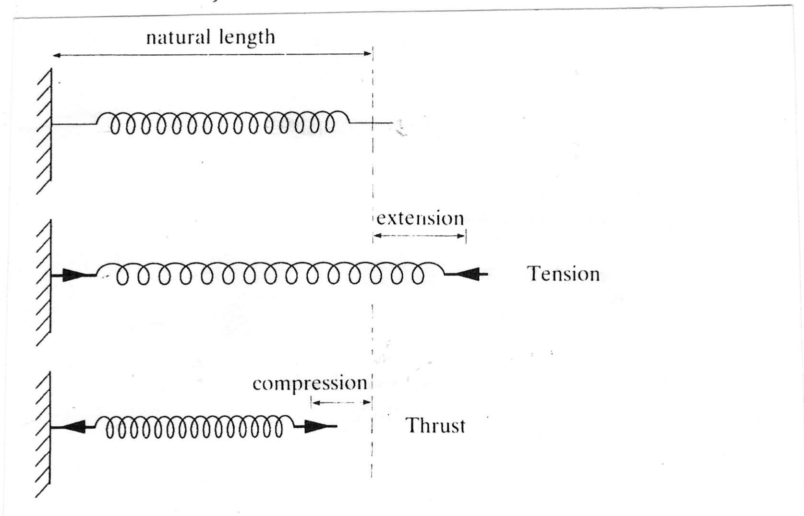
Strings & Springs

So far in situations involving strings it has been assumed that the strings do not stretch when they are under tension. Such strings are called *inextensible*. For some materials this is a good assumption, but for others the length of the string increases significantly under tension. Strings and springs which stretch are said to be *elastic*. *Open coiled springs* are springs which can also be compressed.

The length of a string or spring when there is no force applied to it is called its *natural length*. If it is stretched the increase in length is called its *extension* and if a string is compressed it is said to have a *compression*.

When stretched, a spring exerts an inward force or tension on whatever is attached to its ends. When compressed it exerts an outward force or *thrust* on its ends. An elastic string exerts a tension when stretched, but when slack exerts no force.



Hooke's law

In 1678 Robert Hooke formulated a *Rule or law of nature in every springing body* which, for small extensions relative to the length of the string or spring can be stated as follows:

The tension in an elastic spring or string is proportional to the extension. If a spring is compressed the thrust is proportional to the decrease in length of the spring.

When a string or spring is described as elastic, it means that it is reasonable to apply the modelling assumption that it obeys Hooke's law. A further assumption, that it is light (i.e. has zero mass) is also usual.

There are three different ways in which Hooke's law can be expressed, but it is normally used in the form:

$$T = \frac{\lambda x}{l}$$

Where T is the tension in the string, x is the extension produced and l is the natural length of the string. The constant λ is called the *modulus of elasticity* of the string and will be the same for any string of a given cross section made out of the same material.

Examples

1. A light elastic string of natural length 0.7m and modulus of elasticity 50N has one end fixed and a particle of mass 1.4kg attached to the other. The system hangs vertically in equilibrium. Find the extension in the string.

2. A spring is of natural length 1.5m and modulus 25N. Find the thrust in the spring when it is compressed to a length of 1.2m.

Exercise 2A Pg 33 Q's 2 to 8 evens

3. A particle of mass 0.4kg is attached to the midpoint of a light elastic string of natural length 1m and modulus λN. The string is then stretched between a point A in the top of a doorway and a point B which is on the floor 2m vertically below A.
   a. Find, in terms of λ, the extensions of the two parts of the string
   b. Calculate their values in the case where $\lambda = 9.8$N
   c. Find the minimum value of λ which will ensure that the lower half of the string won't go slack.

Exercise 2A Q's 10 to 14 evens

4. A light elastic string has one end fixed and a body of mass $\sqrt{3}$ kg freely suspended from its other end. With a horizontal force of X N acting on the body, the system is in equilibrium with the string extended to twice its natural length and making an angle of 30° with the downward vertical. Find the modulus of the string and the value of X.

5. A particle of mass 5kg is attached to one end of a light elastic string of natural length 1m and modulus 4g N. The other end of the string is fastened to a fixed point O at the top of a smooth slope that is inclined at $\tan^{-1}(3/4)$ to the horizontal. The particle is held on the slope at a point that is 2.5m from O down a line of greatest slope. If the particle is released from rest, find its initial acceleration towards O. What would the acceleration have been had the slope been rough, $\mu = 0.25$?

Exercise 2A Q's rest of the evens