Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	7	6	/	0	1	Signature	

Paper Reference(s)

6676/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Tuesday 26 June 2007 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the examiner.
Answers without working may not gain full credit.

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1.

$$\frac{dy}{dx} = y\,e^{x^2}.$$

It is given that $y = 0.2$ at $x = 0$.

(a) Use the approximation $\frac{y_1 - y_0}{h} \approx \left(\frac{dy}{dx}\right)_0$, with $h = 0.1$, to obtain an estimate of the value of y at $x = 0.1$.

(2)

(b) Use your answer to part (a) and the approximation $\frac{y_2 - y_0}{2h} \approx \left(\frac{dy}{dx}\right)_1$, with $h = 0.1$, to obtain an estimate of the value of y at $x = 0.2$.

Give your answer to 4 decimal places.

(3)

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Question 1 continued

(Total 5 marks)

Q1

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2. $$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=0.$$

At $x = 0$, $y = 2$ and $\frac{dy}{dx}=-1$.

(a) Find the value of $\frac{d^3y}{dx^3}$ at $x = 0$.

(3)

(b) Express y as a series in ascending powers of x, up to and including the term in x^3.

(4)

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Question 2 continued

(Total 7 marks)

Q2

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3. Given that $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is an eigenvector of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 3 & 4 & p \\ -1 & q & -4 \\ 1 & 1 & 3 \end{pmatrix},$$

(a) find the eigenvalue of **A** corresponding to $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$,

(2)

(b) find the value of p and the value of q.

(4)

The image of the vector $\begin{pmatrix} l \\ m \\ n \end{pmatrix}$ when transformed by **A** is $\begin{pmatrix} 10 \\ -4 \\ 3 \end{pmatrix}$.

(c) Using the values of p and q from part (b), find the values of the constants l, m and n.

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total 10 marks)

Q3

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4. (a) Given that $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta.$$

(2)

(b) Express $32\cos^6\theta$ in the form $p\cos 6\theta + q\cos 4\theta + r\cos 2\theta + s$, where p, q, r and s are integers.

(5)

(c) Hence find the exact value of

$$\int_0^{\frac{\pi}{3}} \cos^6\theta \, \mathrm{d}\theta.$$

(4)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 11 marks)

Q4

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5. Prove by induction that, for $n \in \mathbb{Z}^+$, $\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$.

(5)

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Question 5 continued

(Total 5 marks)

Q5

N 2 2 3 3 6 A 0 1 5 2 8

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6. Given that

$$f(n) = 3^{4n} + 2^{4n+2},$$

(a) show that, for $k \in \mathbb{Z}^+$, $f(k+1) - f(k)$ is divisible by 15, **(4)**

(b) prove that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 5, **(3)**

(c) show that it is not true that, for all positive integers n, $f(n)$ is divisible by 15. **(1)**

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 8 marks)

Q6

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7. The points A, B and C have position vectors, relative to a fixed origin O,

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j},$$

$$\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k},$$

$$\mathbf{c} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k},$$

respectively. The plane Π passes through A, B and C.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

(4)

(b) Show that a cartesian equation of Π is $3x - y + 2z = 7$.

(2)

The line l has equation $(\mathbf{r} - 5\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = \mathbf{0}$. The line l and the plane Π intersect at the point T.

(c) Find the coordinates of T.

(5)

(d) Show that A, B and T lie on the same straight line.

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 14 marks)

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8. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where

$w = u + \mathrm{i}v$, is given by

$$w = \frac{z + \mathrm{i}}{z}, \quad z \neq 0.$$

(a) The transformation T maps the points on the line with equation $y = x$ in the z-plane, other than (0, 0), to points on a line l in the w-plane. Find a cartesian equation of l.

(5)

(b) Show that the image, under T, of the line with equation $x + y + 1 = 0$ in the z-plane is a circle C in the w-plane, where C has cartesian equation

$$u^2 + v^2 - u + v = 0.$$

(7)

(c) On the same Argand diagram, sketch l and C.

(3)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total 15 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

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