edexcel

Mark Scheme (Results)

Summer 2012

GCE Further Pure FP2
(6668) Paper 1

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2012
Publications Code UA032240

Summer 2012
6668 Further Pure 2
FP2 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where} |pq|=|c|$$, leading to x = ...

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where} |pq|=|c| \text{ and } |mn|=|a|$$, leading to x = …

2. Formula

Attempt to use correct formula (with values for a, b and c), leading to x = …

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

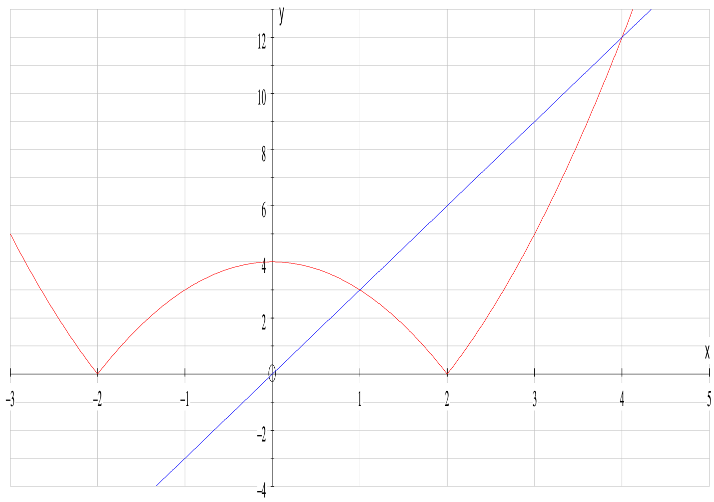
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Summer 2012
6668 Further Pure Mathematics FP2
Mark Scheme

Question Number	Scheme	Marks
1.	$x^2 - 4 = 3x$ and $x^2 - 4 = -3x$, or graphical method, or squaring both sides, leading to $x = ...$	M1
	$(x = -4,\ x = -1)$ $x = 1,\ x = 4$ **seen anywhere**	B1 B1
	Using only 2 critical values to find an inequality	dM1
	$x < 1$ $x > 4$ **both strict, ignore 'and'**	A1
		(5) **5**
	Notes  1st M1 accept $\pm(x^2-4) > 3x$ or $\pm(x^2-4) = 3x$ Require **modulus** of parabola and straight line with positive gradient through origin for graphical method. 1st B1 for x=1, 2nd B1 for x=4 2nd M1 dependent upon first M1 A0 for error in solution of quadratic leading to correct answer.	

Question Number	Scheme	Marks
2.	$y = r\sin\theta = \sin\theta + 2\sin\theta\cos\theta$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta + 2\cos 2\theta$	M1
	$4\cos^2\theta + \cos\theta - 2 = 0$	A1oe
	$\cos\theta = \frac{-1 \pm \sqrt{1+32}}{8}$	M1 A1
	$OP = r = 1 + \frac{-1+\sqrt{1+32}}{4} = \frac{3+\sqrt{33}}{4}$	M1 A1
		(7)
		7

Notes

B1 for $sin\theta + 2\sin\theta cos\theta$ or $sin\theta(1 + 2cos\theta)$

1st M1 for use of Product Rule or Chain Rule (require 2 or condone ½)

1st A1 equation required

2nd M1 Valid attempt at solving 3 term quadratic (usual rules) to give $cos\theta = \cdots$

2nd A1 for exact or 3 dp or better (-0.843.....and 0.593....)

3rd M1 for 1+2x 'their $cos\theta$'

3rd A1 for any form A0 if negative solution not discounted.

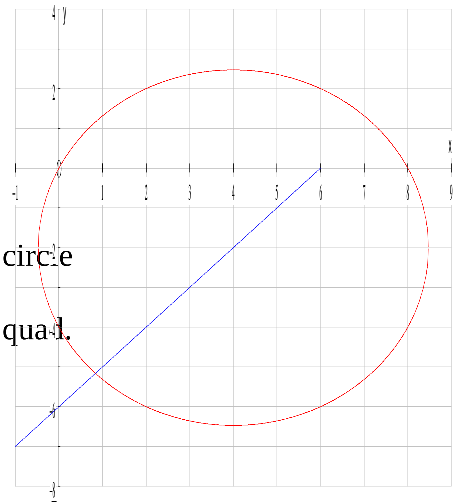
Question Number	Scheme	Marks
3.		
(a)	$r=\sqrt{(-2)^2+(2\sqrt{3})^2}=4$	B1
	$\tan\theta=-\sqrt{3}$ (Also allow M mark for $\tan\theta=\sqrt{3}$)	M1
	M mark can be implied by $\theta=\pm\frac{2\pi}{3}$ or $\theta=\pm\frac{\pi}{3}$	
	$\theta=\frac{2\pi}{3}$	A1
		(3)
(b)	Finding the 4th root of their r: $r=4^{1/4}$ $(=\sqrt{2})$	M1
	For one root, dividing their θ by 4: $\theta=\frac{2\pi}{3}\div 4=\frac{\pi}{6}$	M1
	For another root, add or subtract a multiple of 2π to their θ **and** divide by 4 in correct order.	M1
	$\sqrt{2}(\cos\theta+\mathrm{i}\sin\theta)$, where $\theta=-\frac{5\pi}{6},\ -\frac{\pi}{3},\ \frac{\pi}{6},\ \frac{2\pi}{3}$	A1 A1
		(5)
		8
	Notes	
(a)	M1 Accept $\pm\sqrt{3}$ *or* $\pm\frac{1}{\sqrt{3}}$	
	A1 Accept awrt 2.1. A0 if in degrees.	
(b)	2nd M1 for awrt 0.52	
	1st A1 for two correct values	
	2nd A1 for all correct values values in correct form and no more	

Question Number	Scheme	Marks
4.	$m^2+5m+6=0 \qquad m=-2,\ -3$	M1
	C.F. $(x=)Ae^{-2t}+Be^{-3t}$	A1
	P.I. $x=P\cos t+Q\sin t$	B1
	$\dot{x}=-P\,\mathbf{sin}\,t+Q\,\mathbf{cos}\,t$	
	$\ddot{x}=-P\,\mathbf{cos}\,t-Q\,\mathbf{sin}\,t$	M1
	$(-P\cos t-Q\sin t)+5(-P\sin t+Q\cos t)+6(P\cos t+Q\sin t)=2\cos t-\text{si}$	M1
	$-P+5Q+6P=2$ and $-Q-5P+6Q=-1$, and solve for P and Q	M1
	$P=\dfrac{3}{10}$ and $Q=\dfrac{1}{10}$	A1 A1
	$x=Ae^{-2t}+Be^{-3t}+\dfrac{3}{10}\cos t+\dfrac{1}{10}\sin t$	B1 ft
		(9) **9**
	Notes 1st M1 form quadratic and attempt to solve (usual rules) 1st B1 Accept negative signs for coefficients. Coefficients must be different. 2nd M1for differentiating their trig PI twice 3rd M1 for substituting x, $\dot{x}$ and $\ddot{x}$ expressions 4th M1 Form 2 equations in two unknowns and attempt to solve 1st A1 for one correct, 2nd A1 for two correct 2nd B1 for x=their CF + their PI as functions of t Condone use of the wrong variable (e.g. x instead of t) for all marks except final B1.	

Question Number	Scheme	Marks
5.		
(a)	$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=3+2y\frac{dy}{dx}$ (Using differentiation of product or quotient **and** also differentiation of implicit function)	M1
	$x\frac{d^2y}{dx^2}+(1-2y)\frac{dy}{dx}=3$ **ag**	A1 cso
		(2)
(b)	$\left(x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+.....$	B1
	$.....\left[(1-2y)\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2\right]=0$	M1 A1
	At x = 1: $\frac{dy}{dx}=4$	B1
	$\frac{d^2y}{dx^2}=7$ $\frac{d^3y}{dx^3}=32$	B1, B1
	$(y=)\mathrm{f}(1)+\mathrm{f}'(1)(x-1)+\frac{\mathrm{f}''(1)(x-1)^2}{2}+\frac{\mathrm{f}'''(1)(x-1)^3}{6}...$	M1
	$y=1+4(x-1)+\frac{7}{2}(x-1)^2+\frac{16}{3}(x-1)^3$ (or equiv.)	A1 ft
		(8)
		10
	Notes	
(a)	Finding second derivative and substituting into given answer acceptable	
(b)	1st M1 for differentiating second term to obtain an expression involving $\frac{d^2y}{dx^2}$ and $\left(\frac{dy}{dx}\right)^2$	
	B1B1B1 for 4,7,32 seen respectively	
	2nd M1 require $\mathrm{f}(1)$ or $1, \mathrm{f}'(1)$ etc and x-1and at least first 3 terms	
	A1 for 4 terms following through their constants	
	Condone f(x)= instead of y=	

Question Number	Scheme	Marks
6.		
(a)	$\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right) = \frac{1}{2r}, -\frac{1}{2r+4}$	B1,B1oe
		(2)
(b)	$r=1: \quad \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$	M1
	$r=2: \quad \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$	
	$r=3: \quad \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$	
	$r=n-1: \quad \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$	
	$r=n: \quad \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$	A1
	Summing: $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$	M1 A1
	$= \frac{1}{2}\left(\frac{3(n+1)(n+2) - 2(n+1) - 2(n+2)}{2(n+1)(n+2)}\right) = \frac{n(3n+5)}{4(n+1)(n+2)}$	M1 A1cao
		(6)
(c)	$\sum_{r=1}^{2n} \frac{1}{r(r+2)} = \frac{2n(6n+5)}{4(2n+1)(2n+2)}$	B1oe
	$S_{2n} - S_n \quad = \frac{2n(6n+5)}{4(2n+1)(2n+2)} - \frac{n(3n+5)}{4(n+1)(n+2)}$	M1
	$= \frac{n(6n+5)(n+2) - n(3n+5)(2n+1)}{4(n+1)(n+2)(2n+1)}$	
	$= \frac{n(6n^2 + 17n + 10 - 6n^2 - 13n - 5)}{4(n+1)(n+2)(2n+1)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$ (*ag*)	A1 cso
		(3)
		11
(a)	1st and 2nd B1 Any form is acceptable	
(b)	1st M1 must include at least 4 out of 5 of (r=)1,2,3 and n-1, n 1st A1 require all terms that do not cancel to be accurate 2nd M1 Summed expression involving all terms that do not cancel 2nd A1 Correct expression 3rd M1 for attempt to find single fraction	
(c)	1st M1 for expression for $S_{2n} - S_n$	

Question Number	Scheme	Marks
7.		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ seen	B1
	$3x^3v^2\left(v + x\frac{\mathrm{d}v}{\mathrm{d}x}\right) = x^3 + v^3x^3 \quad \Rightarrow \quad 3v^2x\frac{\mathrm{d}v}{\mathrm{d}x} = 1 - 2v^3$ (**ag**)	M1 A1 cso
		(3)
(b)	$\int \frac{3v^2}{1-2v^3}\,\mathrm{d}v = \int \frac{1}{x}\,\mathrm{d}x$	M1
	$-\frac{1}{2}\ln(1-2v^3) = \ln x \quad (+C)$	M1 A1
	$-\ln(1-2v^3) = \ln x^2 + \ln A$	
	$Ax^2 = \frac{1}{1-2v^3}$	M1
	$1 - \frac{2y^3}{x^3} = \frac{1}{Ax^2}$	
	$y = \sqrt[3]{\frac{x^3 - Bx}{2}}$ or equivalent	dM1 A1cso
		(6)
(c)	Using $y = 2$ at $x = 1$: $\quad 12\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 8$	M1
	At $x = 1$, $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{4}$	A1
		(2)
		11
	Notes	
(a)	M1 for substituting y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ obtaining an expression in v and x only	
(b)	1st M1 for separating variables 2nd M1 for attempting to integrate both sides 1st A1both sides required or equivalent expressions. (Modulus not required.) 3rd M1 Removing logs, dealing correctly with constant 4th M1 dep on 1st M. Substitute $v = \frac{y}{x}$ and rearranging to $y = \mathrm{f}(x)$	
(c)	M1 for finding a numerical value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ A1 for correct numerical answer oe.	

Question Number	Scheme	Marks
8.		
(a)	$\lvert x+\mathrm{i}y-6\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-3\rvert$	M1
	$x^2+(y-6)^2 = 4\left[(x-3)^2+y^2\right]$ $x^2+y^2-12y+36 = 4x^2-24x+36+4y^2$ $3x^2+3y^2-24x+12y=0$	M1 A1
	$(x-4)^2+(y+2)^2=20$	M1
	Centre $(4,-2)$, Radius $\sqrt{20}=2\sqrt{5}=$ awrt 4.47	A1 A1
		(6)
(b)	 Centre in correct quad for their circle	M1
	Passes through O centre in 4th quad.	A1cao
	Half line with positive gradient	B1
	Correct position, clearly through (6, 0)	B1
		(4)
(c)	Equation of line $y=x-6$	B1
	Attempting simultaneous solution of $(x-4)^2+(y+2)^2=20$ and $y=x-6$	M1
	$x=4\pm\sqrt{10}$	A1
	$\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	A1cao
		(4) **14**
	Notes	
(a)	1st M Substituting $z=x+iy$ oe 2nd M implementing modulus of both sides and squaring. Require Re^2 **plus** Im^2 on both sides & no terms in i. Condone 2 instead of 4 here. 3rd M1 for gathering terms and attempting to find centre and / or radius	

Question Number	Scheme	Marks
	2nd A1 for centre, 3rd A1 for radius	
Alt 8(c)	**For geometric approach in this part.**	
	Centre (4,-2) on line, can be implied.	B1
	Use of Pythagoras or trigonometry to find lengths of isosceles triangle	M1
	$x = 4 - \sqrt{10}$	A1
	$\left(4 - \sqrt{10}\right) + \mathrm{i}\left(-2 - \sqrt{10}\right)$	A1cao

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA032240 Summer 2012

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning