c$		
	$=\frac{1}{2}e^{2x}-e^x+\ln(1+e^x)+k$ **AG**	$\frac{1}{2}e^{2x}-e^x+\ln(1+e^x)+k$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 **cso**
			(7)
			[13]

Question Number	Scheme		Marks
7 (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2-8$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2t$		
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{3t^2-8}$	Their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1
	At A, $\mathrm{m}(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T\left(x-(\text{their } 7)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$		
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 cso (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}$ = 55.125 or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]

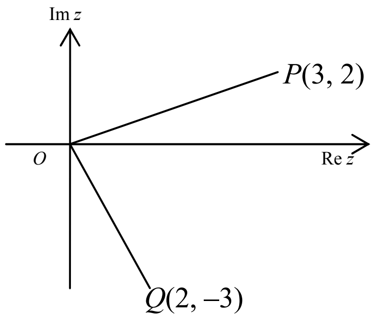
January 2009
6674 Further Pure Mathematics FP1 (legacy)
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$	M1, A1
	Simplifying this expression	M1
	$= \frac{1}{3}n(n^2-4)$ (✱)	A1 (4) cso
(b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) = \frac{1}{3}\times 20\times(20^2-4) - \frac{1}{3}\times 9\times(9^2-4)$	M1
	$= 2409$	A1 (2)
Alt (b)	$\sum_{r=1}^{20}(r^2-r-1) - \sum_{r=1}^{9}(r^2-r-1) =$	
	$\left(\frac{1}{6}\times 20\times 21\times 41 - \frac{1}{2}\times 20\times 21 - 20\right) - \left(\frac{1}{6}\times 9\times 10\times 19 - \frac{1}{2}\times 9\times 10 - 9\right)$ M1	
	$= 2409$ A1	**[6]**
Notes	(a) 1st M: Separating, substituting set results, at least two correct. 2nd M: Either "eliminate" brackets totally or factor x [......] where any product of brackets inside [....] has been reduced to a single bracket 2nd A: ANSWER GIVEN. No wrong working seen; must have been an intermediate step, e.g. $\frac{1}{6}n(2n^2+3n+1-3n-3-6)$. (b) M : Must be $\sum_{r=1}^{20}(....) - \sum_{r=1}^{9}(....)$ applied. If list terms and add, allow M1 if **11 terms** with **at most two wrong**: [89, 109, 131, 155, 181, 209, 239, 271, 305, 341, 379]	

Question Number	Scheme	Marks
2	3 – i is a root (seen anywhere)	B1
	Attempt to multiply out $[x-(3+\mathrm{i})][x-(3-\mathrm{i})]$ $\{= x^2 - 6x + 10\}$	M1
	$\mathrm{f}(x) = (x^2 - 6x + 10)(2x^2 - 2x + 1)$	M1, A1
	$x = \dfrac{2 \pm \sqrt{4-8}}{4}$, $x = \dfrac{1 \pm \mathrm{i}}{2}$	*M1, A1 **[6]**
Notes	1st M: Using the two roots to form a quadratic factor. 2nd M: Complete method to find second quadratic factor $2x^2 + \mathrm{a}x\ (+\ \mathrm{b})$. 3rd *M: Correct method, as far as $x = \ldots$, for solving candidate's second quadratic, DEPENDENT on both previous M marks	
Alt	(i)$\mathrm{f}(x)/\{x-(3+\mathrm{i})\} = 2x^3 + (-8+2\mathrm{i})x^2 + (7-2\mathrm{i})x - 3 + \mathrm{i}$ $\{=\mathrm{g}(x)\}$ $\mathrm{g}(x)/\{x-(3-\mathrm{i})\} = (2x^2 - 2x + 1)$ Attempt at complete process M2; A1	Lines 2 and 3
	(ii)(2)(x – a+ib)(x – a– ib)("$x^2 - 6x + 10$") = f(x) and compare ≥ 1 coeff. M1 Either $-2\mathrm{a} - 6 = -7$, or two of $10(\mathrm{b}^2 + \mathrm{a}^2) = 5$ or $-6(\mathrm{a}^2 + \mathrm{b}^2) - 20\mathrm{a} = -13$, $20 + 2(\mathrm{b}^2 + \mathrm{a}^2) + 24\mathrm{a} = 33$ A1; Complete method for a **and** b, M1; AnswerA1	Lines 3 and 4

Question Number	Scheme	Marks
3	Identifying 3 as critical value e.g. used in soln	B1
	Identifying 0 as critical value e.g. used in soln	B1
	$\frac{x^3+5x-12-4(x-3)}{x-3}>0$ or $(x^3+5x-12)(x-3)>4(x-3)^2$ o.e.	M1
	$\frac{x(x^2+1)}{x-3}>0$ or $(x-3)(x^3+x)>0$	A1
	Using their critical values to **obtain** inequalities.	M1
	$x<0$ or $x>3$	A1 cso
Notes		
	1st M must be a valid opening strategy.	
	Sketching $y=\frac{x}{x-3}$ or $y=\frac{x(x^2+1)}{x-3}$ should mark as scheme.	
	The result $0>x>3$ (poor notation) can gain final M but not A.	
Alt		
	Identifying 3 as critical value e.g. $x=3$ seen as asymp.	B1
	Identifying 0 as critical value e.g. pt of intersection on y-axis of $y=\frac{x^3+5x-12}{x-3}$ and $y=4$	B1
	M1 $y=\frac{x^3+5x-12}{x-3}$ sketched for $x<3$ or $y=\frac{x^3+5x-12}{x-3}$ sketched for $x>3$ A1 All correct including y = 4 drawn	M1, A1
	Using the graph values to obtain one or more inequalities	M1
	$x<0$ or $x>3$	A1

Question Number	Scheme	Marks
4 (a)	At st. pt $f'(x) = 0$, $\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ is undefined	B1 (1)
	or at st. pt, **tan.** // to x-axis, or **tan.** does not cross x-axis, o.e.	
(b)	$f'(x) = -1 - 2x\cos(x^2)$ (may be seen in body of work)	M1, A1
	f(0.6) = 0.0477…, f′(0.6) = –2.123… (may be implied by correct answer)	A1
	Attempt to use $(x_1) = 0.6 - \frac{f(0.6)}{f'(0.6)}$ $\left[0.6 - \frac{0.0477...}{-2.123...}\right]$	M1
	= 0.622 (3 dp) (0.6224795…)	A1 (5)
(c)	$f(0.6215) = 1.77\ldots\times10^{-3} > 0$, $f(0.6225) = -3.807\times\ldots\times10^{-4} < 0$	M1
	Change of sign in f(x) in (0.6215, 0.6225) "so 0.622 correct"	A1 (2)
		[8]
Notes	(b) 2ndM: If the N-R statement applied to 0.6 **not** seen, can be implied if answer correct; otherwise M0	
	If no values for f(0.6), $f'(0.6)$ seen, they can be implied if final answer correct.	
	(c) M: For candidates x_1, calculate $f(x_1 - 0.0005)$ and $f(x_1 + 0.0005)$ (or a tighter interval) A: Requires correct values of f(0.6215) and f(0.6225) (or their acceptable values) [may be rounded, e.g. 2×10^{-3}, or truncated, e.g – 3.80×10^{-4}], sign change stated or >0, <0 seen, and conclusion.	

Question Number	Scheme	Marks
5 (a)	$z_2 = \frac{12-5\text{i}}{3+2\text{i}}\times\frac{3-2\text{i}}{3-2\text{i}} = \frac{36-24\text{i}-15\text{i}-10}{13}$ $= 2 - 3\text{i}$	M1 A1 (2)
(b)	 P: B1, Q: B1ft from (a)	B1, B1ft (2)
(c)	grad. OP × grad. $OQ = (\frac{2}{3}\times-\frac{3}{2})$ $= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2}$ (✱)	M1 A1 (2)
Alt (c)	(i) $\angle \text{POX} = \tan^{-1}\frac{2}{3}$, $\angle \text{QOX} = \tan^{-1}\frac{3}{2}$ $\text{Tan}(\angle \text{POQ}) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1 $\Rightarrow \angle POQ = \frac{\pi}{2}$ (✱) A1	
(d)	$z = \frac{3+2}{2}+\frac{2+(-3)}{2}\text{i}$ $= \frac{5}{2}-\frac{1}{2}\text{i}$	M1 A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2+\left(-\frac{1}{2}\right)^2}$ $= \frac{\sqrt{26}}{2}$ or exact equivalent	M1 A1 (2) **[10]**
Notes	(a) M: Multiplying num. and den. by 3–2i and attempt to simplify num. and denominator . If (c + id)(3 + 2i) = 12 –5i used, need to find 2 equations in c and d and then solve for c and d. (b) Coords seen or clear from labelled axes. S.C: If only P and Q seen(no coords) or correct coords given but P and Q interchanged allow B1B0 (c) If separate arguments are found and then added, allow M1 but **not** A1 for decimals used e.g. 1.570796327.. = ½π . Alts: Appropriate transformation matrix applied to one point M1; A1 Scalar product used correctly M1; 0 and conclusion A1 Pythagoras' theorem, congruent triangles are other methods seen. (d) M: Any complete method for finding centre. A: Must be complex number; coordinates not sufficient. (e) M: Correct method for radius, or diameter, for candidate's answer to (d)	

Question Number	Scheme	Marks
6 (a)	$r=\sqrt{x^2+y^2}$, $y= r\sin\theta$	
	$\therefore \sqrt{x^2+y^2}=\frac{6y}{\sqrt{x^2+y^2}}$ or $x^2+y^2=6y$ o.e.	M1, A1 (2)
(b)	$r= 9\sqrt{6}(1-2\sin^2\theta)$ o.e.	B1 (1)
(c)	$y= r\sin\theta=9\sqrt{6}(\sin\theta-2\sin^3\theta)\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=;\ 9\sqrt{6}\cos\theta(1-6\sin^2\theta)$ o.e.	M1;A1
	Or $y=9\sqrt{6}\sin\theta\cos 2\theta\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=9\sqrt{6}(\cos 2\theta\cos\theta-2\sin\theta\sin 2\theta)$ o.e.	
	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=0$ [$\Rightarrow \cos\theta(1-6\sin^2\theta)=0$] **and** attempt to solve	M1
	$(0\le\theta\le\frac{\pi}{4})$ $\therefore \sin\theta=\frac{1}{\sqrt{6}}$ (✱)	A1 (4)
(d)	$r=9\sqrt{6}\left(1-2\times\frac{1}{6}\right)$	M1
	$=6\sqrt{6}$ or 14.7 (awrt)	A1 (2)
(e)	C_2: tan. // to initial line is $y= r\sin\theta = 6\sqrt{6}\times\frac{1}{\sqrt{6}}=6$	B1
	C_1: Circle, centre (0, 3) (cartesian) or $(3,\frac{\pi}{2})$ (polar), passing through (0,0).	M1
	$\therefore$ tangent // to initial line has eqn $y = 6 \Rightarrow y = 6$ is a common tangent	A1 (3) **[12]**
Notes	(a) M1: Use of $r=\sqrt{x^2+y^2}$ or $r^2 = x^2 + y^2$, and $y= r\sin\theta$ (allow $x= r\sin\theta$) to form cartesian equation. (b) May be scored in (c) (c) 1st M: Finds y and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ Working with $r\cos\theta$ instead of $r\sin\theta$, can score the M marks. If $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}/\frac{\mathrm{d}x}{\mathrm{d}\theta}$ used throughout, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ etc. all marks may be gained (d) M: Using $\sin\theta=\frac{1}{\sqrt{6}}$ to find r (e) Alt. for C_1: M:Find $y=6\sin^2\theta$, $(\frac{\mathrm{d}y}{\mathrm{d}\theta}=12\sin\theta\cos\theta)$ and solve $\frac{\mathrm{d}y}{\mathrm{d}\theta}=0$ A: Find $\theta=\frac{\pi}{2}$ and conclude that $y = 6$, so common tangent	

Question Number	Scheme	Marks
7 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda x\mathrm{e}^x + \lambda\mathrm{e}^x$ Use of the product rule	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \lambda x\mathrm{e}^x + \lambda\mathrm{e}^x + \lambda\mathrm{e}^x$	A1
	$\lambda x\mathrm{e}^x + 2\lambda\mathrm{e}^x + 4\lambda x\mathrm{e}^x + 4\lambda\mathrm{e}^x - 5\lambda x\mathrm{e}^x = 4\mathrm{e}^x$	*M1
	$\lambda = \frac{2}{3}$	A1 (4)
	$(\therefore \text{ P.I. is } \frac{2}{3}x\mathrm{e}^x)$	
(b)	Aux. eqn. $m^2 + 4m - 5 = 0$	
	$(m-1)(m+5) = 0$	
	$m = 1$ or $m = -5$	M1
	C.F. is $y = A\mathrm{e}^x + B\mathrm{e}^{-5x}$	M1 A1
	Gen. soln. is $(y =)\ \frac{2}{3}x\mathrm{e}^x + A\mathrm{e}^x + B\mathrm{e}^{-5x}$ [f.t: Candidate's C.F + P.I.]	A1ft (4)
(c)	$-\frac{2}{3} = A + B$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{3}x\mathrm{e}^x + \frac{2}{3}\mathrm{e}^x + A\mathrm{e}^x - 5B\mathrm{e}^{-5x}$	M1
	$-\frac{4}{3} = \frac{2}{3} + A - 5B$ A1 two correct unsimplified eqns.	A1
	$-2 = A - 5B$	
	$\frac{4}{3} = 6B$	
	$B = \frac{2}{9}, A = -\frac{8}{9}$	M1
	$y = \frac{2}{3}x\mathrm{e}^x - \frac{8}{9}\mathrm{e}^x + \frac{2}{9}\mathrm{e}^{-5x}$	A1 (5)
		[13]
Notes	(a) 2nd M dependent on first M. (b) 1st M: Attempt to solve A.E. 2nd M: Only allow C.F. of form $A\mathrm{e}^{ax} + B\mathrm{e}^{bx}$, where a and b are real. If seen in (a), award marks there. PI must be of form $\lambda x\mathrm{e}^x$ ($\lambda \neq 0$) to gain final A1 f.t. (c) 1st M: Using $x = 0, y = -\tfrac{2}{3}$ in their **general solution**. 2nd M: Differentiating their **general solution** {C.F. + P.I. } (must have term in $\lambda x\mathrm{e}^x$) (condone slips) and using $x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{3}$ to find an equation in A and B. 3rd M: Solving simultaneous equations to find a value of A and a value of B. Can be awarded if only C.F. found. Insist on $y =$ .... in this part.	

Question Number	Scheme	Marks
8 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ o.e.	M1, A1
	$\sin x \times -\frac{1}{t^2}\times\frac{\mathrm{d}t}{\mathrm{d}x} + \frac{1}{t}\cos x = \frac{1}{t^2}$	M1
	$\frac{\mathrm{d}t}{\mathrm{d}x} - t\cot x = -\operatorname{cosec} x$ (✱)	A1 cso(4)
(b)	$\mathrm{I} = \mathrm{e}^{\int -\cot x\,\mathrm{d}x}$	M1
	$= \mathrm{e}^{-\ln\sin x}$	
	$= \frac{1}{\sin x}$ or cosec$x$	A1
	$\frac{1}{\sin x}\frac{\mathrm{d}t}{\mathrm{d}x} - t\frac{\cos x}{\sin^2 x} = -\operatorname{cosec}^2 x$	M1
	$\frac{t}{\sin x} = \int -\cos\mathrm{ec}^2 x\,\mathrm{d}x$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{t}{\sin x}\right) = -\cos\mathrm{ec}^2 x$	A1f.t.
	$\frac{t}{\sin x} = \cot x \ \ (+c)$ o.e.	A1 cso (5)
(c)	$t = \cos x + c\sin x \ \Rightarrow \ y = \frac{1}{\cos x + c\sin x}$ (✱)	M1, A1 (2)
(d)	$\frac{\sqrt{2}}{3} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}}$	M1
	$\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{c}{\sqrt{2}}\right) = 3$	
	$c = 2$	A1
	$x = \frac{\pi}{2}, \quad y = \frac{1}{2}$ ft on their c	A1ft (3)
		[14]
Notes	(a) 1st M: Use of $\frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x}$ (even if integrated 1/t) 2nd M: Substituting for $\frac{\mathrm{d}y}{\mathrm{d}x}, y, y^2$ to form d.e. in x and t only (b) 1st M: For $\mathrm{e}^{\int -\cot x\,\mathrm{d}x}$ (allow $\mathrm{e}^{\int \cot x\,\mathrm{d}x}$) and attempt at integrating 2nd* M: Multiplying by integrating factor (requires at least two terms "correct" for their IF.) (can be implied) 3rdA1f.t: is only for those who have I.F. = sin$x$ or $-$sin$x$ $\frac{\mathrm{d}}{\mathrm{d}x}(t\sin x) = -1$ equivalent integral (c) M: Substituting to find t = 1/y in their solution to (b) (d) M: Using y = $\frac{\sqrt{2}}{3}, x = \frac{\pi}{4}$ to find a value for c.	

January 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
1	$x-3$ is a factor	B1
	$\mathrm{f}(x) = (x-3)(2x^2-2x+1)$	M1 A1
	Attempt to solve quadratic i.e. $x = \dfrac{2 \pm \sqrt{4-8}}{4}$	M1
	$x = \dfrac{1 \pm \mathrm{i}}{2}$	A1 **[5]**

Notes:

First and last terms in second bracket required for first M1
Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2 (a)	$6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n$	M1 A1, B1
	$= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)$ or $n(n+1)(2n+1) + (2n+1)n$	M1
	$= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1)$ *	A1 (5)
(b)	$\sum_{r=1}^{20}(6r^2 + 4r - 1) - \sum_{r=1}^{10}(6r^2 + 4r - 1) = 20 \times 22 \times 41 - 10 \times 12 \times 21$	M1
	$= 15520$	A1 (2) **[7]**

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
Second M1 for attempt to expand and gather terms.
Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3 (a)	$xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
(b)	A has co-ords (5, 5) and B has co-ords (25, 1) Mid point is at (15, 3)	B1 M1A1 (3) **[4]**

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for $n = k$; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for $n = k + 1$ (and by induction true for $n \in \mathbf{Z}^+$)	B1 **[5]**

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n = k$ 'and 'so result true for $n = k+1$' and correct solution for final B1

Question Number	Scheme	Marks
5 (a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$\mathrm{f}'(x)=\frac{3}{2}x^{-\frac{1}{2}}-9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f′ (1.1) = -6.37086…	B1 B1
	$x_1=1.1-\frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) **[9]**

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt -6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	At *n* =1, $u_n = 5\times 6^0 + 1 = 6$ and so result true for *n* = 1	B1
	Assume true for *n* = *k;* $u_k = 5\times 6^{k-1} + 1$,and so $u_{k+1} = 6(5\times 6^{k-1} + 1) - 5$	M1, A1
	$\therefore u_{k+1} = 5\times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5\times 6^k + 1$	A1
	and so result is true for *n* = *k* + 1 and by induction true for $n \geq 1$	B1 **[5]**

Notes:
6 and so result true for *n* = 1 award B1
Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2
Second A1 for $\therefore u_{k+1} = 5\times 6^k + 1$
'Assume true for *n* = *k'* and 'so result is true for *n* = *k* + 1' and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	The determinant is a - 2	M1
	$\mathbf{X}^{-1} = \frac{1}{a-2}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \frac{1}{a-2} = 1$, or $a - \frac{a}{a-2} = 0$, or $-1 + \frac{1}{a-2} = 0$, or $-1 + \frac{2}{a-2} = 1$	M1
	To obtain a =3 only	A1 cso (3) **[6]**
	Alternatives for (b) If they use $\mathbf{X}^2 + \mathbf{I} = \mathbf{X}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1 If they use $\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}$, they can score the B1then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1	

Notes:

(a) Attempt *ad-bc* for first M1

$\frac{1}{\det}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8 (a)	$\frac{dy}{dx} = a^{\frac{1}{2}} x^{-\frac{1}{2}}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$	M1
	The gradient of the tangent is $\frac{1}{q}$	A1
	The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$	M1
	So $yq = x + aq^2$ *	A1 (4)
(b)	R has coordinates (0, aq)	B1
	The line l has equation $y - aq = -qx$	M1A1 (3)
(c)	When $y = 0$ $x = a$ (so line l passes through (a, 0) the focus of the parabola.)	B1 (1)
(d)	Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are ($-a$, $2aq$)	M1:A1 (2) **[10]**

Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1

Use of $y = mx + c$ to find c OK for second M1

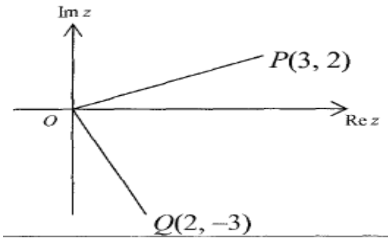
Correct solution only for final A1

(b) -1/(their gradient in part a) in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.

Both coordinates correct for A1.

Question Number		Scheme	Marks
9	(a)	$z_2 = \frac{12-5\mathrm{i}}{3+2\mathrm{i}} \times \frac{3-2\mathrm{i}}{3-2\mathrm{i}} = \frac{36-24\mathrm{i}-15\mathrm{i}-10}{13}$ $= 2-3\mathrm{i}$	M1 A1 (2)
	(b)	 P: B1, Q: B1ft	B1, B1ft (2)
	(c)	grad. OP × grad. OQ = $\frac{2}{3} \times -\frac{3}{2}$ $= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2}$ (✱)	
	OR	$\angle POX = \tan^{-1}\frac{2}{3}$, $\angle QOX = \tan^{-1}\frac{3}{2}$ $\mathrm{Tan}(\angle POQ) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1 $\Rightarrow \angle POQ = \frac{\pi}{2}$ (✱) A1	M1 A1 (2)
	(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}\mathrm{i}$ $= \frac{5}{2} - \frac{1}{2}\mathrm{i}$	M1 A1 (2)
	(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$ $= \frac{\sqrt{26}}{2}$ or exact equivalent	M1 A1 (2) **[10]**

Notes:

(a) $\times\frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10 (a)	**A** represents an enlargement scale factor $3\sqrt{2}$ (centre *O*)	M1 A1
	B represents reflection in the line $y = x$	B1
	C represents a rotation of $\frac{\pi}{4}$, i.e.45° (anticlockwise) (about O)	B1 (4)
(b)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0, 0), (90 , 0) and (51 , 75)	M1A1A1A1 (4)
(e)	Area of $\Delta\, OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$	B1
	Determinant of **E** is –18 or use area scale factor of enlargement So area of $\Delta\, ORS$ is $3375 \div 18 = 187.5$	M1A1 (3) **[14]**

Notes:

(a) Enlargement for M1
$3\sqrt{2}$ for A1

(b) Answer incorrect, require **CD** for M1

(c) Answer given so require **DB** as shown for B1

(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1
Divide by theirs for M1

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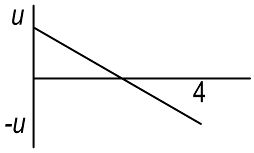
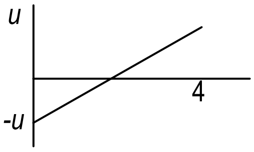
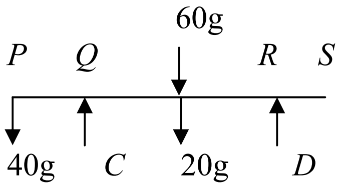
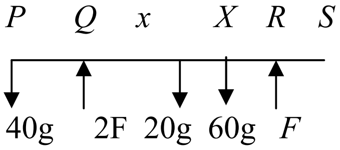
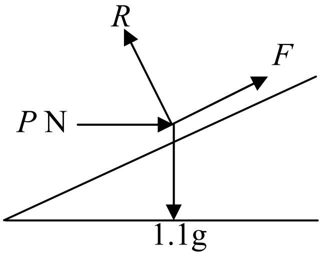
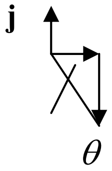
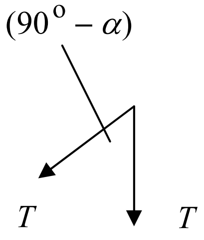
Mark Scheme

Question Number	Scheme	Marks
1	$-6\mathbf{i}+\mathbf{j}=\mathbf{u}+3(2\mathbf{i}-5\mathbf{j})$ $\Rightarrow \mathbf{u}=-12\mathbf{i}+16\mathbf{j}$ $\Rightarrow u=\sqrt{(-12)^2+16^2}=20$	M1 A1 A1 cso M1 A1 **[5]**
2 (a)	 or  shape values	B1 B1 (2)
(b)	$19.6=\frac{1}{2}\times 2\times u$ $u=19.6$	M1 A1 A1 (3) **[5]**
3 (a)	$2u \rightarrow \quad \leftarrow 4u$ $\qquad km2u-4mu=-kmu+mv$ $km \quad m$ $\qquad u(3k-4)=v$ $u \leftarrow \quad \rightarrow v$	M1 A1 A1 (3)
(b)	$k>2\Rightarrow v>0\Rightarrow$ dir^n of motion reversed	M1A1A1 cso (3)
(c)	For B, $m(u(3k-4)--4u)$ $=7mu$	M1 A1 f.t. A1 (3) **[9]**

Question Number	Scheme	Marks
4 (a)		
	$C + D = 120g$	
	$M(Q)$, $80g.0.8 - 40g.0.4 = D.1.6$	M1 A1
	solving	M1 A1
	$C = 90g; D = 30g$	M1
		A1 A1 (7)
(b)		
	$2F + F = 40g + 20g + 60g$	M1 A1
	$M(Q)$, $60gx + 20g.0.8 = 40g.0.4 + F.1.6$	
	solving	M1 A1 M1
	$QX = x = \frac{16}{15}\text{ m} = 1.07\text{m}$	A1 (6)
		[13]

Question Number	Scheme	Marks
5 (a)		B2 -1 e.e.o.o. (labels not needed) (2)
(b)	$F = \frac{1}{2}R$ $(\uparrow),\ R\cos\alpha + F\sin\alpha = mg$ $R = \frac{1.1g}{(\cos\alpha + \frac{1}{2}\sin\alpha)} = 9.8$ N $(\rightarrow),\ P + \frac{1}{2}R\cos\alpha = R\sin\alpha$ $P = R(\sin\alpha - \frac{1}{2}\cos\alpha)$ $= 1.96$	B1 M1 A2 M1 A1 (6) M1 A2 M1 A1 (5) **[13]**

Question Number	Scheme	Marks
6 (a)		
		
	$\tan\theta = \frac{2}{1} \Rightarrow \theta = 63.4^{o}$	M1 A1
	angle is 153.4^{o}	A1 (3)
(b)		B1
	$(4+p)\mathbf{i}+(q-5)\mathbf{j}$	
	$(q-5) = -2(4+p)$	M1 A1
	$2p+q+3=0$ *	A1 (4)
(c)	$q=1 \Rightarrow p=-2$	B1
	$\Rightarrow\ \mathbf{R} = 2\mathbf{i}-4\mathbf{j}$	M1
	$\Rightarrow \lvert\mathbf{R}\rvert = \sqrt{2^2+(-4)^2} = \sqrt{20}$	M1 A1 f.t.
	$\sqrt{20} = m8\sqrt{5}$	M1 A1 f.t.
	$\Rightarrow m = \frac{1}{4}$	A1 cao (7)
		[14]

Question Number	Scheme	Marks
7 (a)	$T - 5g\sin\alpha = 5a$ $15g - T = 15a$ solving for a $a = 0.6g$ solving for T $T = 6g$	M1 A1 M1 A1 M1 A1 M1 A1 (8)
(b)	For Q : $5g -\quad N = 5a$ $N = 2g$	M1 A1 A1 f.t. (3)
(c)	 $F = 2T\cos(\frac{90^{\circ} - \alpha}{2})$ $= 12g\cos 26.56..^{\circ}$ $= 105\text{ N}$	M1 A2 A1 f.t. A1 (5) **[16]**

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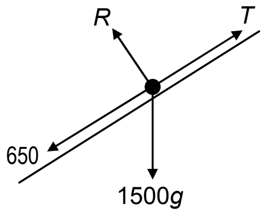
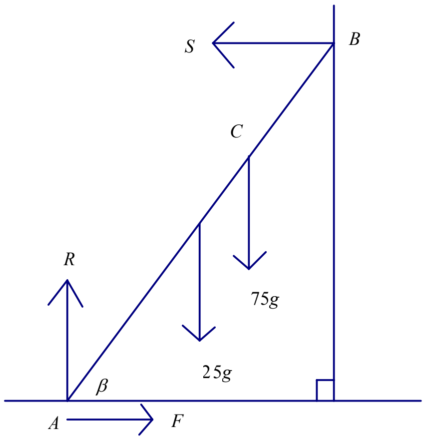
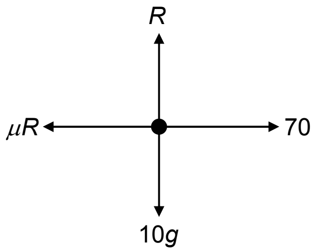
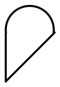
Mark Scheme

Question Number	Scheme	Marks
1	 F = ma parallel to the slope, $T - 1500g\sin\theta - 650 = 1500a$ Tractive force, 30000 = T x 15 $a = \dfrac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ <u>0.2</u> (m s^{-2})	M1* A1 M1* d*M1 A1 (5) **[5]**
2 (a)	 $\mathrm{R}(\uparrow): R = 25g + 75g (= 100g)$ $F = \mu R \Rightarrow F = \dfrac{11}{25} \times 100g$ = 44g (=431)	B1 M1 A1 (3)
(b)	M(A): $25g \times 2\cos\beta + 75g \times 2.8\cos\beta$ $= S \times 4\sin\beta$ $\mathrm{R}(\leftrightarrow): F = S$ $176g\sin\beta = 260g\cos\beta$ $\beta = 56(°)$	M1 A2,1,0 M1A1 A1 (6)
(c)	So that Reece's weight acts directly at the point C.	B1 **[10]**

Question Number	Scheme	Marks
3 (a)	 $R(\updownarrow): R = 10g$	B1
	$F = \mu R \;\Rightarrow\; F = \frac{4}{7}(10g) = 56$	B1
	$\therefore$ WD against friction $= \frac{4}{7}(10g)(50)$	M1
	2800(J)	A1
		(4)
(b)	$70(50) - \text{"2800"} = \tfrac{1}{2}(10)v^2 - \tfrac{1}{2}(10)(2)^2$	M1* A1ft
	$700 = 5v^2 - 20,\; 5v^2 = 720 \Rightarrow v^2 = 144$	d*M1
	Hence, $v = \underline{12}$ (m s^{-1})	A1 cao
		(4)
Or (b)	N2L$(\rightarrow)$: $70 - \frac{4}{7}R = 10a$	M1*
	$70 - \frac{4}{7} \times 10g = 10a$, $(a = 1.4)$	A1ft
	$AB\,(\rightarrow)$: $v^2 = (2)^2 + 2(1.4)(50)$	d*M1
	Hence, $v = \underline{12}$ (m s^{-1})	A1 cao
		(4)
		[8]
4 (a)	$v = 10t - 2t^2,\; s = \int v\,dt$	M1
	$= 5t^2 - \frac{2t^3}{3}(+C)$	A1
	$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)	A1
		(3)
(b)	$\underset{\sim}{s} = \int v\,dt = \underline{\frac{-432t^{-1}}{-1}}(+K) = \underline{\frac{432}{t}}(+K)$	<u>B1</u>
	t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*
	$\Rightarrow K = -36$	A1
	At $t = 10$, $s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1 <u>A1</u>
		(5) **[8]**

Question Number	Scheme	Marks

5 (a)

MR	 108	 18π	 $108+18\pi$
x_i (→) from *AD*	4	6	$\bar{x}$
y_i (↓) from *BD*	6	$-\frac{8}{\pi}$	$\bar{y}$

B1

B1

$AD(\rightarrow)$: $108(4)+18\pi(6)=(108+18\pi)\bar{x}$ M1

$$\bar{x}=\frac{432+108\pi}{108+18\pi}=4.68731...=\underline{4.69}\ \text{(cm) (3 sf)}\ \mathbf{AG}$$ A1

(4)

(b)

y_i (↓) from *BD*	6	$-\frac{8}{\pi}$	$\bar{y}$

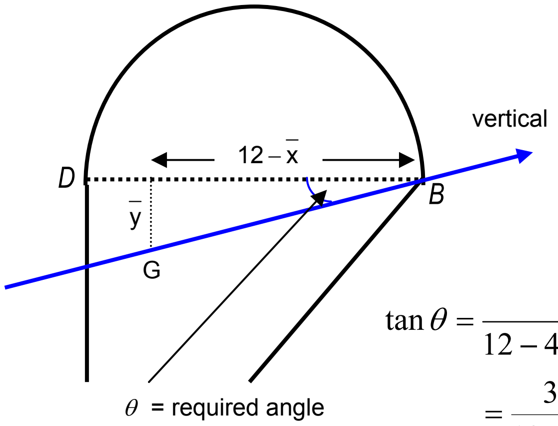
B1 oe

$BD(\downarrow)$: $108(6)+18\pi(-\frac{8}{\pi})=(108+18\pi)\bar{y}$ M1 A1ft

$$\bar{y}=\frac{504}{108+18\pi}=3.06292...=3.06\ \text{(cm) (3 sf)}$$ A1

(4)

(c)



M1

$$\tan\theta=\frac{\bar{y}}{12-4.68731..}$$ dM1

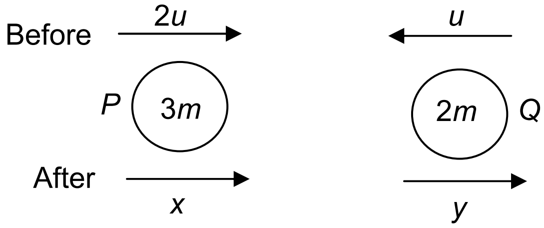
$$=\frac{3.06392..}{12-4.68731..}$$ A1

$\theta=22.72641...=\underline{23}$ (nearest degree) A1

(4)

[12]

Question Number	Scheme	Marks
6 (a)	Horizontal distance: $57.6 = p \text{ x } 3$	M1
	$p = 19.2$	A1
		(2)
(b)	Use $s = ut + \frac{1}{2}at^2$ for vertical displacement.	M1
	$-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$	A1
	$-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$	
	$q = \frac{43.2}{3} = 14.4$ ***AG***	A1 cso
		(3)
(c)	initial speed $\sqrt{p^2 + 14.4^2}$ (with their p)	M1
	$= \sqrt{576} = \underline{\underline{24}}$ (m s^{-1})	A1 cao
		(2)
(d)	$\tan\alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)	B1
		(1)
(e)	When the ball is 4 m above ground:	
	$3.1 = ut + \frac{1}{2}at^2$ used	M1
	$3.1 = 14.4t - \frac{1}{2}gt^2$ o.e $(4.9t^2 - 14.4t + 3.1 = 0)$	A1
	$\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied	M1
	$t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389\ldots$ or $2.70488\ldots$ awrt 0.23 and 2.7	A1
	duration $= 2.70488... - 0.23389...$	M1
	$= 2.47$ or 2.5 (seconds)	A1
		(6)
or 6 (e)	M1A1M1 as above	
	$t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$	A1
	Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e.	M1
	$= 2.47$ or 2.5 (seconds)	A1
		(6)
(f)	Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.	B1
		(1) **[15]**

Question Number	Scheme	Marks
7 (a)	 Correct use of NEL	M1*
	$y - x = e(2u + u)$ o.e.	A1
	CLM (→): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$)	B1
	Hence x = y – 3eu, 4u = 3(y-3eu) + 2y, ($u(9e+4) = 5y$)	d*M1
	Hence, speed of Q $= \underline{\frac{1}{5}(9e+4)u}$ **AG**	A1 cso
		(5)
(b)	$x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$	M1#
	Hence, speed P $= \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e.	A1
	$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e	d#M1
	gives, $e = \frac{3}{12} \Rightarrow \underline{e = \frac{1}{4}}$ **AG**	A1
		(4)
Or (b)	Using NEL correctly with given speeds of P and Q	M1#
	$3eu = \frac{1}{5}(9e+4)u - \frac{1}{2}u$	A1
	$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e	d#M1
	$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.	A1
		(4)
(c)	Time taken by Q from A to the wall $= \underline{\frac{d}{y}} = \left\{\frac{4d}{5u}\right\}$	M1†
	Distance moved by P in this time $= \underline{\frac{u}{2} \times \frac{d}{y}}$ $\left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$	A1
	Distance of P from wall $= d - x\left(\frac{d}{y}\right)$; $= d - \frac{2}{5}d = \underline{\frac{3}{5}d}$ **AG**	d†M1; A1 cso
		(4)
or (c)	Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u$ = 2:5	M1†
	So if Q moves a distance d, P will move a distance $\frac{2}{5}d$	A1
	Distance of P from wall $= d - \frac{2}{5}d ; = \underline{\frac{3}{5}d}$ **AG** cso	d†M1; A1
		(4)

Question Number	Scheme	Marks
(d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	Time for P, $T_{AB} = \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \dfrac{x}{\frac{1}{4}u}$ from their y	B1ft
	Hence $T_{AB} = T_{WB} \Rightarrow \dfrac{\frac{3d}{5} - x}{\frac{1}{2}u} = \dfrac{x}{\frac{1}{4}u}$	M1
	gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x,\ 3x = \frac{3d}{5} \Rightarrow x = \underline{\frac{1}{5}d}$	A1 **cao**
		(4)
or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y	B1ft
	Distance of B from wall $= \dfrac{1}{3} \times \dfrac{3d}{5}; = \dfrac{d}{5}$ their $\dfrac{1}{2+1}$	M1; A1
		(4)
2nd or (d)	After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y	B1ft
	Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$	
	Time from wall to 2nd collision $= \dfrac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y	B1ft
	Distance of B from wall = (their speed)x(their time) $= \dfrac{u}{4} \times \dfrac{4d}{5u}\ ; = \underline{\frac{1}{5}d}$	M1; A1
		(4) **[17]**

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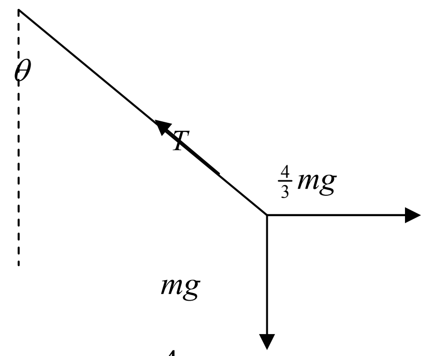
Mark Scheme

Question Number	Scheme	Marks
1	N2L $3a = -\left(9+\frac{15}{(t+1)^2}\right)$	B1
	$3v = -9t+\frac{15}{t+1}(+A)$	M1 A1ft
	$v=0, t=4 \Rightarrow 0=-36+3+A \Rightarrow A=33$	M1 A1
	$v=-3t+\frac{5}{t+1}+11$	
	$t=0 \Rightarrow v=16$	M1 A1 (7) **[7]**
2		
(a)	$(\leftarrow)$ $T\sin\theta = \frac{4}{3}mg$	M1 A1
	$(\uparrow)$ $T\cos\theta = mg$	A1
	$T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$	M1
	Leading to $T = \frac{5}{3}mg$	A1 (5)
(b)	HL $T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a}$ ft their T	M1 A1ft
	$e = \frac{5}{9}a$	
	$E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a}\times\left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$	M1 A1 (4) **[9]**

Question Number	Scheme	Marks
3	$\omega = \frac{80\times 2\pi}{60}$ rad s^{-1} $\left(= \frac{8\pi}{3} \approx 8.377...\right)$	B1
	Accept $v = \frac{16\pi}{75} \approx 0.67\,\text{ms}^{-1}$ as equivalent	
	$(\uparrow)\quad R = mg$	B1
	For least value of μ $\quad(\leftarrow)\quad \mu mg = mr\omega^2$	M1 A1=A1
	$\mu = \frac{0.08}{9.8}\times\left(\frac{8\pi}{3}\right)^2 \approx 0.57$ accept 0.573	M1 A1 (7)
		[7]
4 (a)	$a = 8$	B1
	$T = \frac{25}{2} = \frac{2\pi}{\omega} \quad\Rightarrow\quad \omega = \frac{4\pi}{25}\,(\approx 0.502\ ...)$	M1 A1
	$v^2 = \omega^2\left(a^2 - x^2\right) \quad\Rightarrow\quad v^2 = \left(\frac{4\pi}{25}\right)^2\left(8^2 - 3^2\right)$ ft their a, ω	M1 A1ft
	$v = \frac{4\pi}{25}\sqrt{55} \approx 3.7 \quad \left(\text{m}\,\text{h}^{-1}\right)$ awrt 3.7	M1 A1 (7)
(b)	$x = a\cos\omega t \quad\Rightarrow\quad 3 = 8\cos\left(\frac{4\pi}{25}t\right)$ ft their a, ω	M1 A1ft
	$t \approx 2.3602\ \ldots$	M1
	time is 12 22	A1 (4)
		[11]

Question Number	Scheme	Marks
5 (a)	Let x be the distance from the initial position of B to C GPE lost = EPE gained	
	$mgx\sin 30° = \dfrac{6mgx^2}{2a}$	M1 A1=A1
	Leading to $x = \dfrac{a}{6}$	M1
	$AC = \dfrac{7a}{6}$	A1 (5)
(b)	The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.	
	$(\nwarrow) \quad T = mg\sin 30° = \dfrac{6mge}{a}$	M1
	$e = \dfrac{a}{12}$	A1
	CE $\quad \dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}\left(\dfrac{a}{12}\right)^2 = mg\dfrac{a}{12}\sin 30°$	M1 A1=A1
	Leading to $\quad v = \sqrt{\left(\dfrac{ga}{24}\right)} = \dfrac{\sqrt{6ga}}{12}$	M1 A1 (7)
		[12]
	Alternative approaches to (b) *are considered on the next page.*	

Question Number	Scheme	Marks
5	*Alternative approach to* (b) *using calculus with energy.*	
	Let distance moved by B be x	
	CE $\quad \frac{1}{2}mv^2+\frac{6mg}{2a}x^2=mgx\sin 30°$	M1 A1=A1
	$v^2=gx-\frac{6g}{a}x^2$	
	For maximum v $\quad \frac{\mathrm{d}}{\mathrm{d}x}\left(v^2\right)=2v\frac{\mathrm{d}v}{\mathrm{d}x}=g-\frac{12g}{a}x=0$	M1 A1
	$x=\frac{a}{12}$	
	$v^2=g\left(\frac{a}{12}\right)-\frac{6g}{a}\left(\frac{a}{12}\right)^2=\frac{ga}{24}$	M1
	$v=\sqrt{\left(\frac{ga}{24}\right)}$	A1 (7)
	Alternative approach to (b) *using calculus with Newton's second law.*	
	As before, the centre of the oscillation is when extension is $\frac{a}{12}$	M1 A1
	N2L $\quad mg\sin 30°-T=m\ddot{x}$	
	$\frac{1}{2}mg-\frac{6mg\left(\frac{a}{12}+x\right)}{a}=m\ddot{x}$	M1 A1
	$\ddot{x}=-\frac{6g}{a}x \quad \Rightarrow \quad \omega^2=\frac{6g}{a}$	A1
	$v_{\max}=\omega a=\sqrt{\left(\frac{6g}{a}\right)}\times\frac{a}{12}=\sqrt{\left(\frac{ga}{24}\right)}$	M1 A1 (7)

Question Number	Scheme	Marks
6 (a)	$\int y^2\,\mathrm{d}x=\int\left(4-x^2\right)^2\mathrm{d}x=\int\left(16-8x^2+x^4\right)\mathrm{d}x$	
	$=16x-\frac{8x^3}{3}+\frac{x^5}{5}$	M1 A1
	$\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_0^2=\frac{256}{15}$	M1 A1
	$\int xy^2\,\mathrm{d}x=\int x\left(4-x^2\right)^2\mathrm{d}x=\int\left(16x-8x^3+x^5\right)\mathrm{d}x$	
	$=8x^2-2x^4+\frac{x^6}{6}$	M1 A1
	$\left[8x^2-2x^4+\frac{x^6}{6}\right]_0^2=\frac{32}{3}$	M1A1
	$\overline{x}=\frac{\int xy^2\,\mathrm{d}x}{\int y^2\,\mathrm{d}x}=\frac{32}{3}\times\frac{15}{216}=\frac{5}{8}$ *	M1 A1 (10)
(b)	$A\times\overline{x}=\left(\pi r^2 l\right)\times\frac{l}{2}$	M1
	$\frac{256}{15}\pi\times\frac{5}{8}=\pi\times16l\times\frac{l}{2}$	A1 ft
	Leading to $l=\frac{2\sqrt{3}}{3}$ accept exact equivalents or awrt 1.15	M1 A1 (4)
		[14]

Question Number	Scheme	Marks
7 (a)	Let speed at C be u	
	CE $\frac{1}{2}mu^2-\frac{1}{2}m\left(\frac{ag}{4}\right)=mga\left(1-\cos\theta\right)$	M1 A1
	$u^2=\frac{9ga}{4}-2ga\cos\theta$	
	$mg\cos\theta\ \left(+R\right)=\frac{mu^2}{a}$	M1 A1
	$mg\cos\theta=\frac{9mg}{4}-2mg\cos\theta$ eliminating u	M1
	Leading to $\cos\theta=\frac{3}{4}$ ∗	M1 A1 (7)
(b)	At C $u^2=\frac{9ga}{4}-2ga\times\frac{3}{4}=\frac{3}{4}ga$	B1
	$(\rightarrow)$ $u_x=u\cos\theta=\sqrt{\left(\frac{3ga}{4}\right)}\times\frac{3}{4}=\sqrt{\left(\frac{27ga}{64}\right)}=2.033\sqrt{a}$	M1 A1ft
	$(\downarrow)$ $u_y=u\sin\theta=\sqrt{\left(\frac{3ga}{4}\right)}\times\frac{\sqrt{7}}{4}=\sqrt{\left(\frac{21ga}{64}\right)}=1.792\sqrt{a}$	M1
	$v_y^2=u_y^2+2gh\ \Rightarrow\ v_y^2=\frac{21}{64}ga+2g\times\frac{7}{4}a=\frac{245}{64}ga$	M1 A1
	$\tan\psi=\frac{v_y}{u_x}=\sqrt{\left(\frac{245}{27}\right)}\approx 3.012\ldots$	M1
	$\psi\approx 72^\circ$ awrt 72°	A1 (8)
	Or 1.3^c (1.2502^c) awrt 1.3^c	**[15]**
	Alternative for the last five marks Let speed at P be v.	
	CE $\frac{1}{2}mv^2-\frac{1}{2}m\left(\frac{ag}{4}\right)=mg\times 2a$ or equivalent	M1
	$v^2=\frac{17mga}{4}$	M1 A1
	$\cos\psi=\frac{u_x}{v}=\sqrt{\left(\frac{27}{64}\times\frac{4}{17}\right)}=\sqrt{\left(\frac{27}{272}\right)}\approx 0.315$	M1
	$\psi\approx 72^\circ$ awrt 72°	A1
	Note: The time of flight from C to P is $\frac{\sqrt{235}-\sqrt{21}}{8}\sqrt{\left(\frac{a}{g}\right)}\approx 1.38373\sqrt{\left(\frac{a}{g}\right)}$	

January 2009
6683 Statistics S1
Mark Scheme

Question Number	Scheme	Marks
1		
(a)	$S_{xx} = 57.22 - \dfrac{(21.4)^2}{10} = 11.424$	M1 A1
	$S_{xy} = 313.7 - \dfrac{21.4 \times 96}{10} = 108.26$	A1 (3)
(b)	$b = \dfrac{S_{xy}}{S_{xx}} = 9.4765...$	M1 A1
	$a = \overline{y} - b\overline{x} = 9.6 - 2.14b = (-10.679...)$	M1
	$y = -10.7 + 9.48x$	A1 (4)
(c)	Every (extra) <u>hour</u> spent using the programme produces about <u>9.5 marks improvement</u>	B1ft (1)
(d)	$y = -10.7 + 9.48 \times 3.3, = 20.6$ awrt 21	M1,A1 (2)
(e)	Model may not be valid since [8h is] outside the range [0.5 - 4].	B1 (1) **[11]**
(a)	M1 for a correct expression 1st A1 for AWRT 11.4 for S_{xx} 2nd A1 for AWRT 108 for S_{xy} Correct answers only: One value correct scores M1 and appropriate A1, both correct M1A1A1	
(b)	1st M1 for using their values in correct formula 1st A1 for AWRT 9.5 2nd M1 for correct method for *a* (minus sign required) 2nd A1 for equation with *a* and *b* AWRT 3 sf (e.g. $y = -10.68 + 9.48x$ is fine) Must have a full equation with *a* and *b* correct to awrt 3 sf	
(c)	B1ft for comment conveying the idea of <u>*b* marks per hour</u>. Must mention value of *b* but can ft their value of *b*. No need to mention "extra" but must mention "marks" and "hour(s)" e.g. " …9.5 times per hour …" scores B0	
(d)	M1 for sub $x = 3.3$ into their regression equation from the end of part (b) A1 for awrt 21	
(e)	B1 for a statement that says or implies that it may <u>not</u> be valid because <u>outside the range</u>. They do not have to mention the values concerned here namely 8 h or 0.5 - 4	

Question Number	Scheme	Marks
2	E = take regular exercise $\qquad B$ = always eat breakfast	
(a)	$\text{P}(E \cap B) = \text{P}(E \mid B) \times \text{P}(B)$	M1
	$= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$	A1 (2)
(b)	$\text{P}(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}$ | or $\text{P}(E' \mid B')$ | or $\text{P}(B' \cap E)$ | or $\text{P}(B \cap E')$	M1
	$= \frac{62}{75}$ | $= \frac{13}{25}$ | $= \frac{12}{75}$ | $= \frac{32}{75}$	A1
	$\text{P}(E' \cap B') = 1 - \text{P}(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$	M1 A1 (4)
(c)	$\text{P}(E \mid B) = 0.36 \neq 0.40 = \text{P}(E)$ or $\text{P}(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = \text{P}(E) \times \text{P}(B)$	M1 A1 (2)
	So E and B are <u>not</u> statistically independent	**[8]**

(a) M1 for $\frac{9}{25} \times \frac{2}{3}$ or $\text{P}(E|B) \times \text{P}(B)$ <u>and</u> at least one correct value seen. A1 for 0.24 or exact equiv.

NB $\frac{2}{5} \times \frac{2}{3}$ alone or $\frac{2}{5} \times \frac{9}{25}$ alone scores M0A0. Correct answer scores full marks.

(b) 1st M1 for use of the addition rule. Must have 3 terms and some values, can ft their (a)

<u>Or</u> a full method for P(*E'*|*B'*) requires 1 - P(*E*|*B'*) and equation for P(*E*|*B'*): $(\text{a}) + \frac{x}{3} = \frac{2}{5}$

<u>Or</u> a full method for $\text{P}(B' \cap E)$ <u>or</u> $\text{P}(B \cap E')$ [or other valid method]

2nd M1 for a method leading to answer e.g. $1 - \text{P}(E \cup B)$

<u>or</u> $\text{P}(B') \times \text{P}(E' \mid B')$ <u>or</u> $\text{P}(B') - \text{P}(B' \cap E)$ <u>or</u> $\text{P}(E') - \text{P}(B \cap E')$

<u>Venn Diagram</u> 1st M1 for diagram with attempt at $\frac{2}{5} - \text{P}(B \cap E)$ or $\frac{2}{3} - \text{P}(B \cap E)$. Can ft their (a)

1st A1 for a correct first probability as listed or 32, 18 and 12 on Venn Diagram

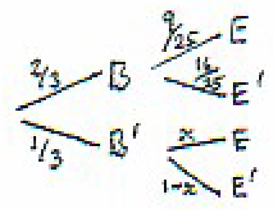
2nd M1 for attempting 75 - their (18 +32 + 12)

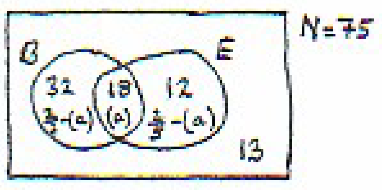
(c) M1 for identifying suitable values to test for independence e.g. P(*E*) = 0.40 and P(*E*|*B*) = 0.36

<u>Or</u> $\text{P}(E) \times \text{P}(B) = \ldots$ and $\text{P}(E \cap B) =$ their (a) [but their (a) $\neq \frac{2}{5} \times \frac{2}{3}$]. Values seen somewhere

A1 for correct values and a correct comment

<u>Diagrams</u> You may see these or find these useful for identifying probabilities.





<u>Common Errors</u>

(a) $\frac{9}{25}$ is M0A0

(b) P(*E*U*B*)= $\frac{53}{75}$ scores M1A0

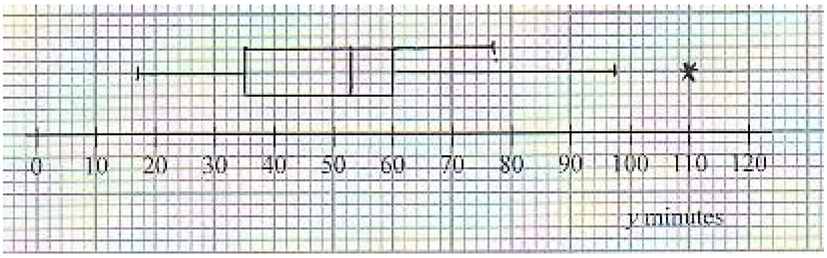
$1 - \text{P}(E \cup B) = \frac{22}{75}$ scores M1A0

(b) $\text{P}(B') \times \text{P}(E') = \frac{1}{3} \times \frac{3}{5}$ scores 0/4

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3 (a)</td><td>$E(X) = 0\times 0.4 + 1\times 0.3 + ... + 3\times 0.1, \quad = \quad 1$</td><td>M1, A1 (2)</td></tr>
<tr><td>(b)</td><td>$F(1.5) = [P(X \le 1.5) =]\ P(X \le 1), \quad = 0.4 + 0.3 = 0.7$</td><td>M1, A1 (2)</td></tr>
<tr><td>(c)</td><td>$E(X^2) = 0^2\times 0.4 + 1^2\times 0.3 + ... + 3^2\times 0.1 \quad , = \ 2$
$\text{Var}(X) = 2 - 1^2 \ , \ = \ 1 \qquad (*)$</td><td>M1, A1
M1, A1cso
(4)</td></tr>
<tr><td>(d)</td><td>$\text{Var}(5-3X) = (-3)^2\ \text{Var}(X), \quad = \ 9$</td><td>M1, A1 (2)</td></tr>
<tr><td>(e)</td><td>
<table>
<tr><th>Total</th><th>Cases</th><th>Probability</th></tr>
<tr><td rowspan="3">4</td><td>$(X=3)\cap(X=1)$</td><td>$0.1\times 0.3 = 0.03$</td></tr>
<tr><td>$(X=1)\cap(X=3)$</td><td>$0.3\times 0.1 = 0.03$</td></tr>
<tr><td>$(X=2)\cap(X=2)$</td><td>$0.2\times 0.2 = 0.04$</td></tr>
<tr><td rowspan="2">5</td><td>$(X=3)\cap(X=2)$</td><td>$0.1\times 0.2 = 0.02$</td></tr>
<tr><td>$(X=2)\cap(X=3)$</td><td>$0.2\times 0.1 = 0.02$</td></tr>
<tr><td>6</td><td>$(X=3)\cap(X=3)$</td><td>$0.1\times 0.1 = 0.01$</td></tr>
</table>
Total probability = 0.03 + 0.03+0.04 +0.02 + 0.02 + 0.01 = 0.15</td><td>B1B1B1
M1
A1
A1 (6)
[16]</td></tr>
<tr><td>(a)</td><td colspan="2">M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k(\ne 1)$ is M0.</td></tr>
<tr><td>(b)</td><td colspan="2">M1 for F(1.5) = P($X \le 1$).[**Beware**: $2\times 0.2 + 3\times 0.1 = 0.7$ but scores M0A0]</td></tr>
<tr><td>(c)

ALT</td><td colspan="2">1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0. Condone calling $E(X^2)$ =Var(X).
1st A1 is for an answer of 2 or a fully correct expression.
2nd M1 for $-\mu^2$, condone 2 – 1, unless clearly 2 - □. Allow $2-\mu^2$, with □ = 1 even if E(X) ≠ 1
2nd A1 for a fully correct solution with no incorrect working seen, **both** Ms required.
$\underline{\sum (x-\mu)^2 \times P(X=x)}$
1st M1 for an attempt at a full list of $(x-\mu)^2$ values and probabilities. 1st A1 if all correct
2nd M1 for at least 2 non-zero terms of $(x-\mu)^2 \times P(X=x)$ seen. 2nd A1 for 0.4 + 0.2 + 0.4 = 1</td></tr>
<tr><td>(d)</td><td colspan="2">M1 for use of the correct formula. -3^2 Var(X) is M0 unless the final answer is >0.
Can follow through their Var(X) for M1</td></tr>
<tr><td>(e)

ALT</td><td colspan="2">1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0
2nd B1 for all cases listed for 2 totals }
3rd B1 for a complete list of all 6 cases }These may be highlighted in a table
Using Cumulative probabilities
1st B1 for one or more cumulative probabilities used e.g.2 then 2 or more or 3 then 1 or more
2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥2; 3, ≥1
M1 for one correct pair of correct probabilities multiplied
1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.
2nd A1 for 0.15 or exact equivalent only as the final answer.</td></tr>
</table>

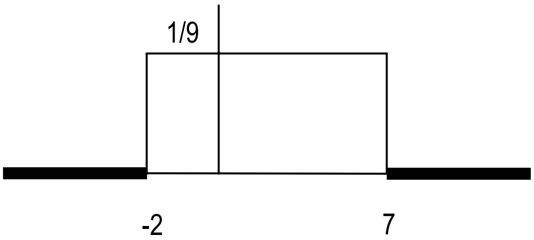
Question Number	Scheme	Marks
4 (a)	$Q_2 = 53, \quad Q_1 = 35, \quad Q_3 = 60$	B1, B1,B1 (3)
(b)	$Q_3 - Q_1 = 25 \Rightarrow Q_1 - 1.5 \times 25 = -2.5$ (no outlier) $Q_3 + 1.5 \times 25 = 97.5$ (so 110 is an outlier)	M1 A1 (2)
(c)		M1 A1ft A1ft (3)
(d)	$\sum y = 461, \sum y^2 = 24\ 219 \therefore\ S_{yy} = 24219 - \frac{461^2}{10}$, $= 2966.9$ (*)	B1, B1, B1cso (3)
(e)	$r = \frac{-18.3}{\sqrt{3463.6 \times 2966.9}}$ or $\frac{-18.3}{3205.64...} = -0.0057$ AWRT - 0.006 or -6×10^{-3}	M1 A1 (2)
(f)	*r* suggests correlation is close to zero so parent's claim is not justified	B1 (1) **[14]**
(a)	1st B1 for median 2nd B1 for lower quartile 3rd B1 for upper quartile	
(b)	M1 for attempt to find one limit A1 for both limits found and correct. No explicit comment about outliers needed.	
(c)	M1 for a box and two whiskers 1st A1ft for correct position of box, median and quartiles. Follow through their values. 2nd A1ft for 17 and 77 or "their" 97.5 and *. If 110 is not an outlier then score A0 here. Penalise no gap between end of whisker and outlier. Must label outlier, needn't be with *. __Accuracy__ should be within the correct square so 97 or 98 will do for 97.5	
(d)	1st B1 for $\sum y$ N.B. $\left(\sum y\right)^2 = 212521$ and can imply this mark 2nd B1 for $\sum y^2$ or at least three correct terms of $\sum\left(y - \bar{y}\right)^2$ seen. 3rd B1 for complete correct expression seen leading to 2966.9. So all 10 terms of $\sum\left(y - \bar{y}\right)^2$	
(e)	M1 for attempt at correct expression for *r*. Can ft their S_{yy} for M1.	
(f)	B1 for comment <u>rejecting</u> parent's claim on basis of <u>weak or zero</u> correlation Typical error is "negative correlation so comment is true" which scores B0 Weak negative or weak positive correlation is OK as the basis for their rejection.	

Question Number	Scheme	Marks
5 (a)	8-10 hours: width = 10.5 - 7.5 = 3 represented by 1.5cm	
	16-25 hours: width = 25.5 - 15.5 = 10 so represented by <u>5 cm</u>	B1
	8- 10 hours: height = fd = 18/3 = 6 represented by 3 cm	M1
	16-25 hours: height = fd = 15/10 = 1.5 represented by <u>0.75 cm</u>	A1 (3)
(b)	$Q_2 = 7.5 + \frac{(52-36)}{18}\times 3 = 10.2$	M1 A1
	$Q_1 = 5.5 + \frac{(26-20)}{16}\times 2[= 6.25 \text{ or } 6.3]$ or $5.5+\frac{(26.25-20)}{16}\times 2$ [=6.3]	A1
	$Q_3 = 10.5 + \frac{(78-54)}{25}\times 5[= 15.3]$ or $10.5 + \frac{(78.75-54)}{25}\times 5$ [=15.45 \15.5]	A1
	IQR = (15.3 - 6.3) = <u>9</u>	A1ft (5)
(c)	$\sum fx = 1333.5 \Rightarrow \bar{x} = \frac{1333.5}{104} =$ AWRT <u>12.8</u>	M1 A1
	$\sum fx^2 = 27254 \Rightarrow \sigma_x = \sqrt{\frac{27254}{104} - \bar{x}^2} = \sqrt{262.05 - \bar{x}^2}$ AWRT <u>9.88</u>	M1 A1 (4)
(d)	$Q_3 - Q_2 [= 5.1] > Q_2 - Q_1 \ [= 3.9]$ or $Q_2 < \bar{x}$	B1ft
	So data is positively skew	dB1 (2)
(e)	Use median and IQR,	B1
	since data is skewed <u>or</u> not affected by extreme values or outliers	B1 (2) **[16]**
(a)	M1 For attempting both frequency densities $\frac{18}{3}(=6)$ and $\frac{15}{10}$, <u>and</u> $\frac{15}{10}\times \text{SF}$, where SF $\neq 1$ NB Wrong class widths(2 and 9) gives $\frac{h}{1.66...} = \frac{3}{9} \to h = \frac{5}{9}$ or 0.55… and scores M1A0	
(b)	M1 for identifying correct interval and a correct fraction e.g. $\frac{\frac{1}{2}(104)-36}{18}$. Condone 52.5 or 53 1st A1 for 10.2 for median. Using (n + 1) allow awrt 10.3 2nd A1 for a correct expression for either Q_1 or Q_3 (allow 26.25 and 78.75) 3rd A1 for correct expressions for both Q_1 and Q_3 4th A1ft for IQR, ft their quartiles. Using (n + 1) gives 6.28 and 15.45	<u>NB</u>: <u>Must see</u> <u>some</u> <u>method</u>
(c)	1st M1 for attempting $\sum fx$ and $\bar{x}$ 2nd M1 for attempting $\sum fx^2$ and $\sigma_x, \sqrt{\ }$ is needed for M1. Allow s = awrt 9.93	
(d)	1st B1ft for suitable test, values need not be seen but statement must be compatible with values used. Follow through their values 2nd dB1 Dependent upon their test showing positive and for stating positive skew If their test shows negative skew they can score 1st B1 but lose the second	
(e)	1st B1 for choosing median and IQR. Must mention <u>both</u>. }<u>Award independently</u> 2nd B1 for suitable reason } e.g. "use median because data is skewed" scores B0B1 since IQR is not mentioned	

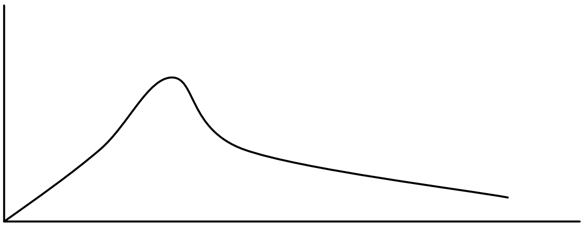
Question Number	Scheme	Marks
6 (a)	$P(X<39) = P\left(Z<\frac{39-30}{5}\right)$ $= P(Z<1.8) = \underline{0.9641}$ (allow awrt 0.964)	M1 A1 (2)
(b)	$P(X<d) = P\left(Z<\frac{d-30}{5}\right) = 0.1151$ $1-0.1151 = 0.8849$ $\Rightarrow \quad z=-1.2$ (allow ± 1.2) $\therefore \frac{d-30}{5} = -1.2$ $\underline{d=24}$	M1 B1 M1A1 (4)
(c)	$P(X>e) = 0.1151$ so $e = \mu + (\mu$ - their $d)$ <u>or</u> $\frac{e-30}{5} = 1.2$ or − their z $\underline{e=36}$	M1 A1 (2)
(d)	$P(d<X<e) = 1-2\times0.1151$ $= 0.7698$ AWRT <u>0.770</u>	M1 A1 (2) **[10]**
	Answer only scores all marks in each section BUT check (b) and (c) are in correct order	
(a)	M1 for standardising with σ, $z = \pm\frac{39-30}{5}$ is OK A1 for 0.9641 or awrt 0.964 but if they go on to calculate 1 – 0.9641 they get M1A0	
(b)	1st M1 for attempting 1- 0.1151. Must be seen in (b) in connection with finding d B1 for $z = \pm$ 1.2. They must state $z = \pm$ 1.2 or imply it is a z value by its use. This mark is only available in part (b). 2nd M1 for $\left(\frac{d-30}{5}\right) =$ their negative z value (or equivalent)	
(c)	M1 for a full method to find e. If they used $z = 1.2$ in (b) they can get M1 for $z = \pm$ 1.2 here If they use symmetry about the mean $\mu + (\mu$ - their $d)$ then ft their d for M1 Must explicitly <u>see</u> the method used unless the answer is correct.	
(d)	M1 for a complete method or use of a correct expression e.g. "their 0.8849" - 0.1151 <u>or</u> **If their d < their e** using their values with $P(X<e) - P(X<d)$ If their $d \geq$ their e then they can only score from an argument like 1 – 2x0.1151 A negative probability or probability > 1 for part (d) scores M0A0	

January 2009
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1	The random variable X is the number of daisies in a square.	
	Poisson(3)	B1
(a)	$1 - \mathrm{P}(X \le 2) = 1 - 0.4232 \qquad 1 - e^{-3}(1 + 3 + \frac{3^2}{2!})$	M1
	$= 0.5768$	A1
		(3)
(b)	$\mathrm{P}(X \le 6) - \mathrm{P}(X \le 4) = 0.9665 - 0.8153 \qquad e^{-3}\left(\frac{3^5}{5!} + \frac{3^6}{6!}\right)$	M1
	$= 0.1512$	A1
		(2)
(c)	$\mu = 3.69$	B1
	$\mathrm{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$	M1
	$= 3.73/3.72/3.71$ accept $s^2 = 3.77$	A1
		(3)
(d)	For a Poisson model , Mean = Variance ; For these data $3.69 \approx 3.73$ $\Rightarrow$ Poisson model	B1
		(1)
(e)	$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$ allow their mean or var	M1
	Awrt 0.193 or 0.194	A1 ft
		(2)

Question Number	Scheme	Marks
2 (a)	$\mathrm{f}(x)=\begin{cases}\frac{1}{9} & -2\le x\le 7\\ 0 & otherwise\end{cases}$	B1 B1 (2)
(b)		B1 B1 (2)
(c)	E(X) = <u>2.5</u> Var (X) = $\frac{1}{12}(7+2)^2$ or <u>6.75</u> both	B1
	E (X^2) = Var (X) + E(X)^2	M1
	= 6.75 + 2.5^2 = 13	A1
	alternative	(3)
	$\int_{-2}^{7} x^2 f(x)dx = \left[\frac{x^3}{27}\right]_{-2}^{7}$ $\int$ x^{2}"f(x)" attempt to integrate and use limits of -2 and 7	B1 M1
	= 13	A1
(d)	P(-0.2 $<X<$ 0.6) = $\frac{1}{9}\times 0.8$	M1
	= $\frac{4}{45}$ or 0.0889 0r equiv awrt 0.089	A1
		(2)

Question Number	Scheme	Marks
3 (a)	$X \sim \text{B}(20, 0.3)$	M1
	$\text{P}(X \leq 2) = 0.0355$	
	$\text{P}(X \geq 11) = 1 - 0.9829 = 0.0171$	
	Critical region is $(X \leq 2) \cup (X \geq 11)$	A1 A1 (3)
(b)	Significance level = 0.0355 + 0.0171, = 0.0526 or 5.26%	M1 A1 (2)
(c)	Insufficient evidence to reject H_0 **Or** sufficient evidence to accept H_0 /not significant	B1 ft
	$x = 3$ (or the value) is not in the critical region or $0.1071 > 0.025$	B1 ft (2)
	Do not allow inconsistent comments	

Question Number	Scheme	Marks
4 (a)	$\int_0^{10} kt\mathrm{dt} = 1$ or Area of triangle = 1	M1
	$\left[\frac{kt^2}{2}\right]_0^{10} = 1$ or 10 x0.5 x 10k =1 or linear equation in k	M1
	$50k = 1$	
	$k = \frac{1}{50}$ cso	A1
		(3)
(b)	$\int_6^{10} ktdt = \left[\frac{\mathrm{kt}^2}{2}\right]_6^{10}$	M1
	$= \frac{16}{25}$	A1
		(2)
(c)	$\mathrm{E}(T) = \int_0^{10} kt^2 dt = \left[\frac{\mathrm{kt}^3}{3}\right]_0^{10}$	M1
	$= 6\frac{2}{3}$	A1
	$\mathrm{Var\ (T)} = \int_0^{10} kt^3 dt - \left(6\frac{2}{3}\right)^2 = \left[\frac{kt^4}{4}\right]_0^{10}; -\left(6\frac{2}{3}\right)^2$	M1;M1dep
	$= 50 - \left(6\frac{2}{3}\right)^2$	
	$= 5\frac{5}{9}$	A1
		(5)
(d)	10	B1
		(1)
(e)		B1
		(1)

Question Number	Scheme	Marks
5 (a)	X represents the number of defective components.	
	P $(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1
		(2)
(b)	P $(X \geq 2) = 1 - \text{P}(X \leq 1)$	M1
	$= 1 - (p)^{10} - \text{(a)}$	A1√
	$= 0.0043$	A1
		(3)
(c)	$X \sim \text{Po}(2.5)$	B1B1
	$\text{P}(1 \leq X \leq 4) = \text{P}(X \leq 4) - \text{P}(X = 0)$	M1
	$= 0.8912 - 0.0821$	
	$= 0.809$	A1
		(4)
	Normal distribution used. B1for mean only	
	Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier. (a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0	
	For any other values of p which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0	

Question Number	Scheme	Marks
6 (a)(i)	$H_0 : \lambda = 7$ $\quad H_1 : \lambda > 7$	B1
	X = number of visits. $X \sim \text{Po}(7)$	B1
	$P(X \geq 10) = 1 - P(X \leq 9)$ $\quad 1 - P(X \leq 10) = 0.0985$	M1
	$= 0.1695$ $\quad 1 - P(X \leq 9) = 0.1695$	
	CR $X \geq 11$	A1
	$0.1695 > 0.10$, $\quad$ CR $X \geq 11$	
	Not significant or it is not in the critical region or do not reject H_0	M1
	The rate of visits on a Saturday is not greater/ is unchanged	A1 no ft
(ii)	$X = 11$	B1 (7)
(b)	(The visits occur) randomly/ independently or singly or constant rate	B1 (1)
(c)	[$H_0 : \lambda = 7$ $\quad H_1 : \lambda > 7$ $\quad$ (or $H_0 : \lambda = 14$ $\quad H_1 : \lambda > 14$)]	
	$X \sim$N;(14,14)	B1;B1
	$P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right)$ $\quad$ +/- 0.5, stand	M1 M1
	$= P(z \geq 1.47)$	
	$= 0.0708$ $\quad$ or $z = 1.2816$	A1dep both M
	$0.0708 < 0.10$ therefore significant. The rate of visits is greater on a Saturday	A1dep 2nd M (6)

Question Number	Scheme	Marks
7 (a)	$F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9}\,\mathrm{dx} = \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right]_1^x$	M1A1
	$=\left[-\frac{1}{9}x^2 + \frac{8}{9}x\right] - \left[-\frac{1}{9} + \frac{8}{9}\right]$	
	$= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$	A1
		(3)
(b)	$F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$	B1B1√
		(2)
(c)	$F(x) = 0.75$; or $F(2.5) = -\frac{1}{9}\times 2.5^2 + \frac{8}{9}\times 2.5 - \frac{7}{9}$	M1;
	$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$	
	$4x^2 - 32^x + 55 = 0$	
	$-x^2 + 8x - 13.75 = 0$	
	$x = 2.5$ $= 0.75$ cso	A1
	and $F(x) = 0.25$	
	$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$	M1
	$-x^2 + 8x - 7 = 2.25$	
	$-x^2 + 8x - 9.25 = 0$ quadratic 3 terms =0	M1 dep M1 dep
	$x = \frac{-8 \pm \sqrt{8^2 - 4\times -1\times -9.25}}{2\times -1}$	A1
	$x = 1.40$	
		(6)
(d)	$Q_3 - Q_2 > Q_2 - Q_1$	M1
	Or mode = 1 and mode < median	
	Or mean = 2 and median < mode	
	Sketch of pdf here or be referred to if in a different part of the question	
	Box plot with Q_1, Q_2, Q_3 values marked on	
	Positive skew	A1
		(2)

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6689 Decision D1
Mark Scheme

Question Number	Scheme	Marks

1

(a) e.g.

M	L	J	H	K	T	R	I	M1
J	H	I	K	M	L	T	R	A1
H	J	I	K	M	L	R	T	A1ft
H	I	J	K	L	M	R	T	A1ft
H	I	J	K	L	M	R	T	A1cso

(5)

Sort complete.

(b)

1st choice $\left[\frac{1+8}{2}\right] \rightarrow 5$ Lauren reject right — M1 A1

2nd choice $\left[\frac{1+4}{2}\right] \rightarrow 3$ John reject right — A1ft

3rd choice $\left[\frac{1+2}{2}\right] \rightarrow 2$ Imogen reject right

4th choice 1 Hannah reject

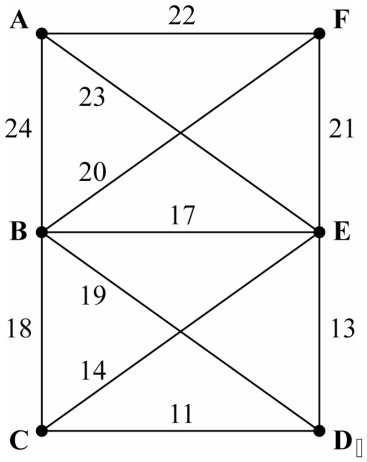
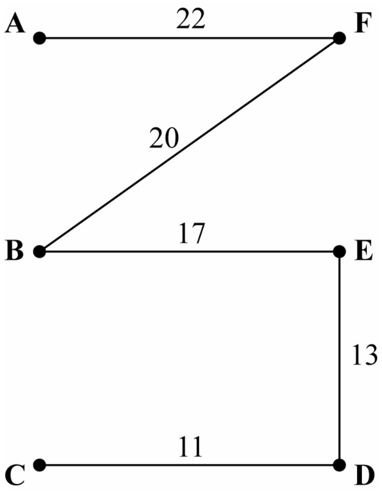
List now empty so Hugo not in list — A1 (4)

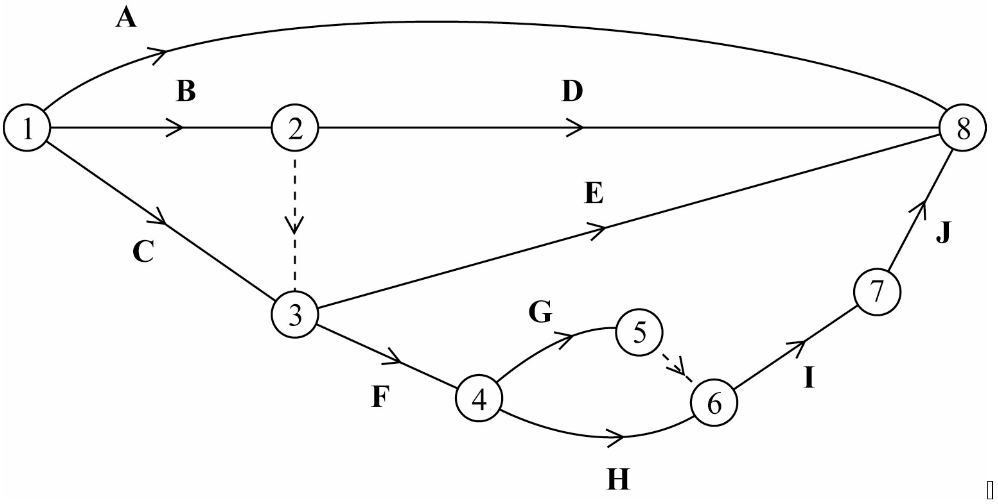
[9]

Notes:

(a) 1M1: quick sort, pivots, p, chosen and two sublists one <p one >p.
If choosing 1 pivot per iteration only M1 only.
1A1: first pass correct and next pivots chosen correctly/consistently.
2A1ft: second pass correct, next pivots correctly/consistently chosen.
3A1ft: third pass correct, next pivots correctly/consistently chosen.
4A1: all correct, cso.

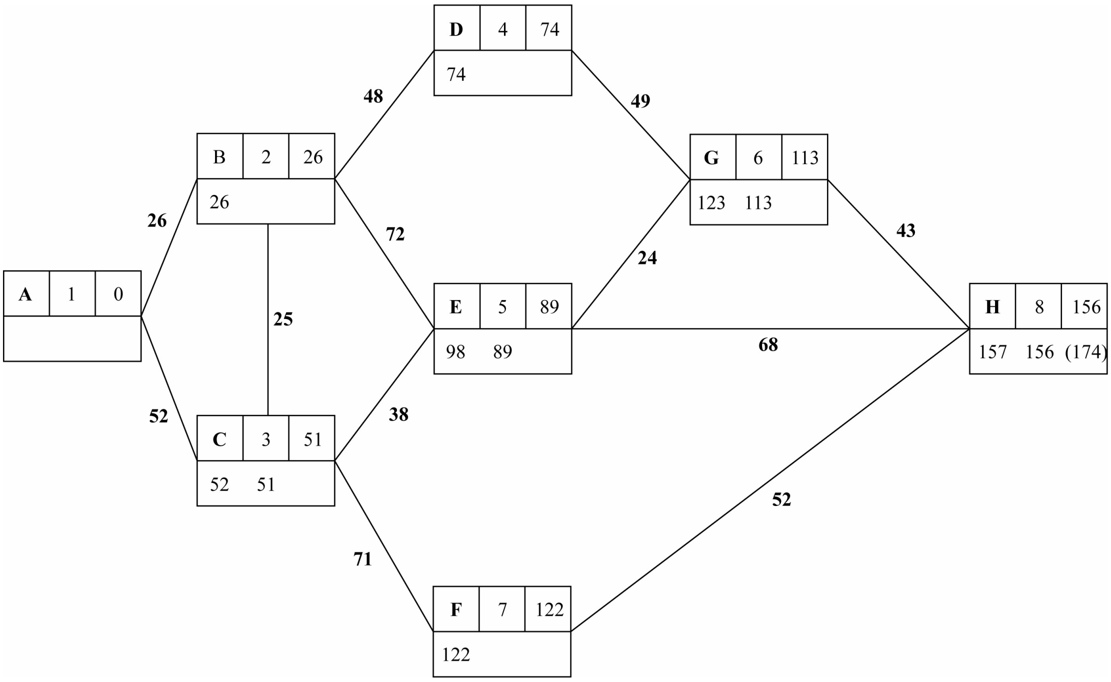
(b) 1M1: binary search, choosing pivot, rejecting half list. If using unsorted list, M0. Accept choice of K for M1 only.
1A1: first pass correct, condone 'sticky'pivot here, bod.
2A1ft: second pass correct, pivot rejected.
3A1: cso.

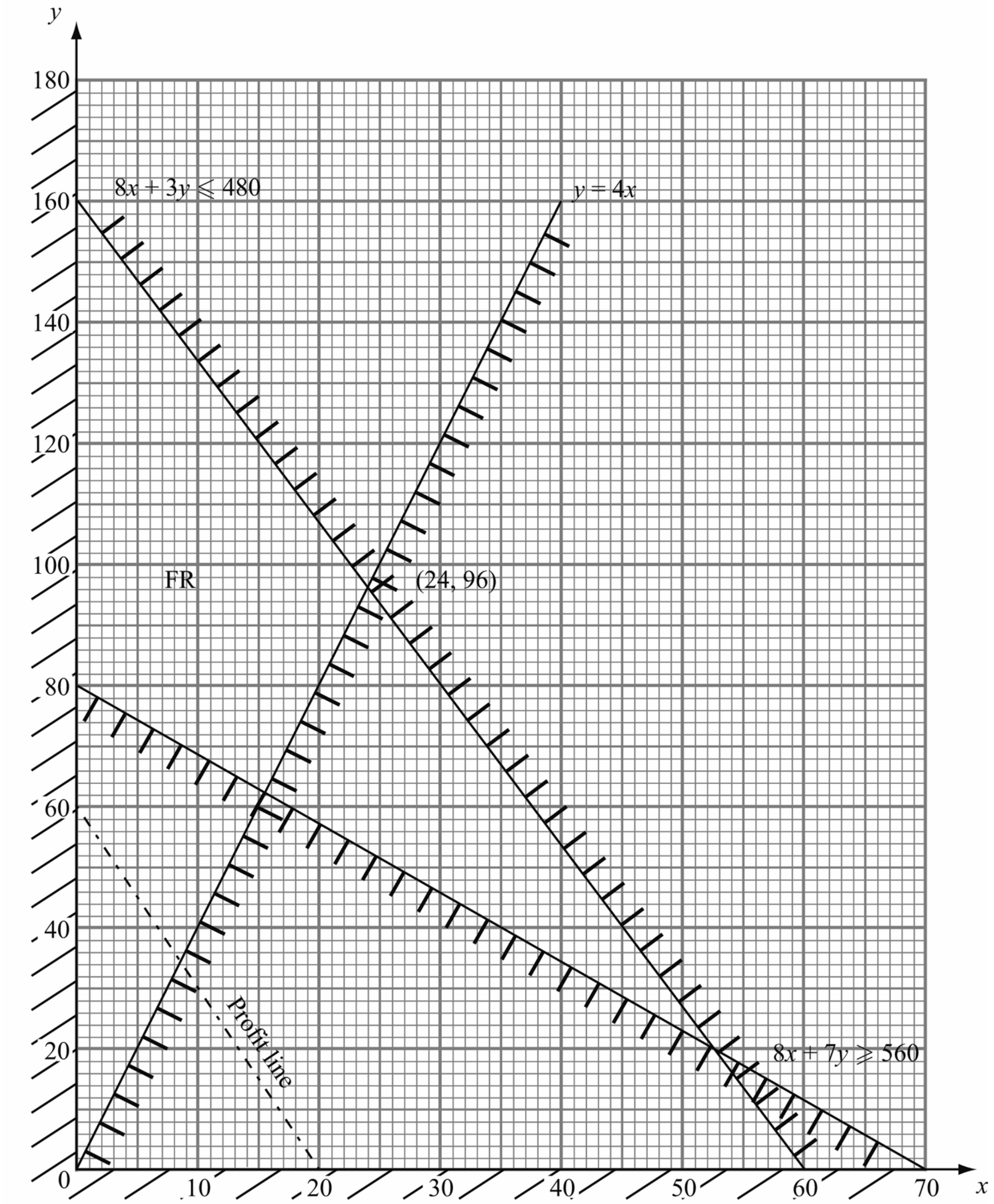
Question Number	Scheme	Marks
2 (a)		M1 A1 A1 (3)
(b)	CD, DE, reject CE, BE, reject BC, reject BD, BF, reject EF, AF 11 13 14 17 18 19 20 21 22	M1 A1 A1 (3)
	 Weight of tree 83 (m) **Notes:** (a) 1M1: More than 10 arcs 1A1: all arcs correct 2A1: all values correct (b) 1M1: First three arcs correctly chosen 1A1: All used acrs selected correctly 2A1: All rejected arcs selected in correct order (c) 1B1: CAO for arcs – numbers not needed. NO ft. 2B1: CAO 83, condone units	B1 B1 (2) **[8]**

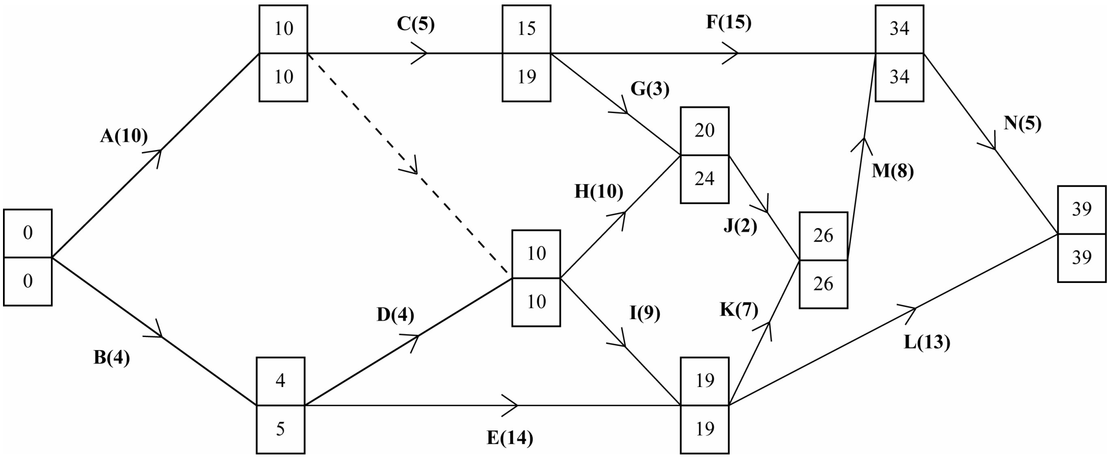
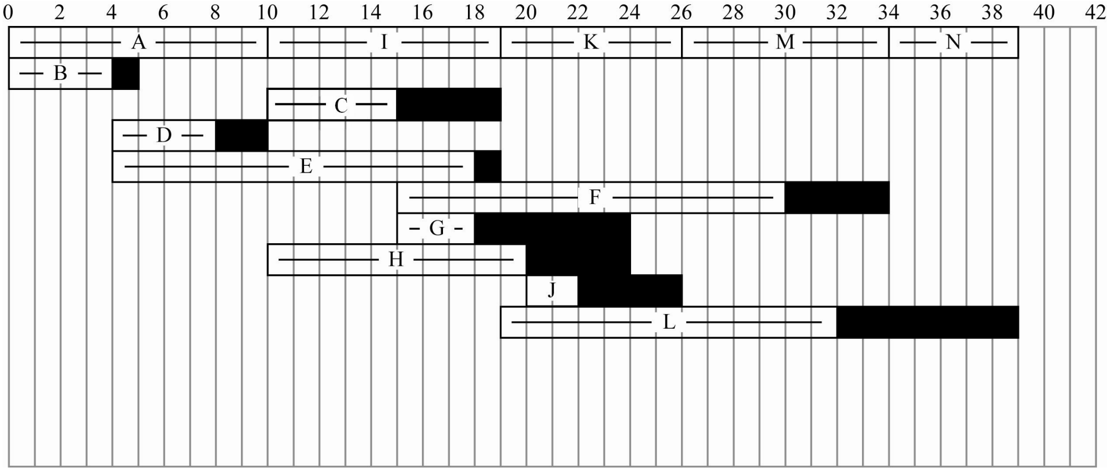
Question Number	Scheme	Marks
3 (a)		M1 A1 A1 A1 A1 (5)
(b)	1st dummy – D depends on B only, but E and F depend on B and C 2nd dummy – G and H both must be able to be described uniquely in terms of the events at each end.	B1 B1 (2) **[7]**
	Notes: (a) 1M1: one start and A to C and one of D, E or F drawn correctly 1A1: 1st dummy (+arrow) and D, E and F drawn correctly 2A1: G, H, I and J drawn in correct place 3A1: second dummy (+arrow) drawn in a correct place 4A1: cso. all arrows and one finish. (b) 1B1: cao, but B, C, D, E and/or F referred to, generous 2B1: cao, but generous.	

Question Number	Scheme	Marks
4		
(a)	Alternating path B – 3 = A – 5 change status B = 3 – A = 5	M1 A1
	A = 5 B = 3 C = 2 D = 1 E = 6 F unmatched	A1 (3)
(b)	e.g. C is the only person able to do 2 and the only person able to do 4. Or D, E and F between them can only be allocated to 1 and 6.	B2, 1, 0 (2)
(c)	Alternating path F – 6 = E – 1 = D – 2 = C – 4 change status F = 6 – E = 1 – D = 2 – C = 4	M1 A1
	A = 5 B = 3 C = 4 D = 2 E = 1 F = 6	A1 (3)
		[8]
	Notes: (a) 1M1: Path from B to 5. 1A1: Correct path including change status 2A1: CAO my matching, may be drawn but if so 5 lines only and clear. (b) 1B1: Close, a correct relevant, productive statement bod generous 2B1: A Good clear answer generous (c) 1M1: Path from F to 4. No ft. 1A1: Correct path penalise lack of change status once only 2A1: CAO may be drawn but if so 6 lines only and clear	

Question Number	Scheme	Marks
5		
(a)	Odd vertices C, D, E, G	B1
	CD + EG = 17 + 19 = 36 ←	M1 A1
	CE + DG = 12 + 25 = 37	
	CG + DE = 28 + 13 = 41	A1
	Length = 543 + 36 = 579 (km)	A1ft (5)
(b)	CE (12) is the shortest	M1
	So repeat CE (12)	A1ft
	Start and finish at D and G	A1ft (3)
		[8]
	Notes: (a) 1B1: cao (may be implicit) 1M1: Three pairings of their four odd nodes 1A1: one row correct 2A1: all correct 3A1ft: 543 + their least = a number. Condone lack of km (b) 1M1ft: Identifies their shortest from a choice of at least 2 rows. 1A1ft: indicates their intent to repeat shortest. 2A1ft: correct for their least.	

Question Number	Scheme	Marks
Q6 (a)		M1 A1 A1ft
	Shortest route: A B C E G H Length: 156 (km)	A1 A1ft (5)
(b)	New route: A B E G H Length: 165 (km)	B1 B1 (2) **[7]**
	Notes: (a) 1M1: Dijkstra's algorithm, small replacing larger in at least one of the sets of working values at C, E, G or H 1A1: Values correct at vertices A to E. 2A1ft: Values correct at vertices F to H, penalise order only once. 3A1: cao 4A1ft: 156ft (b) 1B1: cao ABEGH 2B1: 165 Special Case Accept 166 if ABDGH listed as the path.	

Question Number	Scheme	Marks
7 (a)		B1 B1 B1 (lines) B1 (shading) B1 (R found) B1 (labels) (6)
(b)	Point testing or Profit line method Minimum point (0, 80); Value of 80 Maximum point (24, 96); Value of 168	M1 A1 B1 A1 B1 A1 (6) **[12]**

Question Number	Scheme	Marks
8 (a)		M1 A1 M1 A1 (4)
(b)	A, I, K, M, N; Length 39	B2,1,0; B1 (3)
(c)	Float on F is 34 – 15 – 15 = 4 Float on G is 24 – 15 – 3 = 6	M1 A1 B1 (3)
(d)		M1 A1 M1 A1 (4)
(e)	e.g. At time 14 ½ there are 4 tasks I, E, H and C must be happening.	B2,1,0 (2) **[16]**

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Order Code UA020941 January 2009

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