Ex 1A

(1) $2x - 1 < 4(x-3)$

$2x - 1 < 4x - 12$

$-2x < -11$

$x > \frac{-11}{-2}$

$x > 5.5$

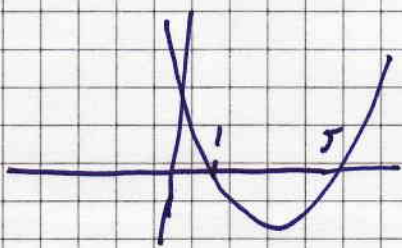
(2) $5(x-2) \geq 2(2x+7)$

$5x - 10 \geq 4x + 14$

$x \geq 24$

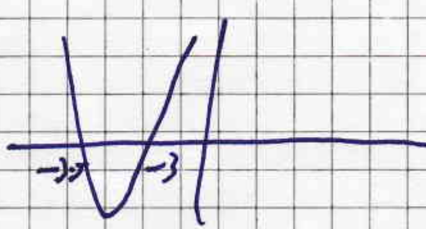
(3) $(x-1)(x-5) > 0$

$x < 1$ or $x > 5$




(4) $(x+3)(2x+7) \leq 0$

$-3.5 \leq x \leq -3$

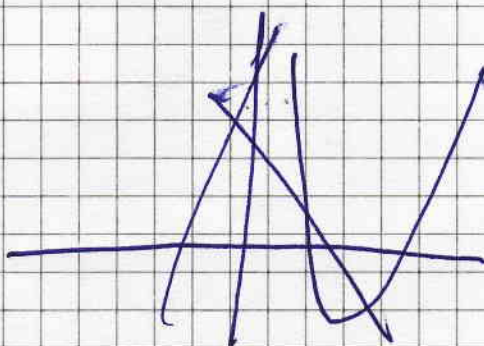


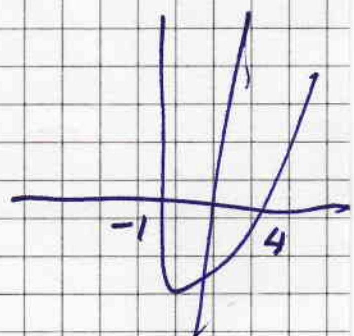

(5) $x^2 < 3x + 4$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

$-1 < x < 4$






(6) $\frac{3}{x-1} > 1$

$\frac{3}{x-1} - 1 > 0$

$\times (x-1)^2$

$\frac{3}{\cancel{x-1}} \times (x-1)^{\cancel{2}} - (x-1)^2 > 0$

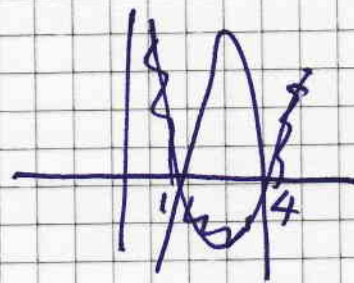
$3x - 3 - (x^2 - 2x + 1) > 0$

(6) cont

$\cancel{3x}\ -x^2+5x-4>0$

$(x-1)(4-x)>0$

$\cancel{x<1 \text{ or } x>4}\quad 1<x<4$




(7) $x^2 < \frac{1}{x}$

$x^2 - \frac{1}{x} < 0$

$\times x^2\quad x^4 - x < 0$

$x(x^3-1) < 0$

sketch $y = x(x^3-1)$

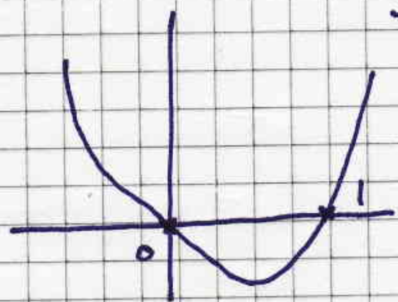
when $y=0$ $x=0$

or $x^3-1=0$

$x^3=1$

$x=1.$

when $x=0$ $y=0$




$0<x<1$

(8) $\frac{x}{x-1} < 2$

$\frac{x}{x-1} - 2 < 0$

$\times (x-1)^2$

$x(x-1) - 2(x-1)^2 < 0$

$x^2-x-2(x^2-2x+1) < 0$

$x^2-x-2x^2+4x-2 < 0$

$-x^2+3x-2 < 0$

$\cancel{(x+1)(-x+2)}$

$-(x^2-3x+2) < 0$

$-((x-2)(x-1)) < 0$

$(2-x)(x-1) < 0$

$x<1$ or $x>2$

(9) $\dfrac{x-1}{x+1} > 2$

$\dfrac{x-1}{x+1} - 2 > 0$

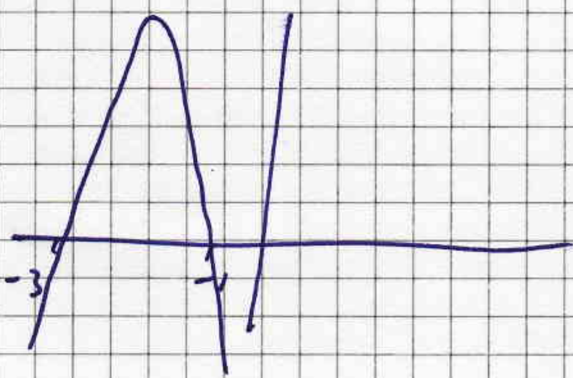
$\times (x+1)^2$

$(x-1)(x+1) - 2(x+1)^2 > 0$

$(x+1)[(x-1) - 2(x+1)] > 0$

$(x+1)(-x-3) > 0$

$-1(x+1)(x+3) > 0$




$-3 < x < -1$

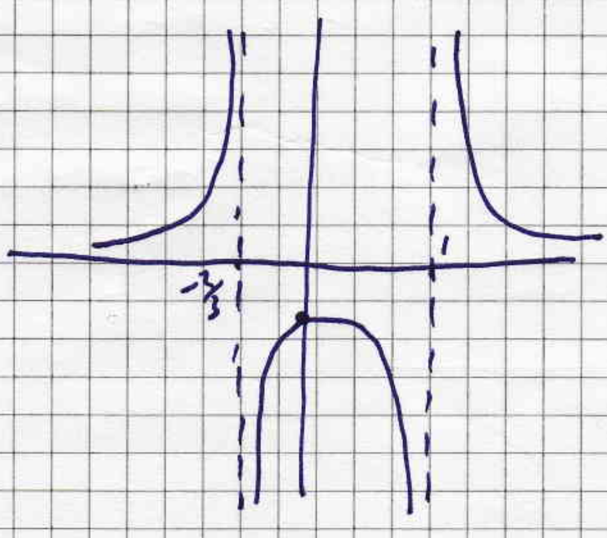
(10) $\dfrac{1}{3x^2 - x - 2} < 0$

$\dfrac{1}{(3x+2)(x-1)} < 0$

Consider graph $y = \dfrac{1}{(3x+2)(x-1)}$

when $x = 0$ $y = \dfrac{1}{2 \times -1} = -\dfrac{1}{2}$ $(0, -0.5)$

denominator is zero when $x = 1$ and $x = -\frac{2}{3}$ $\Rightarrow$ vertical asymptotes

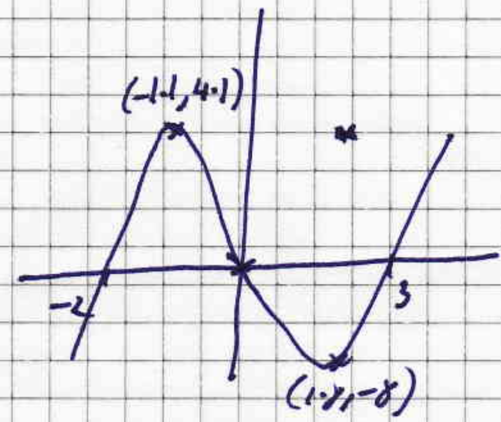


(11) $x^3 - x^2 \geq 6x$

$x^3 - x^2 - 6x \geq 0$

$x(x^2 - x - 6) \geq 0$

$x(x+2)(x-3) \geq 0$




graph $y = x^3 - x^2 - 6x$

$\frac{dy}{dx} = 3x^2 - 2x - 6 = 0$

$x = 1.8$ or $x = -1.1$

$x = 1.8$ $y = (1.8)^3 - (1.8)^2 - 6(1.8) = -8.208$ $(1.8, -8.2)$

$x = -1.1$ $y = 4.059$ $(-1.1, 4.059)$

$\therefore$ $-2 \leq x \leq 0$ or $x \geq 3$.

(12) $\frac{x^2+10}{x} > 7$

$\times x^2$ $(x^2+10)x > 7x^2$

$(x^2+10)x - 7x^2 > 0$

$x[x^2+10-7x^2] > 0$

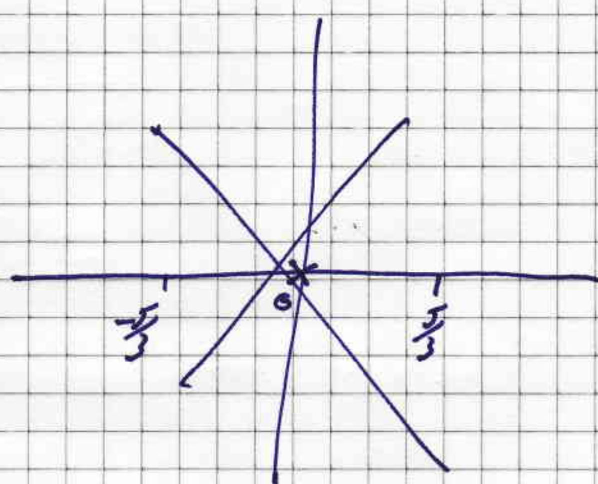
$x[10-6x^2] > 0$

$2x[5-3x^2] > 0$

$x=0$ $5-3x^2=0$

$x^2 = \frac{5}{3}$

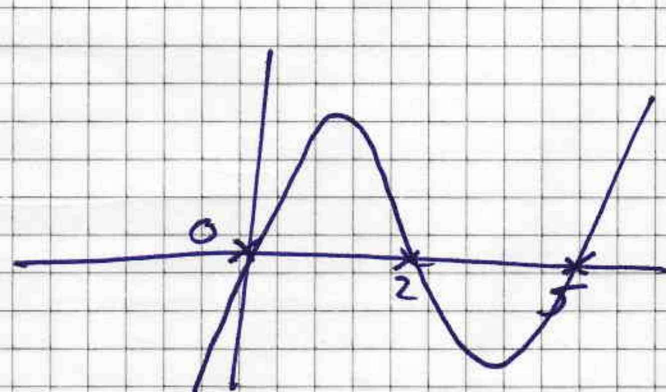
$x = \pm\frac{5}{3}$




$x^3 - 7x^2 + 10x > 0$

$x[x^2 - 7x + 10] > 0$

$x(x-5)(x-2) > 0$




$0 < x < 2$ or $x > 5$

(13) $\dfrac{4x-1}{x+1} < 2x$

$\times (x+1)^2$

$(4x-1)(x+1) < 2x(x+1)^2$

$(4x-1)(x+1) - 2x(x+1)^2 < 0$

$(x+1)[4x-1-2x(x+1)] < 0$

$(x+1)[4x-1-2x^2-2x] < 0$

$(x+1)[-2x^2+2x-1] < 0$

$\Downarrow$

No real soln

~~$\therefore x+1<0$~~

~~$x<-1$~~

$\therefore$ crosses x axis @ $x=-1$

$f'(x) = -6x^2+1 = 0$

$x^2 = \frac{1}{6}$

$x = \pm\frac{1}{6}$

$x > -1$

$4x^2+4x-x-1 - 2x(x^2+2x+1) < 0$

$4x^2+4x-x-1-2x^3-4x^2-2x < 0$

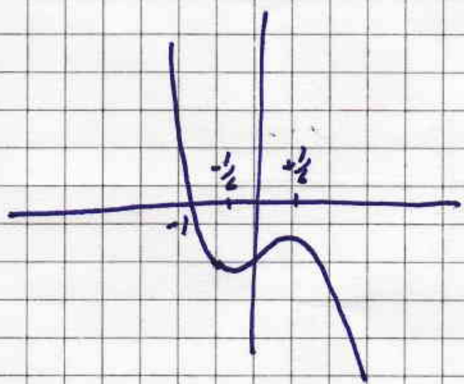
$-2x^3+x-1 < 0$

$f(x) = -2x^3+x-1$

$f(-1) = -2(-1)^3+(-1)-1 = +2-2=0$

$\therefore x+1$ is a factor

$$
\begin{array}{r|l}
 & -2x^2+2x-1 \\
\hline
x+1 & -2x^3+0x^2+x-1 \\
 & \underline{-2x^3-2x^2} \\
 & 2x^2+x \\
 & \underline{2x^2+2x} \\
 & -x-1 \\
 & \underline{-x-1} \\
 & \cdot\ \cdot
\end{array}
$$



(14) $\dfrac{x+3}{x-1} < \dfrac{x-3}{x+1}$

$\times (x-1)^2(x+1)^2$

$(x+3)(x-1)(x+1)^2 - (x-3)(x+1)(x-1)^2 < 0$

$(x-1)(x+1)\left[(x+3)(x+1) - (x-3)(x-1)\right] < 0$

$(x-1)(x+1)\left[x^2+4x+3 - x^2+4x-3\right] < 0$

$8x(x-1)(x+1) < 0$

Cubic, +ve $x^3$ crosses @ $(1,0)$, $(-1,0)$, $(0,0)$


$\therefore\ 0 < x < 1$ or $x < -1$

(15) $\dfrac{4x+8}{x-1} > 3$

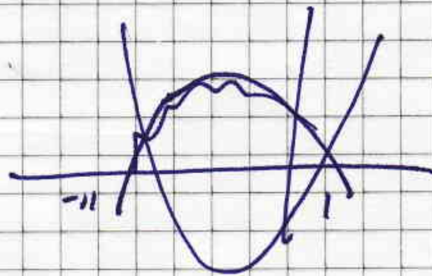
$\times(x-1)^2\quad (4x+8)(x-1) - 3(x-1)^2 > 0$

$4x^2+4x-8 - 3x^2+6x-3 > 0$

$x^2+10x-11 > 0$

$(x-1)(x+11) > 0$

~~then x<1~~ $x<-11$ or $x>1$



(16) $f(x) = x^3 - 7x - 6$

if $x+1$ is a factor then $f(-1)=0$

$f(-1) = (-1)^3 - 7(-1) - 6$

$= -1 + 7 - 6$

$= 0$ $\therefore$ $x+1$ is a factor.

$$\begin{array}{r|l} & x^2 - x - 6 \\ x+1 & x^3 + 0x^2 - 7x - 6 \\ & \underline{x^3 + x^2} \\ & \quad -x^2 - 7x \\ & \quad \underline{-x^2 - x} \\ & \qquad -6x - 6 \\ & \qquad \underline{-6x - 6} \\ & \qquad \quad \cdot \quad \cdot \end{array}$$

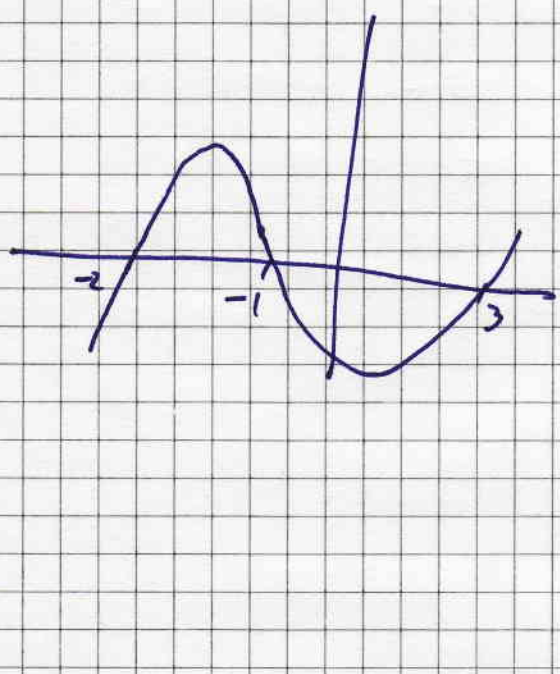
$\therefore$ $(x+1)(x^2-x-6)$ are factors

$(x+1)(x-3)(x+2)$

Hence if $x^3 - 7x - 6 \geq 0$

then $(x+1)(x-3)(x+2) \geq 0$

cubic, +ve $x^3$, crosses $x$ axis @ $(-1,0)$, $(-2,0)$, $(3,0)$




$-2 \leq x \leq -1$ or $x \geq 3$

(17) $$\frac{2}{x-2} > \frac{3}{x+1}$$

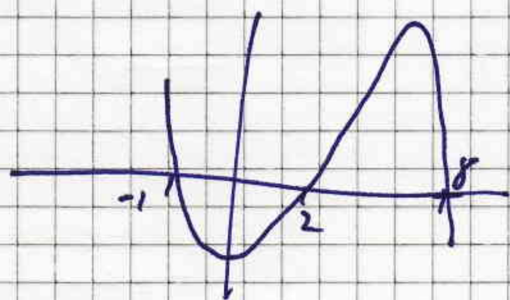
$x(x-2)^2(x+1)^2$

$$2(x-2)(x+1)^2 - 3(x+1)(x-2)^2 > 0$$

$$(x-2)(x+1)\left[2(x+1) - 3(x-2)\right] > 0$$

$$(x-2)(x+1)\left[8-x\right] > 0$$

Cubic, $-$ve $x^3$ ($\wedge$) cuts $x$ axis @ $(-1,0)$, $(2,0)$, $(8,0)$




$x < -1$ or $2 < x < 8$

(18) $$-1 \leq \frac{2x}{x^2+1} \leq 1$$

Consider $\frac{2x}{x^2+1} \leq 1$ $\qquad$ $x^2+1$ is always +ve so ok to $\times$ mult

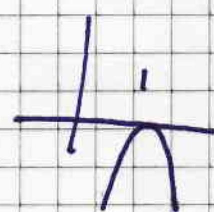
$$2x \leq x^2+1$$

$$-x^2+2x-1 \leq 0$$

$$-(x^2-2x+1) \leq 0$$

$$-(x-1)(x-1) \leq 0$$ Repeated root @ $(1,0)$, $-$ve $x^2$

$\therefore x \in \mathbb{R}$




Now consider $-1 \leq \frac{2x}{x^2+1}$

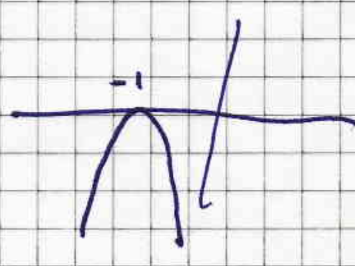
$$-(x^2+1) \leq 2x$$

$$-x^2-2x-1 \leq 0$$

$$-(x^2+2x+1) \leq 0$$

$$-(x+1)(x+1) \leq 0$$ Repeated root @ $(-1,0)$, $-$ve $x^2$

$\therefore x \in \mathbb{R}$




$\therefore$ Function is $\leq 1$ for all real $x$ and $\geq -1$ for all real $x$.

(19) $\dfrac{x^2+7x+10}{x+1} > 2x+7$

$\times(x+1)^2$

$$(x+2)(x+5)(x+1) - (2x+7)(x+1)^2 > 0$$

$$(x+1)\left[x^2+7x+10-(2x+7)(x+1)\right] > 0$$

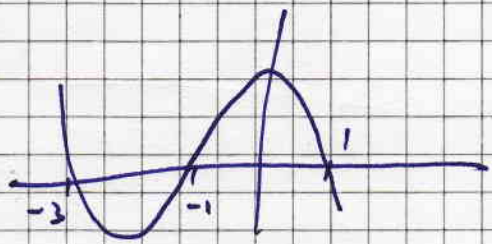
$$(x+1)\left[x^2+7x+10-2x^2-9x-7\right] > 0$$

$$(x+1)\left[-x^2-2x+3\right] > 0$$

$$-1(x+1)(x^2+2x-3) > 0$$

$$-1(x+1)(x-1)(x+3) > 0$$

Cubic, $-\text{ve } x^3$ cuts $x$ axis @ $(-3,0)$, $(-1,0)$, $(1,0)$




$x<-3$ or $-1<x<1$

(20) $$-1 < \frac{2-x}{2+x} \leq 1$$

Consider $\frac{2-x}{2+x} \leq 1$

$\times(2+x)^2$

$$(2-x)(2+x) - (2+x)^2 \leq 0$$

$$(2+x)[2-x-2-x] \leq 0$$

$$(2+x)(-2x) \leq 0$$

Quadratic, -ve $x^2$ term cuts @ $(0,0)$ & $(-2,0)$

$$x \leq -2 \text{ or } x \geq 0$$


Now consider $-1 < \frac{2-x}{2+x}$

$$-1(2+x)^2 < (2-x)(2+x)$$

$$-1(2+x)^2 - (2-x)(2+x) < 0$$

$$-1(2+x)[2+x+2-x] < 0$$

$$-4(2+x) < 0$$

$$-8 - 4x < 0$$

$$-4x < 8$$

$$x > -2$$

So for $\frac{2-x}{2+x} \leq 1$ $x$ must either be $\geq 0$ or $\leq -2$

but for $-1 < \frac{2-x}{2+x}$ $x$ must be $> -2$ $\therefore$ $x \neq -2$ to

to satisfy both inequalities $\therefore$ $x \geq 0$.