Energy

The energy of a body is a measure of the capacity which the body has to do work. When a force does work on a body it changes the energy of the body. Energy can exist in a number of forms, but we shall consider two main types: kinetic energy (KE) and potential energy (PE).

Kinetic Energy

The kinetic energy of a body is that energy which it possesses by virtue of its motion. When a force does work on a body so as to increase its speed, then the work done is a measure of the increase in the kinetic energy of the body.

The kinetic energy of a body of mass m kg moving with velocity v ms^{-1} is given by

$$KE = \tfrac{1}{2} mv^2 \text{ joules}$$

The work done by a constant force F N, acting on a body of mass m kg, travelling with an initial velocity u ms^{-1}, and after moving a distance s metres has a speed v ms^{-1}, gives rise to the following expression:

$$\begin{aligned} \text{Work done} &= \text{Final KE} - \text{Initial KE} \\ &= \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 \end{aligned}$$

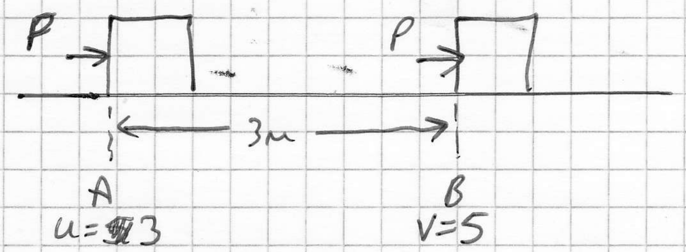
Eg6 A constant force pushes a body of mass 500g in a straight line across a smooth horizontal surface. The body passes through a point A with speed 3ms^{-1}, and later, through a point B with a speed of 5ms^{-1}, point B being 3m from A. For the motion of the body from A to B, find

(a) the increase in KE of the body,
(b) the work done by the force,
(c) the magnitude of the force.

Eg7 A and B are two points 6m apart on a horizontal surface. A particle of mass 400g passes through A with a speed of 8ms^{-1}, and through B with a speed of 5ms^{-1}. The resistance against which the particle moves is constant in magnitude. For the motion of the particle from A to B, find

(a) the loss in KE of the particle,
(b) the work done against the resistances,
(c) the magnitude of the resistance.

Eg6 Increase



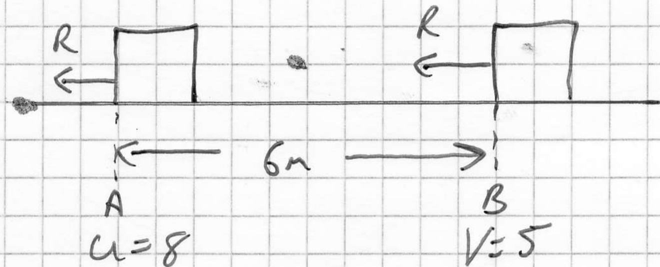
(a) increase in $KE = \frac{1}{2}M(V^2-u^2) = \frac{1}{2}\times 0.5(5^2-3^2) = 4$ J

(b) wd = Increase in KE = 4 J

(c) wd $= P\times s$ $\quad 4 = P\times 3$

$P = \frac{4}{3}$ N

Eg7



(a) loss in $KE = \frac{1}{2}M(V^2-u^2) = \frac{1}{2}\times 0.4(5^2-8^2) = -7.8$ J.

(b) wd v's Res = 7.8 J

(c) ~~7.8 = R~~ $\quad 7.8 = R\times 6$

$R = 1.3$ N

Eg8 (a) $PE = 6\times 9.8\times 1.2 = 70.56$ J.

(b) $PE = 2\times 9.8\times {}^{-}0.75 = -14.7$ J

(c) $PE = 10\times 9.8\times 5\sin 60 = 98\times 5\times \frac{\sqrt{3}}{2} = 245\sqrt{3}$ J

Potential Energy

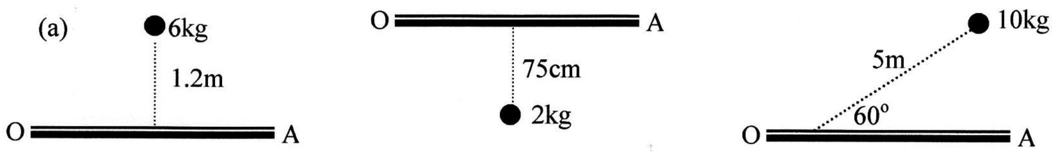
The potential energy of a body is that energy it possesses by virtue of its position. When a body of mass m kg is raised vertically a distance h metres, the work done against gravity is

$$PE = mgh \text{ joules}$$

The work done against gravity is a measure of the *increase* in the potential energy of the body, ie the capacity of the body to do work is increased.
When a body is lowered vertically its potential energy is decreased.
There is no zero of potential energy, although an arbitrary level may be used from which *changes* in the PE of a body may be measured.

Eg8 Calculate the gravitational PE, relative to the reference level OA, for each of the objects shown

(a)



Exercise 3B Pg 67 Q's 1& 2 if you wish, then the rest.

Conservation of energy
When a particle is moving freely under gravity, its total energy is constant so that at any instant,

$$\text{Loss in PE} = \text{Gain in KE}$$

Eg9 Tarzan is about to swing through the jungle on a vine of length 15m. Initially the vine makes an angle of 60° to the horizontal. If he weighs 90kg, find
(a) the decrease in his PE when the vine has reached the vertical,
(b) his KE and hence his speed when the vine is vertical.

Change of energy of a particle
If the energy possessed by a particle changes during the motion being considered it must be as a result of some force doing work on that particle.
If a resistance is acting on the particle, the resistance is working against the motion and will cause a loss of energy.

i.e. *PE lost = KE gained + wd against resistances*
or *KE lost = PE gained + wd against resistances*

Eg10 From the point A situated at the bottom of a rough inclined plane, a body is projected with a speed of 5.6ms^{-1} along and up a line of greatest slope. The plane is inclined at $\tan^{-1}(4/3)$ to the horizontal. If $\mu = 4/7$ and the body first comes to rest at a point B, find, by energy considerations, the distance AB.

Eg11 A cyclist reaches the top of a hill with a speed of 4ms^{-1}. He descends 40m and then ascends 35m to the top of the next incline. His speed is now 3ms^{-1}. The cyclist and his bike have a combined mass of 90kg. The total distance he travels from the top of the first incline to the top of the next is 750m and there is a constant resistance to motion of 15N. Find the work done by the cyclist.

Exercise 3C Pg 71 Odds

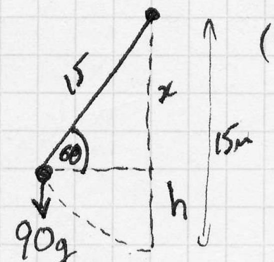
eg5 / eg 8

(a) $Mgh = 6 \times 9.8 \times 1.2 = 70.56$ J

(b) $Mgh = 2 \times 9.8 \times {}^{-}0.75 = -14.7$ J

(c) $Mgh = 10 \times 9.8 \times 5\sin 60 = 490 \times \frac{\sqrt{3}}{2} = 245\sqrt{3}$ J

eg9 / eg6



(a) $x = 15\sin 60 = 13$ m

$\therefore h = 15 - 13 = 2$ m.

Vertical height lost.

P.E. Lost $= Mgh = 90 \times 9.8 \times 2 = 1764$ Joules.

(b) No work done against resistances $\therefore$ loss in PE = gain in KE

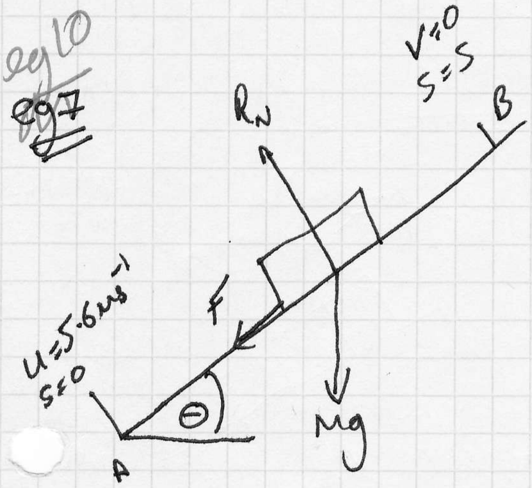
$\therefore$ Tarzans K.E $= 1764$ Joules.

hence $1764 = \frac{1}{2} 90 V^2$

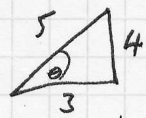
$V^2 = 39.2$

$V = 6.26\ ms^{-1}$

eg10 / eg7



$\tan\theta = \frac{4}{3}$



$\therefore \sin\theta = \frac{4}{5}$ $\quad \cos\theta = \frac{3}{5}$

body projected up slope $\therefore$ loss in KE = gain in PE + wd against resistances

$$0 - \frac{1}{2}M(5.6)^2 = \left[Mgs\sin\theta - 0 \right] + Fs$$

Sign can be ignored.

but $F = \mu R_N$

$F = \frac{4}{7} \cdot Mg\cos\theta$

$= \frac{4}{7} \times M \times 9.8 \times \frac{3}{5}$

$= 3.36$

$\therefore \quad 15.68\cancel{M} = 7.84\cancel{M}s + 3.36\cancel{M}s$

$15.68 = 11.2s$

$\therefore s = \frac{15.68}{11.2} = 1.4$ metres.

eg 11 (From text book pg 157)

$$KE \text{ lost} = \frac{1}{2} \times 90(4^2 - 3^2) = 315 \text{ J}$$

$$PE \text{ lost} = 90 \times 9.8 \times 5 = 4410 \text{ J}$$

$\therefore$ total energy lost by cyclist $= 4410 + 315 = 4725$ J.

Now work done against resistances $= F \times S = 15 \times 750 = 11250$ J.

Cyclist does work against resistances, but is "helped" by energy loss above

$\therefore$ wd by cyclist = wd v's Res − energy lost

$= 11250 - 4725$

$= 6525$ J.