Ex 2A

(1) $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ using $\frac{1}{r(r+1)} \equiv \frac{1}{r} - \frac{1}{r+1}$

$r = 1, 2, 3, .., n$

$r=1 \quad \frac{1}{1(2)} = \frac{1}{1} - \cancel{\frac{1}{2}}$

$r=2 \quad \frac{1}{2(3)} = \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$

$r=3 \quad \frac{1}{3(4)} = \cancel{\frac{1}{3}} - \frac{1}{4}$

$r=n-1 \quad \frac{1}{(n-1)(n)} = \frac{1}{n-1} - \cancel{\frac{1}{n}}$

$r=n \quad \frac{1}{n(n+1)} = \cancel{\frac{1}{n}} - \frac{1}{n+1}$

adding. $\left[\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \dots \frac{1}{n(n+1)}\right] = -\frac{1}{n+1} + 1$

$\therefore \sum_{r=1}^{n} \frac{1}{r(r+1)} \equiv 1 - \frac{1}{n+1}.$

(2) $\sum_{r=1}^{n} (2r+1)$ using $2r+1 \equiv (r+1)^2 - r^2$

$r = 1, 2, 3, \dots, n$

$r=1 \quad 2(1)+1 \equiv (\cancel{2})^2 - (1)^2$

$r=2 \quad 2(2)+1 = (\cancel{3})^2 - (\cancel{2})^2$

$r=3 \quad 2(3)+1 = (\cancel{4})^2 - (\cancel{3})^2$

$r=4 \quad 2(4)+1 = (5)^2 - (\cancel{4})^2$

$r=n-1 \quad 2(n-1)+1 = (\cancel{n})^2 - (n-1)^2$

$r=n \quad 2(n)+1 = (n+1)^2 - (\cancel{n})^2$

Adding. $2\left[1+2+3+4+\dots+(n-1)+(n)\right]+1 = (n+1)^2 - 1$

$\sum_{r=1}^{n} 2r+1 = n^2 + 2n + 1 - 1$

$= n^2 + 2n$

(3) $\sum_{r=1}^{n} \frac{1}{4r^2-1}$ using $\frac{2}{4r^2-1} \equiv \frac{1}{2r-1} - \frac{1}{2r+1}$

$r=1$ $\quad \frac{2}{4(1)^2-1} \equiv \frac{1}{1} - \cancel{\frac{1}{3}}$

$r=2$ $\quad \frac{2}{4(2)^2-1} \equiv \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}$

$r=3$ $\quad \frac{2}{4(3)^2-1} \equiv \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}$

$r=4$ $\quad \frac{2}{4(4)^2-1} \equiv \cancel{\frac{1}{7}} - \frac{1}{9}$

$r=n-1$ $\quad \frac{2}{4(n-1)^2-1} \equiv \frac{1}{2(n-1)-1} - \frac{1}{2(n-1)+1} = \frac{1}{2n-3} - \cancel{\frac{1}{2n-1}}$

$r=n$ $\quad \frac{2}{4(n)^2-1} \equiv \cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}$

Adding

$$2\left[\frac{1}{4(1)^2-1} + \frac{1}{4(2)^2-1} + \frac{1}{4(3)^2-1} + \dots + \frac{1}{4(n-1)^2-1} + \frac{1}{4n^2-1}\right] = 1 - \frac{1}{2n+1}$$

$$2\sum_{r=1}^{n} \frac{1}{4r^2-1} \equiv 1 - \frac{1}{2n+1}$$

$$2\sum_{r=1}^{n} \frac{1}{4r^2-1} \equiv \frac{2n+1-1}{2n+1}$$

$$\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}n}{2n+1} = \frac{n}{2n+1}$$

④ $\sum_{r=1}^{n} r(3r-1)$ using $r^2(r+1) - (r-1)^2(r) \equiv 3r^2 - r$

$r=1$ $\quad \cancel{1^2(2)} - 0^2(1) = 3(1)^2 - 1$

$r=2$ $\quad \cancel{2^2(3)} - \cancel{1^2(2)} = 3(2)^2 - 2$

$r=3$ $\quad \cancel{3^2(4)} - \cancel{2^2(3)} = 3(3)^2 - 3$

$r=4$ $\quad 4^2(5) - \cancel{3^2(4)} = 3(4)^2 - 4$

$r=n-1$ $\quad \cancel{(n-1)^2(n)} - (n-2)^2(n-1) = 3(n-1)^2 - (n-1)$

$r=n$ $\quad n^2(n+1) - \cancel{(n-1)^2(n)} = 3n^2 - n$

adding. $\quad n^2(n+1) \equiv \sum_{r=1}^{n} 3r^2 - r$

⑤ $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ using $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$

$r=1$ $\quad \cancel{\frac{1}{2}} - \frac{0}{1} \equiv \frac{1}{1(1+1)}$

$r=2$ $\quad \cancel{\frac{2}{3}} - \cancel{\frac{1}{2}} = \frac{1}{2(2+1)}$

$r=3$ $\quad \cancel{\frac{3}{4}} - \cancel{\frac{2}{3}} = \frac{1}{3(3+1)}$

$r=4$ $\quad \frac{4}{5} - \cancel{\frac{3}{4}} = \frac{1}{4(4+1)}$

$r=n-1$ $\quad \cancel{\frac{n-1}{n}} - \frac{n-2}{n-1} = \frac{1}{(n-1)n}$

$r=n$ $\quad \frac{n}{n+1} - \cancel{\frac{n-1}{n}} = \frac{1}{n(n+1)}$

adding $\quad \frac{n}{n+1} - 0 \equiv \sum_{r=1}^{n} \frac{1}{r(r+1)}$

(6) $\sum_{r=1}^{n} r(r+1)(r+2)$ using $4r(r+1)(r+2) \equiv r(r+1)(r+2)(r+3) - (r-1)(r)(r+1)(r+2)$

$r=1 \quad 4(1)(2)(3) \equiv 1(2)\cancel{(3)}(4) - 0(1)(2)(3)$

$r=2 \quad 4(2)(3)(4) \equiv 2(3)\cancel{(4)}(5) - 1\cancel{(2)}(3)(4)$

$r=3 \quad 4(3)(4)(5) \equiv 3(4)\cancel{(5)}(6) - 2\cancel{(3)}(4)(5)$

$r=4 \quad 4(4)(5)(6) \equiv 4(5)(6)(7) - 3\cancel{(4)}(5)(6)$

$r=n-1 \quad 4(n-1)(n)(n+1) \equiv (n-1)(n)\cancel{(n+1)}(n+2) - (n-2)(n-1)(n)(n+1)$

$r=n \quad 4(n)(n+1)(n+2) \equiv n(n+1)(n+2)(n+3) - (n-1)(n)\cancel{(n+1)}(n+2)$

adding $\quad 4\sum_{r=1}^{n} r(r+1)(r+2) \equiv n(n+1)(n+2)(n+3) - 0$

$$\sum_{r=1}^{n} r(r+1)(r+2) \equiv \frac{n}{4}(n+1)(n+2)(n+3)$$

(7) $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ using $\frac{2}{r(r+1)(r+2)} \equiv \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$

$r=1 \quad \frac{2}{1(2)(3)} \equiv \frac{1}{1(2)} - \cancel{\frac{1}{(2)(3)}}$

$r=2 \quad \frac{2}{2(3)(4)} \equiv \cancel{\frac{1}{2(3)}} - \cancel{\frac{1}{(3)(4)}}$

$r=3 \quad \frac{2}{3(4)(5)} \equiv \cancel{\frac{1}{3(4)}} - \frac{1}{(4)(5)}$

$r=n-1 \quad \frac{2}{(n-1)(n)(n+1)} \equiv \frac{1}{(n-1)(n)} - \cancel{\frac{1}{n(n+1)}}$

$r=n \quad \frac{2}{n(n+1)(n+2)} \equiv \cancel{\frac{1}{n(n+1)}} - \frac{1}{(n+1)(n+2)}$

Adding $\quad 2\left[\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots \frac{1}{n(n+1)(n+2)}\right] = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$

(7) cont'd

$$2\sum_{r=1}^{n} \frac{1}{n(n+1)(n+2)} = \frac{(n+1)(n+2)-2}{2(n+1)(n+2)}$$

$$\sum_{r=1}^{n} \frac{1}{n(n+1)(n+2)} = \frac{n^2+3n}{4(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

(8) $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$ using $\frac{2r+1}{r^2(r+1)^2} \equiv \frac{1}{r^2} - \frac{1}{(r+1)^2}$

$r=1$ $\quad \frac{2(1)+1}{1^2(2)^2} = \frac{1}{1^2} - \cancel{\frac{1}{2^2}}$

$r=2$ $\quad \frac{2(2)+1}{2^2(3)^2} = \cancel{\frac{1}{2^2}} - \cancel{\frac{1}{3^2}}$

$r=3$ $\quad \frac{2(3)+1}{3^2(4)^2} = \cancel{\frac{1}{3^2}} - \cancel{\frac{1}{4^2}}$

$r=4$ $\quad \frac{2(4)+1}{4^2(5)^2} = \cancel{\frac{1}{4^2}} - \frac{1}{5^2}$

$r=n-1$ $\quad \frac{2(n-1)+1}{(n-1)^2(n)^2} = \frac{1}{(n-1)^2} - \cancel{\frac{1}{n^2}}$

$r=n$ $\quad \frac{2n+1}{n^2(n+1)^2} = \cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2}$

adding $\quad \sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}$

$$= \frac{(n+1)^2-1}{(n+1)^2}$$

$$= \frac{n^2+2n}{(n+1)^2}$$

$$= \frac{n(n+2)}{(n+1)^2}$$

(9) $\sum_{r=1}^{n} r(r+1)$ using $(r+1)^3 - r^3 \equiv 3r^2+3r+1$

$r=1$ $\quad (1+1)^3 - 1^3 = 3(1)^2 + 3(1) + 1 = \cancel{2^3} - 1^3$

$r=2$ $\quad (2+1)^3 - 2^3 = 3(2)^2 + 3(2) + 1 = \cancel{3^3} - \cancel{2^3}$

$r=3$ $\quad (3+1)^3 - 3^3 = 3(3)^2 + 3(3) + 1 = \cancel{4^3} - \cancel{3^3}$

$r=4$ $\quad (4+1)^3 - 4^3 = 3(4)^2 + 3(4) + 1 = 5^3 - \cancel{4^3}$

$r=n-1$ $\quad 3(n-1)^2 + 3(n-1) + 1 = \cancel{n^3} - (n-1)^3$

$r=n$ $\quad 3n^2 + 3n + 1 = (n+1)^3 - \cancel{n^3}$

Adding. $\quad 3\left[\sum_{r=1}^{n} r(r+1)\right] + 1n = (n+1)^3 - 1$

$$3\left[\sum_{r=1}^{n} r(r+1)\right] + 1n = (n+1)(n^2+2n+1) - 1$$

$$3\left[\sum_{r=1}^{n} r(r+1)\right] + n = n^3 + 2n^2 + n + n^2 + 2n + 1 - 1$$

$$3\left[\sum_{r=1}^{n} r(r+1)\right] = n^3 + 3n^2 + 2n$$

$$\sum_{r=1}^{n} r(r+1) = \frac{n}{3}(n^2+3n+2) = \frac{n}{3}(n+2)(n+1)$$

(10). $\quad \frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}$

LHS $\quad \frac{1}{r!} - \frac{1}{(r+1)r!}$

$$= \frac{1}{r!}\left[1 - \frac{1}{r+1}\right]$$

$$= \frac{1}{r!}\left[\frac{r+1-1}{r+1}\right]$$

$$= \frac{1}{r!}\left(\frac{r}{r+1}\right) = \frac{r}{(r+1)r!} = \frac{r}{(r+1)!} = \text{RHS}$$

(10) contd.

$$\sum_{r=1}^{n} \frac{r}{(r+1)!}$$

$r=1$ $\quad \frac{1}{(1+1)!} = \frac{1}{1!} - \cancel{\frac{1}{2!}}$

$r=2$ $\quad \frac{2}{(2+1)!} = \cancel{\frac{1}{2!}} - \cancel{\frac{1}{3!}}$

$r=3$ $\quad \frac{3}{(3+1)!} = \cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}}$

$r=4$ $\quad \frac{4}{(4+1)!} = \cancel{\frac{1}{4!}} - \frac{1}{5!}$

$r=n-1$ $\quad \frac{n-1}{n!} = \frac{1}{(n-1)!} - \cancel{\frac{1}{n!}}$

$r=n$ $\quad \frac{n}{(n+1)!} = \cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}$

adding $\quad \sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$