Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6676/01)

Question Number	Scheme	Marks
1.	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 0 + \cos 0.6 \ (= 0.825335\ldots)$ May be implicit	B1
	$y_1 \approx 0.05\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 + y_0$ $(= 0.05 \times 0.825335\ldots + 0.6)$	M1
	$y_1 \approx 0.641266\ldots$ $= 0.6413$ (4 d.p.) Allow awrt	A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 0.05 + \cos 0.641266\ldots$ [or $0.05 + \cos(0.6 + 0.05\cos 0.6)$]	A1ft
	$= 0.851338\ldots$ $y_2 \approx 0.05\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_1 + y_1$ $(= 0.05 \times 0.851338\ldots + 0.641266\ldots)$ Requires use of the differential equation to find $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_1$	M1
	$y_2 \approx 0.683833\ldots$ $= 0.6838$ (4 d.p.)	A1
		(6)

<u>Degree mode in calculator</u>:
Gives answers: 0.6500   (0.64999…)
                0.7025   (0.70248…)
This can score B1 M1 A0 A1ft M1 A0
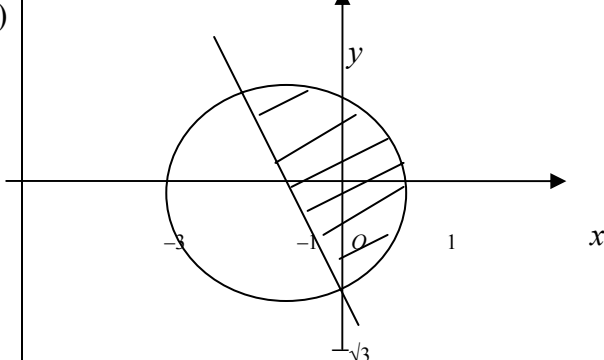
Question Number	Scheme	Marks
2. (a)	$\begin{pmatrix} 1 & p & 2 \\ 0 & 3 & q \\ 2 & p & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1+2p+2 \\ 6+q \\ 2+2p+1 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda \\ \lambda \end{pmatrix}$ is M1 A1 (2 eqns implied) $\begin{pmatrix} 3+2p \\ 6+q \\ 3+2p \end{pmatrix} \Rightarrow 6+q = 2(3+2p)$ is M1 A1 (2 eqns, use of parameter implied) $1+2p+2 = \lambda$ $6+q=2\lambda$ M: Two equations, one in p, one in q $\therefore$ $6+q = 6+4p \Rightarrow q = 4p$ (*)	M1 A1 A1 (3)

(b)	$\begin{vmatrix} -4 & p & 2 \\ 0 & -2 & 4p \\ 2 & p & -4 \end{vmatrix} = 0$ or $\begin{vmatrix} 1-\lambda & p & 2 \\ 0 & 3-\lambda & 4p \\ 2 & p & 1-\lambda \end{vmatrix} = 0$ (or with q instead of $4p$)
	
	 $[-4(8-4p^2) - p(0-8p) + 2(0+4) = 0]$ $p^2 = 1$ or $pq = 4$
	    $p<0$ $p=-1$ $q=-4$ M: Use $q = 4p$ to find value of p and of q
	                     A1: Positive values must be rejected
		M1
		
		 A1
		 dM1 A1 (4)

(c)	$-4x-y+2z = 0,\; -2y-4z = 0,\; 2x-y-4z = 0$ Any 2 eqns, with value of p
	             $2x = -y = 2z$ (or 2 separate equations)
	
	         E.vector is $k\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ (Any non-zero value of k)
		M1
		 M1
		
		 A1 (3)

		(10)
	(a) Assuming a value for λ, e.g. $\lambda = 1$, gives M1 A0 A0. (a) Assuming result and working 'backwards': $\begin{pmatrix} 1 & p & 2 \\ 0 & 3 & 4p \\ 2 & p & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3+2p \\ 6+4p \\ 3+2p \end{pmatrix} = (3+2p)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, gives M1 A0 A0 (b) <u>Alternative</u>: $\begin{pmatrix} 1 & p & 2 \\ 0 & 3 & 4p \\ 2 & p & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 5\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -4 & p & 2 \\ 0 & -2 & 4p \\ 2 & p & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (or q instead of $4p$) $x + py + 2z = 5x,\; 3y + 4pz = 5y,\; 2x + py + z = 5z$ $py + 2z = 4x$ (i), $2pz = y$ (ii), $2x + py = 4z$ (iii) From (i) and (iii) $py = 2z$ From (ii) $p^2 = 1$ (or equiv. in terms of p and/or q) $p<0,\, p=-1,\, q=-4$ A1: Positive values must be rejected (b) Using the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ scores <u>no marks</u> in this part.	M1 A1 dM1 A1

Question Number	Scheme	Marks
3. (a)	$$\left(x^2+1\right)\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}=4y\frac{dy}{dx}+(1-2x)\frac{d^2y}{dx^2}-2\frac{dy}{dx}$$	M1 A1
	$$\left(x^2+1\right)\frac{d^3y}{dx^3}=(1-4x)\frac{d^2y}{dx^2}+(4y-2)\frac{dy}{dx} \qquad (*)$$	A1 (3)
(b)	$$\left(\frac{d^2y}{dx^2}\right)_0=3$$	B1
	$$\left(\frac{d^3y}{dx^3}\right)_0=5 \qquad\qquad \text{Follow through: } \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}+2$$	B1ft
	$$y=1+x+\frac{3}{2}x^2+\frac{5}{6}x^3....$$	M1 A1 (4)
(c)	$x=-0.5, \quad y\approx 1-0.5+0.375-0.104166...$ $\qquad\qquad = 0.77 \text{ (2 d.p.)} \qquad\qquad\qquad\qquad \text{[awrt 0.77]}$	B1 (1)
		(8)
	(a) M: Use of product rule (at least once) and implicit differentiation (at least once). (b) M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)	

4.	(a)	$\lvert (x-3) + iy \rvert = 2\lvert x + iy \rvert \Rightarrow (x-3)^2 + y^2 = 4x^2 + 4y^2$	M1 A1
		$\therefore\ x^2 + y^2 + 2x - 3 = 0$	
		$(x+1)^2 + y^2 = 4$	M1
		Centre $(-1, 0)$, radius 2	A1, A1 (5)

(b)

Circle, centre on *x*-axis   B1

$C(-1, 0), r = 2$ dB1ft
Follow through centre and radius, but dependent on first B1.
There must be indication of their '–3', '–1' or '1' on the *x*-axis and no contradictory evidence for their radius.

B1 dB1

Straight line   B1
Straight line through $(-1, 0)$, or perp. bisector of $(-3, 0)$ and $(0, \sqrt{3})$.   B1
Straight line through point of int. of circle & –ve *y*-axis, or through $(0, -\sqrt{3})$ B1

B1 B1 B1 (5)

	(c)	Shading (only) inside circle	B1
		Inside correct circle and all of the correct side of the correct line… this mark is dependent on <u>all</u> the previous B marks in parts (b) and (c).	dB1 (2)
			(12)

		(a) 1st M: Use $z = x + iy$, and attempt square of modulus of each side.
		Not squaring the 2 on the RHS would be M1 A0.
		2nd M: Attempting to express in the form $(x-a)^2 + (y-b)^2 = k$,
		or attempting centre and radius from the form $x^2 + y^2 + 2gx + 2fy + c = 0$

Question Number	Scheme	Marks
5. (a) $\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$		M1
$\qquad\qquad t(1+k) = 2t(k-4)$		dM1
$\qquad\qquad\qquad k = 9$		A1 (3)
(b) $\det \mathbf{A} = k^2 + 2(1-k)$ $\qquad\qquad$ (Must be seen in part (b))		M1
$\qquad\quad = (k-1)^2 + 1$, which is always positive		M1
$\qquad\quad$ **A** is non-singular		A1cso (3)
(c) $\mathbf{A}^{-1} = \dfrac{1}{k^2 - 2k + 2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$		M1 A1 (2)
(d) $k = 3, \quad \mathbf{A}^{-1} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$		B1
$\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$		M1 A1 (3)
Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4, -2x + 3y = -3$ $\quad$ B1		
M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2, y = -0.2$ (o.e.)		
		(11)
(b) 2nd M: Alternative is to use quadratic formula on the quadratic equation, or to use the discriminant, with a <u>comment</u> about 'no real roots', or 'can't equal zero', or a comment about the condition for singularity. $\left(x = \dfrac{2 \pm \sqrt{4-8}}{2} \right)$ A1 Conclusion. (c) M: Need $\dfrac{1}{\text{their det A}}$, k's unchanged and attempt to change sign for either -2 (leaving as top right) or $1-k$ (leaving as bottom left). (d) M: Requires an attempt to multiply the matrices.		

Question Number	Scheme	Marks
6. (a)	$(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta \quad \therefore$ true for $n = 1$	B1
	Assume true for $n = k$, $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$	
	$(\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$	M1
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1
	(Can be achieved either from the line above or the line below)	
	$= \cos(k+1)\theta + i\sin(k+1)\theta$	A1
	Requires full justification of $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$	
	($\therefore$ true for $n = k+1$ if true for $n = k$) $\quad \therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1cso (5)
(b)	$\cos 5\theta = \mathrm{Re}\left[(\cos\theta + i\sin\theta)^5\right]$	
	$= \cos^5\theta + 10\cos^3\theta\, i^2\sin^2\theta + 5\cos\theta\, i^4\sin^4\theta$	M1 A1
	$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
	$= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1 - \cos^2\theta)^2$	M1
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \qquad\qquad$ (*)	A1cso (5)
(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \quad \Rightarrow \quad \cos 5\theta = 0$	M1
	$5\theta = \dfrac{\pi}{2} \qquad\qquad \theta = \dfrac{\pi}{10}$	A1
	$x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root $\qquad$ (*)	A1 (3)
		(13)
	(a) <u>Alternative</u>:	
	For the 2nd M mark: $\left(e^{ik\theta}\right)\left(e^{i\theta}\right) = e^{i\theta(k+1)}$	
	(b) <u>Alternative</u>:	
	$\left(z + \dfrac{1}{z}\right)^5 = z^5 + 5z^4\left(\dfrac{1}{z}\right) + 10z^3\left(\dfrac{1}{z}\right)^2 + 10z^2\left(\dfrac{1}{z}\right)^3 + 5z\left(\dfrac{1}{z}\right)^4 + \left(\dfrac{1}{z}\right)^5$	M1
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	A1
	$(2\cos\theta)^5 = \ldots\ldots$ and attempt to put $\cos 3\theta$ in powers of $\cos\theta$	M1
	Correct method (or formula) for putting $\cos 3\theta$ in powers of $\cos\theta$	M1
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	A1cso
	(c) <u>Alternatives</u>:	
	(i) Substitute given root into $x^4 - 5x^2 + 5$:	
	$\left(2\cos\dfrac{\pi}{10}\right)^4 - 5\left(2\cos\dfrac{\pi}{10}\right)^2 + 5 = 2^4\left(\cos\dfrac{\pi}{10}\right)^4 - 5\times 2^2\left(\cos\dfrac{\pi}{10}\right)^2 + 5$	M1
	'Multiply by $\cos\theta$' and use result from part (b): $\ldots = \cos\dfrac{5\pi}{10}$	A1
	$= 0$ and conclusion	A1
	(ii) Use $5\theta = \dfrac{\pi}{2}$ in result from part (b)	M1
	$16\left(\cos\dfrac{\pi}{10}\right)^5 - 20\left(\cos\dfrac{\pi}{10}\right)^3 + 5\left(\cos\dfrac{\pi}{10}\right)$ and divide by $\cos\theta$	A1
	$= 0$ and conclusion	A1

Question Number	Scheme	Marks
7. (a)	$\overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k},\quad \overrightarrow{PR} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1
	$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	M1 A1 (3)
(b)	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} - \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ [may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$]	M1
	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$ o.e. ft from (a)	A1ft (2)
(c)	$3x + y - z = 4$ (i), $x - 2y - 5z = 6$ (ii)	
	(i) $\times 2$ + (ii) $7x - 7z = 14$, $x = z + 2$ (M: Eliminate one variable)	M1
	In (ii) $z + 2 - 2y - 5z = 6$, $y + 2 = -2z$ (M: Substitute back)	M1
	$\therefore x = z + 2$ and $y + 2 = -2z$ o.e. $(y = 2 - 2x)$	A1
	(Two correct '3-term' equations)	
	$\dfrac{x-2}{(1)} = \dfrac{y+2}{-2} = \dfrac{z}{(1)}$ o.e. (M: Form cartesian equations)	M1 A1 (5)
(d)	Writing down direction vector of $\overrightarrow{PS}$ from part (c).	M1
	$\overrightarrow{QR} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} = \overrightarrow{PS}$ $\therefore PS\,/\!/\,QR$ (or cross-product = 0)	A1 (2)
(e)	$\overrightarrow{PT} = 4\mathbf{i} + 2\mathbf{j}$ (or $\overrightarrow{QT} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{RT} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$)	
	Volume $= \dfrac{1}{3}\left|\overrightarrow{PQ} \times \overrightarrow{PR}.\overrightarrow{PT}\right| = \dfrac{1}{3}\left|(3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j})\right|$ ft from (a)	M1 A1ft
	(Instead of $\overrightarrow{PQ} \times \overrightarrow{PR}$, it could be $\overrightarrow{PQ} \times \overrightarrow{QR}$ or $\overrightarrow{PR} \times \overrightarrow{QR}$)	
	$= \dfrac{1}{3}(12 + 2)$	
	$= 4\tfrac{2}{3}$ o.e.	A1 (3)
		(15)
	(a) If both vectors are 'reversed', B0 M1 A1 is possible	
	(c) Alternative:	
	Direction of line: $\begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 7\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ M2 A1	
	Through $P\,(1, 0, -1)$: $\dfrac{x-1}{1} = \dfrac{y}{-2} = \dfrac{z+1}{1}$ M1 A1	
	(e) Alternative:	
	$\dfrac{1}{3}\begin{vmatrix} 4 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix}$ gives M1 A1 directly. Here ft from 1st line of part (a).	
	Special case:	
	$\dfrac{1}{6}$ or $\dfrac{1}{2}$ instead of $\dfrac{1}{3}$, but method otherwise correct: M1 A0 A0	